Cosmos Automaton: A Deterministic Fractal Automaton Generating Primes

Birke Heeren[0000-0001-6731-583X]

birke.heeren@cosmos-automaton.de

Abstract: This paper introduces the Cosmos Automaton (CA), a new type of automaton. It is generating fractal patterns based on prime number distribution and prime gaps. Unlike the classical Sieve of Eratosthenes, which requires a finite upper bound, the CA is round based, starting with a step and terminating each round with completing the step and thus can run infinitely in theoretical operation. We demonstrate that this mechanism resolves the "infinite loop" problem of static sieves and constructs the fractal structure of prime distributions bottom-up via additive operations. Three fractal processes are applied to the symbol tape and the fractal dimension is calculated. In order to show the efficiency and feasibility of the proposed algorithm, we apply the Chinese Remainder Theorem to the automaton's state transitions. We present a constructive framework that, if accepted as faithfully modeling the primes, implies the infinitude of twin primes. We show via mathematical induction that the population of twin prime templates grows geometrically with each step, ensuring that they occur within the automaton's stability zone – which is free of the parity problem – infinitely many times. As we do not use any densities, the lumpiness of twin primes does not matter.

Keywords: Prime Numbers; Deterministic Sieves; Automata Theory; Symbolic Dynamics; Fractal Structures; Twin Prime Conjecture; Complex Systems

1 Introduction

Primes have long challenged mathematicians with the possible nature of primes distribution: Probabilistic (chaotic, random) or deterministic (by a still hidden generative rule). Classical analytical approaches and heuristic models (Hardy & Littlewood, 1923) often rely on probabilistic assumptions. Significant analytical breakthroughs, such as Green and Tao's theorem on arithmetic progressions (2008) and the bounded gap proofs by Zhang (2014) and Maynard (2015), have established deep structural properties of primes. However, these approaches remain predominantly static, describing prime patterns at fixed points in time.

In contrast, work in complex systems and fractal geometry (Wolf, 1989; Lapidus, 2006; Cattani, 2010; Bilokon, 2025) suggests that the apparent randomness is the result of high-dimensional deterministic order that can be modeled dynamically.

The known algorithm for the search of primes is the Sieve of Eratosthenes (ES). However, the distribution of primes is not explained and moreover the classical method suffers from a critical computational limitation: It requires a finite upper bound U. To identify primes, the algorithm must mark multiples of p up to U. If the upper bound is removed ($U \to \infty$), the ES deadlocks at the very first step, entering an infinite loop of marking multiples of 2 without ever advancing to check the number 3.

In this paper we present the Cosmos Automaton (CA), a constructive framework for finding primes, that resolves the limitation of classical sieves. The CA is round based and terminates after each round. It always has a lower bound and no upper bound. Instead of projecting deletions forward into an infinite future (like the Eratosthenes "pen"), the CA encodes the sieving rule into a local, finite periodic pattern that grows recursively (like a "rolling stamp"). This allows the CA theoretically to generate primes and prime gaps indefinitely from an unbounded operation (plus 1). The algorithm operates via additive striding, where the automaton deterministically targets specific indices using counters.

It is important to note, that the Cosmos Automaton in its basic mode is NOT meant to produce large or large numbers of primes. This is because its rolling stamp increases with prime primorial. It becomes too large for any computer on earth very quickly (figure 1.1). The Cosmos Automaton is important in its THEORETICAL operation.

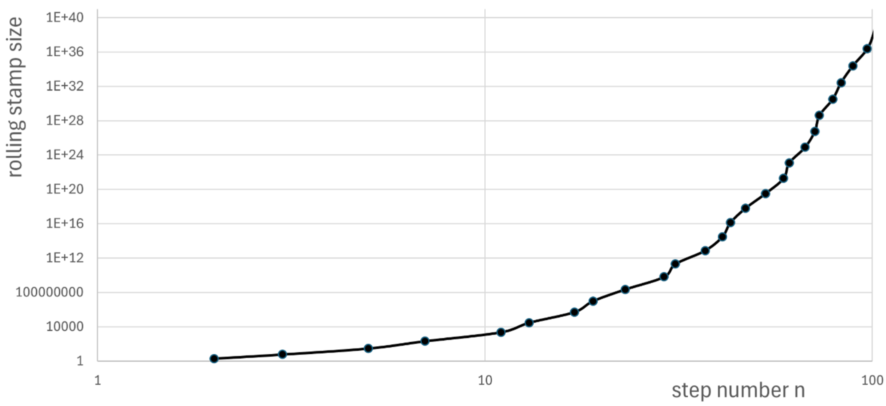


Figure 1.1 size of "rolling stamp" in number of symbols

This constructive approach aligns with recent topological frameworks. For instance Bilokon (2025) suggests that prime generation can be modeled via automata theory. The CA realizes this: The symbolic tape acts as a growing Fractal String (Lapidus, 2006), constructing a self-similar “dust” (Mandelbrot, 1987) of prime residues bottom-up. By modeling the system as a feed-forward automaton it is shown that the “beats” or interference patterns of the prime cycles are deterministic and predictable.

We apply this framework to the Twin Prime Conjecture. By analyzing the population dynamics of the symbolic tape, we demonstrate the structural persistence of the twin prime template. We provide an inductive argument based on the Chinese Remainder Theorem (CRT) showing that for *every* new prime p the population of twin templates grows by a factor of (p-2), ensuring a growing number of survivors. The conveyor belt of the CP tape necessitates that surviving twin templates move into the automaton’s stability zone – which is free of the parity problem and become actual twin primes, confirming their infinitude.

The remaining of this paper is organized as follows: In section 2 methodology the alphabet of Symbols and tape construction are given. The CA algorithm is defined and proved in the main section 3. Section 4 experiments, section 5 deals with the computation of the fractal dimension of the prime generated patterns. An application of the CA algorithm to show the infinitude of twin primes is given in section 6. Section 7 higher mathematics.

2 Methodology

2.1 Alphabet of Symbols

Natural numbers are indexed by the following letters: P for a prime, M (multiple) for a composite and L for all natural numbers (prime, composite or 1). The symbol “ONE” means the number 1 which is neither a composite nor a prime number.

2.2 Construction plan

The CA has two registers and one tape similar to Turing (1936), figure 2.1.

- Register N has one square and contains the step number n.
- Register BP has one square and contains the *encoding* of step number n.
- Tape CPn starts with one square and grows new squares but stays always of finite size. It contains a periodic word of symbols.

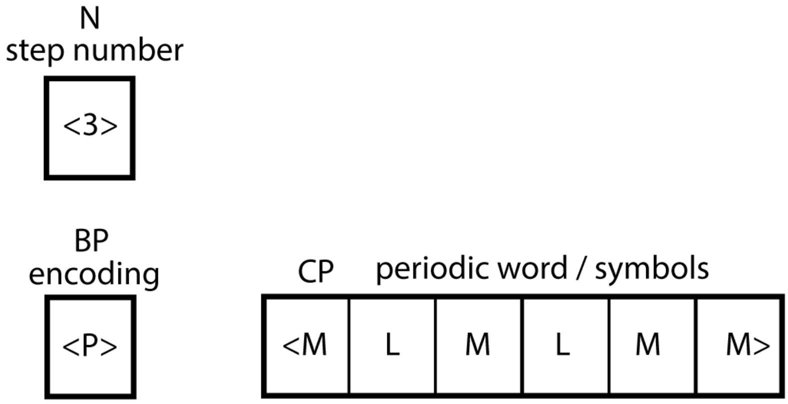


Figure 2.1 Registers and tape

All three elements are kept synchronized by automaton operations. We write the register/tape contents with angle brackets.

2.3 Tape operations

- Pai: This is the plus 1 operation for register N. N = N +1

The Cosmos Automaton starts at $N_n = 1$ and does $N_{n+1} = N_n + 1$, therefore the states of the automaton are qua definition isomorphic to the set of natural numbers $\mathbb{N}$.

- Ain: This is the encoding on register BP. BP = $\begin{cases} <P> \text{ if } CP[1] = <L> \\ <M> \text{ if } CP[1] = <M> \end{cases}$

- Taw: These are the fractal procedures for Tape CP.

$$CP = \begin{cases} Mem_n(Bet_n(Resch_1(CP))) \text{ if } BP = <P> \\ Resch_1(CP) \text{ if } BP = <M> \end{cases}$$

→ $Resch_1$: Remove the first symbol CP[1] and append it to the end of CP.

→ Bet_n: Copy all symbols of CP and append them n-1 times to the end of CP. This insures that the structure (lumpiness) of L's is distributed over CP in such way that there are similar lumps and gaps at the start (left) in the middle and at the end (right) of the tape CP.

→ Mem_n: Stride over CP with plus n and turn any L's hit to M.

2.4 Modes

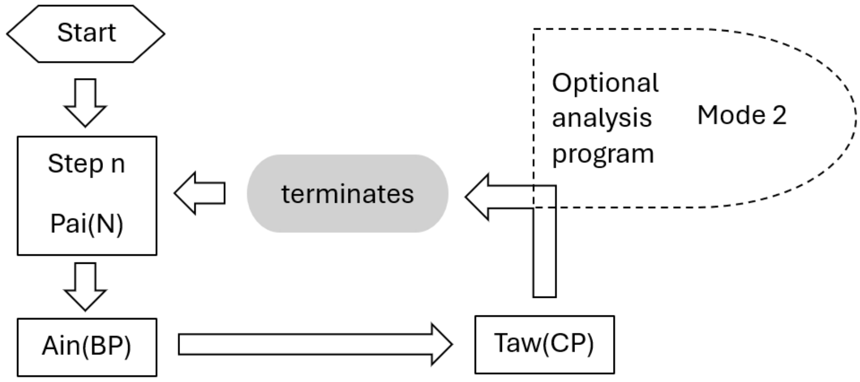


Figure 2.2   Flow diagram of Cosmos Automaton

Mode 1 is the recorder mode and mode 2 has an optional analysis program. The Cosmos Automaton is not designed to be Turing-complete in the sense of universal computability. Its rules are deliberately finite and specialized.

3 Algorithm – Main part

3.1 Description

The Cosmos Automaton starts with a tiny boom, a singularity and then inflates CP as the natural numbers step in N and the encoding takes place in BP.

N_{start} = <1>
BP_{start} = <ONE>
CP_{start} = <L>

N = Pai(N) = <2>
BP = Ain(BP) = <P>
CP = Taw(CP) = <LM>

N = Pai(N) = <3>
BP = Ain(BP) = <P>
CP = Taw(CP) = <MLMLMM>

N = Pai(N) = <4>
BP = Ain(BP) = <M>
CP = Taw(CP) = <LMLMMM>

N = Pai(N) = <5>

BP = Ain(BP) = <P>
CP = Taw(CP) = <MLMMMLMLMMMLMLMMMLMMMMMLMLMMMM>

CP is locally finite but globally unbounded. It represents a *potential infinity*: at no point is the tape actually infinite, yet its growth never ceases. For brevity, we denote the state of the registers/tape at step n simply as N, BP, CP – as is done in coding – implying that these variables are updated iteratively.

We classify the Cosmos Automaton as a **Cybernetic Automaton**. Unlike classical linear automata, it operates on a strict feedback loop: The structural state of the pattern tape (CP) determines the qualitative property of the number (Prime/Composite), while the number value (N) and its qualitative property (BP) conversely control the modulation and geometric expansion of the structure. This self-referential cycle drives the emergent complexity of the prime distribution.

The Cosmos Automaton does not compute primes from an external definition. It models a cybernetic process in which number, structure, and qualitative distinction co-emerge through feedback. Thus the natural numbers emerge through Pai(N) (compare with Peano, 1889), the prime/composite property emerges through Ain(BP) and the fractal structure emerges through Taw(CP). We show that this emergent notion of primeness is able to produce the classical definition.

3.2 Foundation of Primness

In CP there are three fractal procedures: Resch, Bet, and Mem. At every step Resch keeps tape CP in lock-step with registers N and BP. Every prime number sends out a pulse train (figure 3.1) with its own length n. This pulse train is added to the pulse trains of all previous prime numbers. The superposition (beat, Schwebung) is the resulting pattern of size prime# (prime primorial). This is reflected in the fractal procedures Bet, that brings the pattern to the appropriate length and Mem, which turns the multiples from L to M unless they are already of type M. It ensures that all composite numbers will be identified correctly. Any L reaching BP is not a multiple of all smaller numbers and therefore can be encoded as prime.

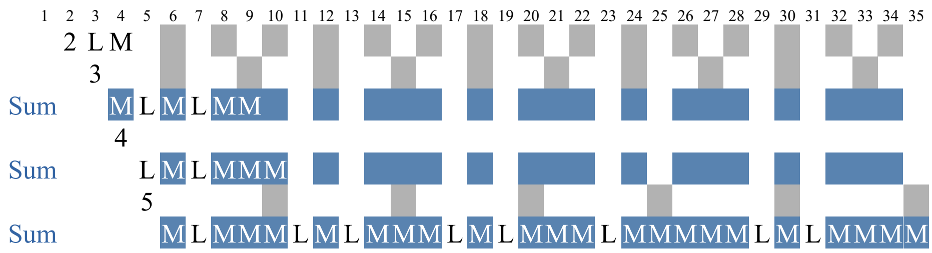


Figure 3.1 CP as pulse trains with constructive interference (sum)

CP can be described as a function of step number n by equations 1, 2, and 3. The functions f and g are defined in their parentheses, they return “0” or “not 0”, the sum is aggregated and the result is processed logically.

$$CP_{(1)} = \langle L \rangle \tag{1}$$

$$CP_{(n>1)} = \sum_{k=n+1}^{n\#+n} f(k \, mod \, n) \vee \sum_{j=2}^{n-1} g(k \, mod \, j) \tag{2}$$

with $$pulse \, CP_{(n>1)} = \begin{Bmatrix} M \, if \, f \vee g = 0 \\ L \, otherwise \end{Bmatrix} \tag{3}$$

n	CP	encodes
2	<LM>	{3,4, … , ∞}
3	<MLMLMM>	{4,5,6,7,8,9, … , ∞}
4	<LMLMMM>	{5,6,7,8,9,10, … , ∞}

Figure 3.2 Examples of equations 2 and 3.

The prime number definition PD_p can be derived by equations 4 to 9. They are derived from the physical analogy of interfering pulse trains. A prime number is identified as a coordinate where the sum of all preceding pulse trains remains zero and is used for encoding (Ain) at CP[1]. Equation 7 calculates for all natural numbers smaller than p, if they are a divisor of p.

Instead of accepting the definition of prime numbers as an axiom, we have provided a new foundation to identify prime numbers as a class of numbers by the Cosmos Automaton and derive the traditional definition of prime numbers through pulse equations.

$$p = n - 1 > 1 \tag{4}$$

$$Prime \, Definition_{(p)} = CP_{(n-1)} \tag{5}$$

$$PD_{(p)} = \sum_{k=n-1+1}^{\text{none}} f(k \, mod \, n-1) \vee \sum_{j=2}^{n-1-1} g(k \, mod \, j) \tag{6}$$

$$PD_{(p)} = f(n \, mod \, n-1) \vee \sum_{j=2}^{n-2} g(n \, mod \, j) \tag{7}$$

$$PD_{(p)} = \sum_{j=2}^{n-2} g(n \, mod \, j) \tag{8}$$

$$pulse\ PD(p) = \begin{cases} ONE\ if\ p=1 \\ M\ if\ g=0 \\ otherwise\ P \end{cases} \quad (9)$$

n	PD
2	<P>
3	<P>
4	<M>
5	<P>
100	<M>

Figure 3.3 Examples of equations 8 and 9

3.3 Cosmos Sieves

Let us construct sieves from the Cosmos Automaton.

	BP			
	P	L	M	CP
N	2	3	4	head
		5	6	
		7	8	
		9	10	
		11	12	
		13	14	
		15	16	
		17	18	
		19	20	
		21	22	
		23	24	
		25	26	
		27	28	
		29	30	
		31	32	
		33	34	
		35	36	
		etc. → ∞		
		width=2		

	BP							
	P	M	L	M	L	M	M	CP
N	3	4	5	6	7	8	9	head
		10	11	12	13	14	15	
		16	17	18	19	20	21	
		22	23	24	25	26	27	
		28	29	30	31	32	33	
		34	35	36	37	38	39	
		40	41	42	43	44	45	
		46	47	48	49	50	51	
		52	53	54	55	56	57	
		58	59	60	61	62	63	
		64	65	66	67	68	69	
		70	71	72	73	74	75	
		76	77	78	79	80	81	
		82	83	84	85	86	87	
		88	89	90	91	92	93	
		94	95	96	97	98	99	
		100	101	102	103	104	105	
		etc. → ∞						
		width=6						

	BP							
	P	L	M	L	M	M	M	CP
N	4	5	6	7	8	9	10	head
		11	12	13	14	15	16	
		17	18	19	20	21	22	
		23	24	25	26	27	28	
		29	30	31	32	33	34	
		35	36	37	38	39	40	
		41	42	43	44	45	46	
		47	48	49	50	51	52	
		53	54	55	56	57	58	
		59	60	61	62	63	64	
		65	66	67	68	69	70	
		71	72	73	74	75	76	
		77	78	79	80	81	82	
		83	84	85	86	87	88	
		89	90	91	92	93	94	
		95	96	97	98	99	100	
		101	102	103	104	105	106	
		etc. → ∞						
		width=6						

	BP																															
	P	M	L	M	M	M	L	M	L	M	M	M	L	M	L	M	M	M	L	M	M	M	M	M	L	M	L	M	M	M	M	CP
N	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	head
		36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	
		66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	
		96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	
		etc. → ∞																														
		width=30																														

Figure 3.4 Cosmos Sieves

At first sight the Cosmos sieves could be mistaken for Eratosthenes sieves, but there are important differences. Cosmos sieves have a lower bound, no upper bound and consist of vertical columns, which can be described by linear equations. The M-columns are composites only, the L-columns contain both, composite and prime numbers, therefore they are envelopes (Hüllkurven) to prime numbers.

$f_{(x)}$ = width * x + head number = envelope equations with $x \in \mathbb{N}_0$.

$f_{(x)} = 2x + 3$	$f_{(x)} = 30x + 11$	$f_{(x)} = 30x + 7$
$f_{(x)} = 6x + 5$	$f_{(x)} = 30x + 17$	$f_{(x)} = 30x + 13$
$f_{(x)} = 6x + 7$	$f_{(x)} = 30x + 23$	$f_{(x)} = 30x + 19$
	$f_{(x)} = 30x + 29$	$f_{(x)} = 30x + 31$

Figure 3.5 Envelope equations

Dirichlet's Theorem on Arithmetic Progressions guarantees that envelope equations contain infinitely many prime numbers.

3.4 Proof of Cosmos Automaton

Lemma 1

Remember **Pai**: This is the plus 1 operation for tape N. N = N +1 and

Resch$_1$: Remove the first square CP[1] and append it to the end of CP

CP is periodical, thus it can be displayed as a circle (figure 3.6). Starting at the yellow triangle it reads <MLMLMM>. Removing the first square and appending it to the end leaves <LMLMMM> starting at blue triangle which is visibly a plus one operation.

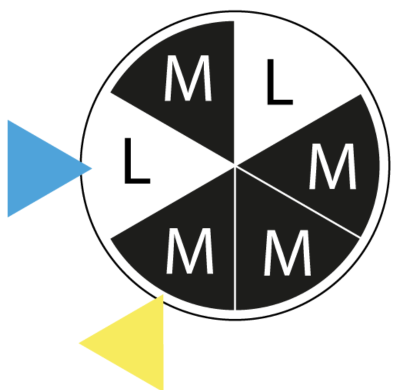


Figure 3.6 CP as circle

Lemma 2

Remember **Bet$_n$**: Copy all squares of CP and append them n-1 times to CP.
CP is periodical, thus instead of displaying just one period <LM>, we can also write: (LM LM LM LM LM LM LM LM LM … ∞). Thus doing Bet$_n$ just changes the periodicity but not the positions of Ls and Ms: (LMLMLM LMLMLM LMLMLM … ∞)

Lemma 3

Remember **Mem$_n$**: Stride over CP with plus n and turn any L's hit to M. Since CP is periodical (figure 3.6) it acts as a rolling stamp. Turning multiples of p in the rolling stamp from L to M turns all composites of p to M.

Base Case

At step number n = 2 the tape CP$_n$ contains the symbols <LM>. This means for all indexes k and c (figures 3.4 and 3.5)

$$k = 2 * x + 3 \; with \; x \in \mathbb{N}_0 \qquad \text{prime candidates} \tag{10}$$

$$c = 2 * x + 4 \; with \; x \in \mathbb{N}_0 \qquad \text{composites} \tag{11}$$

Inductive Hypothesis

Assume that at step number n, the pattern CPn correctly identifies prime candidates and composites.

Inductive Step

$$CP = \begin{cases} Mem_n(Bet_n(Resch_1\,(CP)))\,if\; BP = <P> \\ Resch_1\,(CP)\,if\; BP = <M> \end{cases}$$

Taw consists of either Resch$_1$, Bet$_n$, and Mem$_n$ if BP = <P> or Resch$_1$ if BP = <M>.

Resch$_1$: Remove the first square CP[1] and append it to the right end of CP (see lemma 1)

Bet$_n$: Copy all squares of CP and append them n-1 times to CP (see lemma 2)

Mem$_n$: Stride over CP with plus n and turn any L's hit to M (see lemma 3)

Thus the symbols on tape CPn correctly identify prime candidates and composites.

3.5 Stability Zone

The CA Stability Zone is defined as the interval [n+1, 2n-1] on the left side of CP (equation 12, figure 3.7).

$$SZ=[n+1,2n-1] \tag{12}$$

The Stability Zone develops (figure 3.7), because each step n does a first stride of size n (indicated in yellow) and therefore all L's inside the SZ (indicated in blue) are stable survivors, that will become prime numbers in future steps. This also means there is no parity-problem inside the Stability Zone of the Cosmos Automaton. The Stability Zone **advances two squares** with each step n, because of the combined effect of $Resch_1$ (one square) and the growth of the SZ of one square. This also means (Figure 3.7) as soon as the first L of a twin is inside the SZ, the other L of the twin can not be eliminated any more. Even if the first twin L were to enter the SZ on a prime step than the stride would hit (yellow) between the two L's. And in the next step the second L already enters the Stability Zone. This can be seen from step n = 3 to n = 4. (Figure 3.7).

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	2	L	n																
		3		L	n														
			4	L		L	n												
				5		L			n										
					6	L				L	n								
						7				L		L	n						
							8			L		L			n				
								9		L		L				L	n		
									10	L		L				L		L	n
										11		L				L		L	

Figure 3.7 Stability Zone in blue with L's as stable survivors.

Lemma 4 Completeness of the Stability Zone

For every step n ≥ 3, any symbol marked <L> within the Stability Zone SZ = [n + 1, 2n - 1] corresponds to a prime number.

Proof:

Let $\chi \in$ SZ be a number corresponding to an index marked <L>. Suppose, for the sake of contradiction, that χ is composite.

A fundamental property of composite numbers states that if χ is composite, it must have a prime factor p such that $p \le \sqrt{x}$.

Since $\chi < 2n$ (by definition of the SZ), it follows that $p<\sqrt{2n}$.

For all $n \geq 3$, the inequality $\sqrt{2n}<n$ holds. Therefore, the prime factor p is strictly smaller than the current step-number n ($p < n$).

Since the Cosmos Automaton operates sequentially (1, 2, …, n) the sieving operation Mem for the prime p has already been executed in a previous step. This operation marks all multiples of p as <M>. Consequently, χ (being a multiple of p) must have been marked as <M>.

This contradicts the assumption that χ is marked <L>. Thus, χ cannot be composite and must be prime.

q.e.d.

3.6 Letters a, b, c, and d

a:= <LMLMMM> (13)

b:= <LMMMMM> (14)

c:= <MMLMMM> (15)

d:= <MMMMMM> (16)

In figure 3.4 we can see that n = 4 and CP = <LMLMMM>. We define this sequence as letter a (equation 13). This fundamental block of length 6 defines the DNA structure for all subsequent steps of CA. We define four distinct states (“letters”) based on the possible configurations of L and M starting from a. As Taw acts on the symbols it also acts on the letters. Because of n > 4 only one L of letter a can be turned to M within one step (figure 3.8).

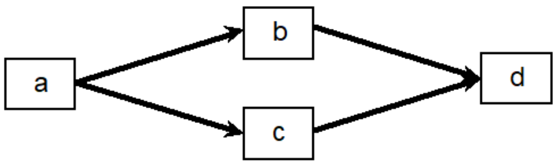


Figure 3.8 Transition rules of letters.

Letter d and clusters of letter d are prime gaps. As the CA walks from step to step all prime gaps are multiplied (Bet_n) and become part of the next sieve. Thus, any prime gap that was ever discovered will appear repeatedly to infinity and can only become larger.

It holds true that no matter in which sequence the letters a, b, c, and d are arranged it is impossible to build the pattern of three consecutive primes <LMLML> beyond 3, 5, and 7.

In the CA symbolic framework, letter a carries the twin prime template <LML>. Thus only letter a can produce actual twin primes in subsequent steps.

4 Experiments

4.1 "Frozen Window" for Stability Zone

To empirically verify the persistence of twin prime templates up to high step numbers, we encountered the limitation of primorial growth in CP. The full symbolic tape for n = 250000, thus 250000# would exceed the number of atoms in the universe. However, to analyze the Stability Zone, it is not necessary to store the entire CP tape. One only needs a pattern segment large enough to cover the interval [n+1, 2n-1].

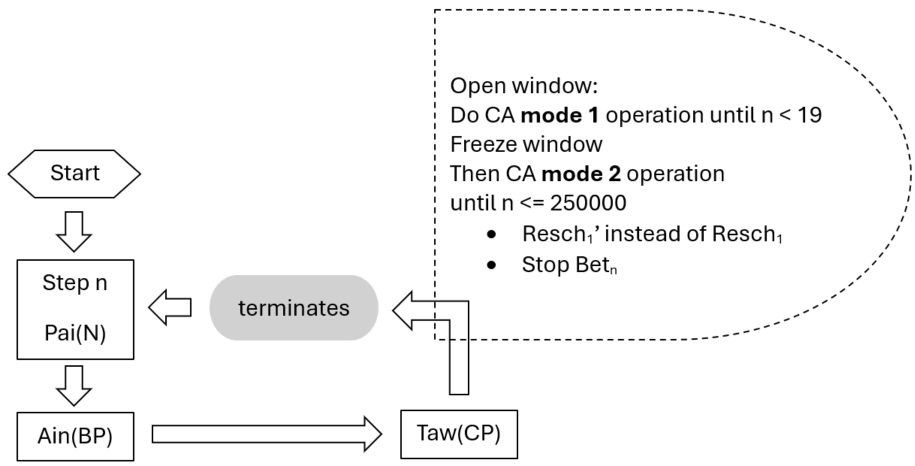


Figure 4.1 Pseudocode for mode 2 analysis "Frozen Window"

$Resch_1$: Remove the first square CP[1] and append it to the end of CP

$Resch_1$': Remove the first square CP[1]

At step n = 18 the pattern-width is 510510. For our target simulation of n = 250000 the upper-bound of the Stability Zone is 2*n = 500000. Since the size of the frozen window with 510510 > 500000 the pattern at step 18 provides a sufficient size to simulate the sieving process for the Stability Zone.

The result is a counter-intuitive perspective: While twin primes are known to be less frequent overall at higher numbers, their amount increases within the ever-enlarging CA Stability Zone (figures 4.2 and 4.4).

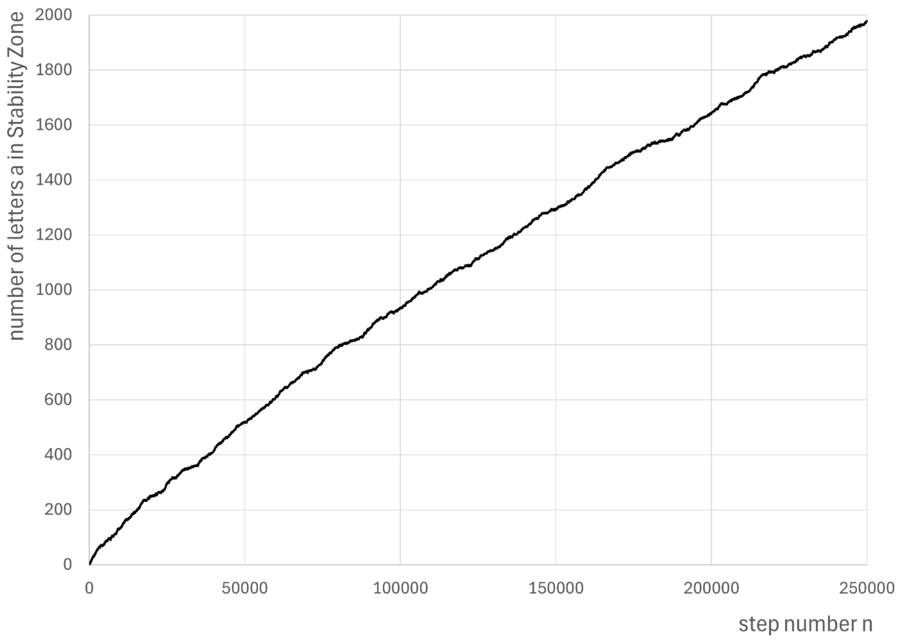


Figure 4.2 Experiment "Frozen Window": amount of letter a

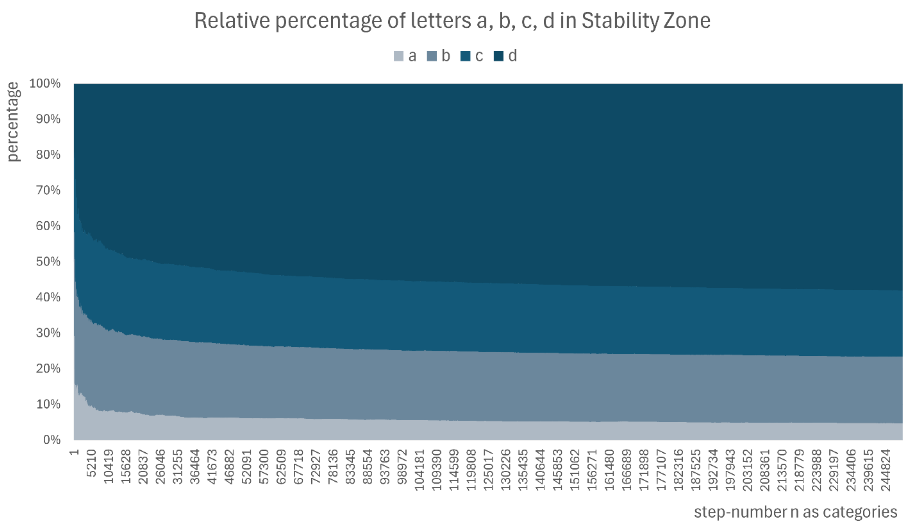


Figure 4.3 Experiment "Frozen Window": percentage of letters

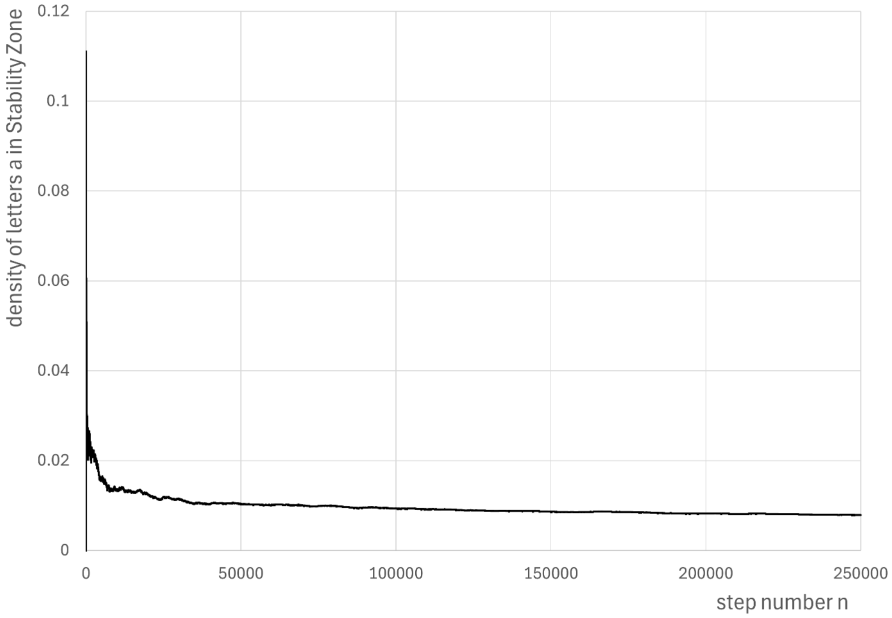


Figure 4.4   Experiment "Frozen Window": density of letter a

The density curve in Figure 4.4 indicates that the proportion of twin-prime templates, thus letter a inside the Stability Zone decreases only mildly and appears to converge to a positive limit. In other words, within the CA's Stability Zone the model predicts a persistent, non-vanishing density of twin-prime templates, compatible with the conjectured infinitude of twin primes.

Figure 4.5 plots the ratio

$$\frac{\ln(a+b+c)}{\ln(a+b+c+d)} \tag{17}$$

, which can be interpreted as an effective fractal dimension of the symbolic support of letters a, b, c within the Stability Zone. The curve stabilizes near a value of about 0.92, hinting at a self-similar, fractal-like structure of the stable region in the CA phase space rather than a full-dimensional, homogeneous distribution.

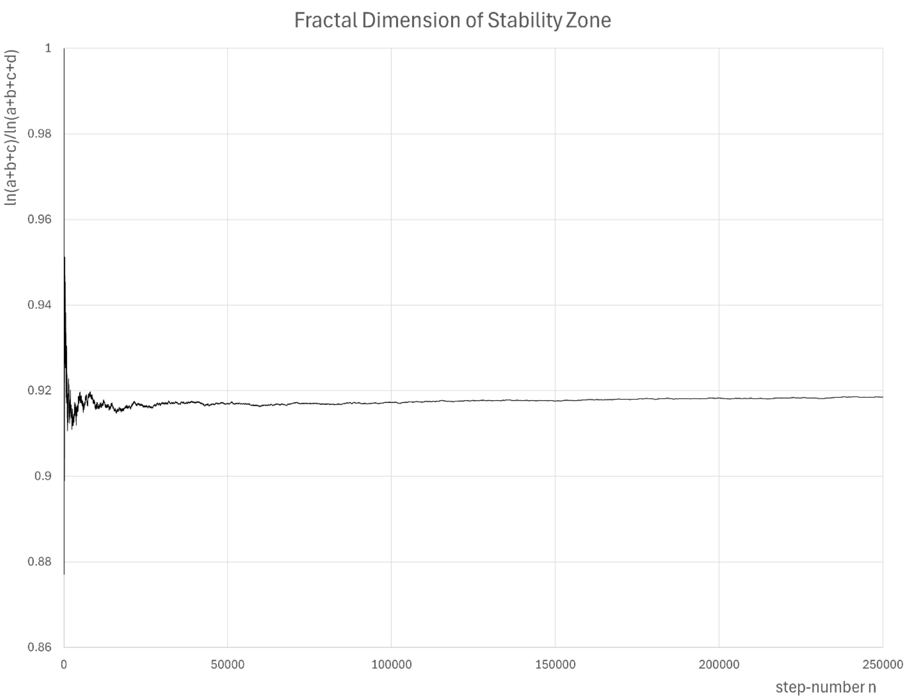


Figure 4.5 Experiment "Frozen Window": fractal Dimension

4.2 Experiment "BLOX"

For using full larger sieves, the BLOX variant of the Cosmos Automaton was developed. Each sieve is a file that contains only L columns (= L envelope). The computation started with one file at step n = 3 containing the numbers 5 and 7 and ended through successive computation at step n = 29 with 667 files, each containing about 1.6 million L entries in sieve blox29.

The BLOX computation is done by the equivalence of the CA. But BLOX does not grow primes, but envelopes of primes, therefore prime candidates, which need to be confirmed by the usual primality tests, which were done with BLOX19. Therefore BLOX is like a traditional segmented sieve.

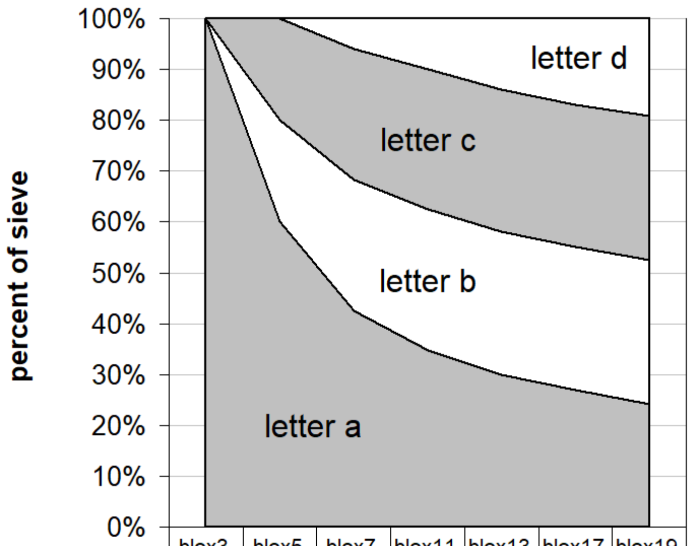
The BLOX was implemented in Java, just like the CA. From step number n = 3 to n = 5 five copies of the L-columns are made in the following way. The sieve-width of n = 3 that is 6 is used. Five and composites of five are cut.

5	cut
7	7
5 + 1*6 = 11	11
7 + 1*6 = 13	13
5 + 2*6 = 17	17
7 + 2*6 = 19	19
5 + 3*6 = 23	23
7 + 3*6 = 25	cut
5 + 4*6 = 29	29
7 + 4*6 = 31	31

Figure 4.6   From BLOX3 to BLOX5

BLOX was also implemented with letters a, b, c, and d. The output allowed to count the letters until BLOX19. The densities (percent of sieve) where computed by BLOX and also as a control up to n=17 by the CA implementation (figure 4.7).

It is interesting to note, that the letters b and c develop with the same percentage. We will come back to that later when investigating twin primes (which can appear only in letter a).



	blox3	blox5	blox7	blox11	blox13	blox17	blox19
letter d	0	0	0,06	0,1	0,14	0,17	0,19
letter c	0	0,2	0,26	0,28	0,28	0,28	0,28
letter b	0	0,2	0,26	0,28	0,28	0,28	0,28
letter a	1	0,6	0,43	0,35	0,3	0,27	0,24

sieves

Figure 4.7 BLOX for letter counting, confirmed by CA up to n = 17

5 Fractal dimension of CP

First, we calculated the fractal dimension from the current CA implementation. Second, we calculated the fractal dimension from files from a previous implementation of CA from 2024. Third, we calculated the fractal dimension from CA-Dust (named by Schwebinghaus 2025) figure 5.1. All three calculations are visible in figure 5.2.

The L's are the segments in this CA-Dust. This corresponds to the fractal strings described by Lapidus (2006). When an L is swapped into an M, it means deleting a segment in analogy to Cantor-Dust (Zeitler and Pagon, 2000).

For box-counting we used all primorial window sizes possible (that can be fit into the length of the pattern CP) and equation 17. With k being the number of different box-sizes, epsilon being the box-size and N the number of L's in one box.

$$D=\frac{\sum_{i=1}^{k}\left(\ln(1/\varepsilon_i)-\overline{\ln(1/\varepsilon)}\right)\left(\ln(N_{(\varepsilon_i)})-\overline{\ln(N_{(\varepsilon)})}\right)}{\sum_{i=1}^{k}\left(\ln(1/\varepsilon_i)-\overline{\ln(1/\varepsilon)}\right)^2} \tag{18}$$

The equation 17 calculates D = covariance (x, y)/variance(x) where x is ln (1/epsilon) and y = ln (N) and thus the linear regression slope. Here the terms with an overbar denote the arithmetic mean of the values across all measured box sizes k. This is an approximation of equation 18.

$$D=\lim_{\varepsilon\to 0}\frac{\ln N_{(\varepsilon)}}{\ln(1/\varepsilon)} \tag{19}$$

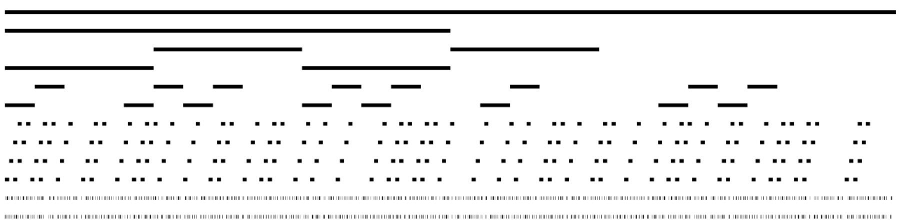


Figure 5.1 CA-Dust

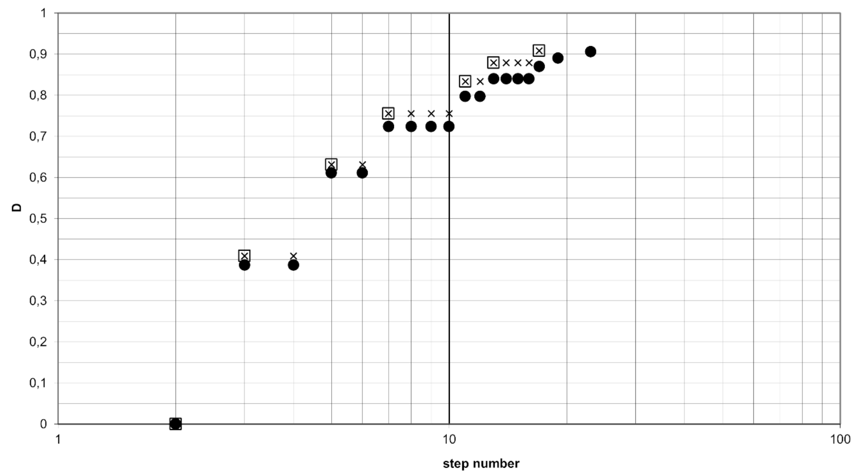


Figure 5.2 Fractal dimension, x = box counting CA, squares = box-counting from files from previous implementation, circles = CA-Dust

See also chapter 7.3 equation 34 and 35.

step-number n	CP	segment	segment	fractal dimension d_c
1	L	[0,1]		ln1/ln1 = NaN
2	LM	[0, 1/2]		ln1/ln2 = 0
3	MLMLMM	[1/6, 2/6]	[3/6, 4/6]	ln2/ln6 = 0.387
4	LMLMMM	[0, 1/6]	[2/6, 3/6]	ln2/ln6 = 0.387
5	8 L of 30			ln8/ln30 = 0.611
6	8 L of 30			ln8/ln30 = 0.611
7	48 L of 210			ln48/ln210 = 0.724
8	48 L of 210			ln48/ln210 = 0.724
9	48 L of 210			ln48/ln210 = 0.724
10	48 L of 210			ln48/ln210 = 0.724
11	480 L of 2310			0.797
12	480 L of 2310			0.797
13	5760 L of 30030			0.840
14	5760 L of 30030			0.840
15	5760 L of 30030			0.840
16	5760 L of 30030			0.840
17	92160 L of 510510			0.870
18	92160 L of 510510			0.870
19	1658880 L of 9699690			0.890
20	1658880 L of 9699690			0.890
21	1658880 L of 9699690			0.890
22	1658880 L of 9699690			0.890
23	36495360 L of 223092870			0.906

Figure 5.3 Calculation of fractal dimension of CA-Dust

6 Infinitude of twin primes – the Hydra Effect

We now address a central question: Can our Cosmos Automaton ever eliminate all twin prime templates (thus letter a)? We demonstrate that the automaton's recursive growth prevents this extinction through a mechanism we term the **Hydra Effect**.

6.1 Population Dynamics

Let $N_{n*}(a)$ be the number of twin prime templates (letter a) existing in the pattern CP_{n*}. In the transition to step n (with new prime p_n), the CA mode 1 performs three operations in Taw: First Resch, then

1. **Growth** (Bet): The pattern is copied p_n times. The intermediate population of templates becomes

$$N_{temp} = N_{n*}(a) \cdot p_n \tag{20}$$

2. **Elimination** (Mem): The algorithm performs an additive stride of size p_n to mark composites. We must determine how many of these p_n clones are destroyed.

6.2 The Deterministic Survival Rate

For any specific template a at position x:

The clones are located at $x + w$, $x + 2w$, … (where $w = P_{n-1}\#$). The condition for the new prime p_n to eliminate an L at relative position k is given by the congruence:

$$x + k \cdot w \equiv 0 \left(mod\ p_n \right) \tag{21}$$

Since p_n is a new prime, it is coprime to the previous CP_n length $w(\gcd(w, p_n) = 1)$. According to the Chinese Remainder Theorem, this linear congruence has exactly one solution for k in the range $[0, p_n - 1]$.

This implies a strict geometric rule, the "**left**" L of the template is hit **exactly once** and the "**right**" L of the template is hit **exactly once**. Since the distance between two L's of one twin is 2 and step size is ≥ 5, a single stride cannot hit both simultaneously.

Therefore, for every single template a from the previous step, **exactly 2 clones are destroyed** (converted to letters b and c) and **p_n – 2 clones survive as a**.

6.3 The Hydra Equation

The population of twin prime templates (letters a) evolves according to the relation:

$$|a|_n = |a|_{n^*} \cdot (p_n - 2) \tag{22}$$

This leads to a geometric explosion of candidates. Like the heads of the mythical Hydra. For **every** head (template) the automaton cuts off at n', the recursive expansion at n grows multiple new ones.

n	Growth	Elimination	total	CA count
4	1			
5	5	2	3	3
7	21	6	15	15
11	165	30	135	135
13	1755	270	1485	1485
17	25245	2970	22275	22275

Figure 6.1 Hydra calculation versus CA count of letter a

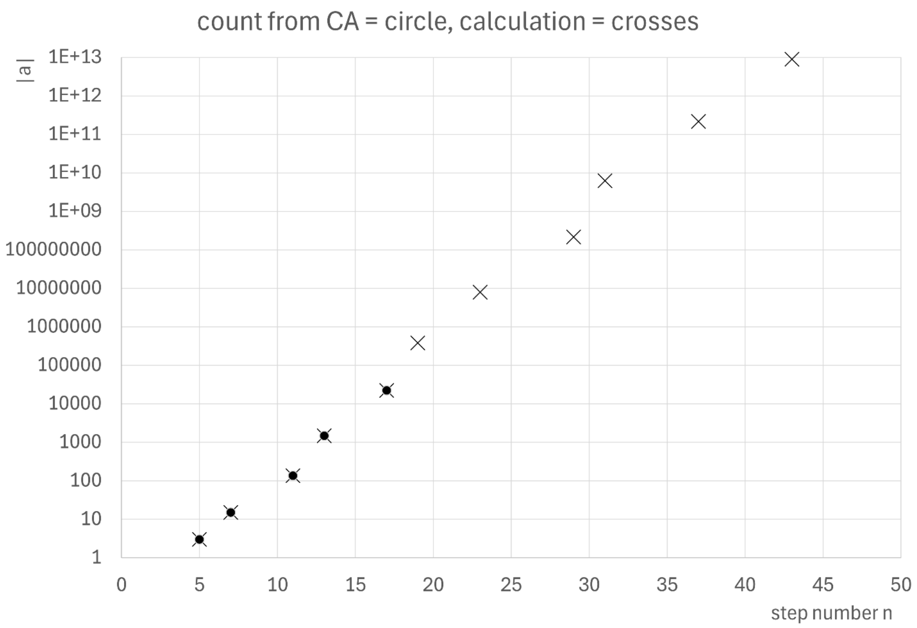


Figure 6.2 Hydra numbers of letter a

6.4 Return via conveyor belt, Lemma 4

Let $|a|_n$ be an infinite number of surviving copies of letter a produced by Cosmos Automaton, then by Resch the letters a (and thus the twin template inside a) are moved one step to the left with each step m inside CP. The Stability Zone is linearly growing one step to the right with each step n. Then there are infinitely many points in n (even if not at every step, because of lumpiness) where at least one copy of letter a enters the Stability Zone and becomes a stable survivor which will be encoded as prime P at some step of the infinite future steps.

We even can leave out the SZ and the lemma 4 still works, because all symbols are moved by the conveyor belt CPn tape to register BPn and are encoded.

We have proven via the Hydra Effect that the population of Twin Prime templates grows geometrically without error terms. While the distribution of these templates within the period P_n# is not uniform, the periodicity of the CA ensures that the Stability Zone [n+1,2n-1] cannot traverse the symbolic tape indefinitely without encountering these templates. Since the tape is generated deterministically and the template count is non-vanishing and unbounded, the intersection of the Stability Zone and the Template Set must occur infinitely often.

7 Higher Mathematics

7.1 Morphism in Symbolic Dynamics

The alphabet $\sum$ = {a, b, c, d } (23)

	Becomes a	Becomes b	Becomes c	Becomes d
a	p - 2	1	1	0
b	0	p - 1	0	1
c	0	0	p - 1	1
d	0	0	0	p

Figure 7.1 Transition Matrix M_p

$$\sigma_p(a)=a^{p-2}bc \tag{24}$$

$$\sigma_p(b)=b^{p-1}d \tag{25}$$

$$\sigma_p(c)=c^{p-1}d \tag{26}$$

$$\sigma_p(d) = d^p \tag{27}$$

The population vector at step n* is

$$v_{n*} = (|a|, |b|, |c|, |d|) \tag{28}$$

The population at the next prime step n is obtained by

$$v_n = v_{n*} \cdot M_p \tag{29}$$

This matrix is an Upper Triangular Matrix. The Eigenvalues are the entries on the main diagonal.

$$M_p = \begin{pmatrix} p-2 & 1 & 1 & 0 \\ 0 & p-1 & 0 & 1 \\ 0 & 0 & p-1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{30}$$

7.2 Normalized Transition Matrix and Relative Density

We analyze the density of the templates and normalize the transition matrix by dividing it by the growth factor p. The yields the Stochastic Matrix W_p, which describes the probability of the letters surviving relative to the expanding tape length.

$$W_p = \frac{1}{p} \cdot M_p = \begin{pmatrix} \frac{p-2}{p} & \frac{1}{p} & \frac{1}{p} & 0 \\ 0 & \frac{p-1}{p} & 0 & \frac{1}{p} \\ 0 & 0 & \frac{p-1}{p} & \frac{1}{p} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{31}$$

The relative density of letter d tends toward 1 (100%). This reflects the Prime Number Theorem: Almost all numbers are composites as n → ∞.

The single prime templates b and c decay slowly.

The twin prime templates decay twice as fast as the singles, but the factor is strictly positive.

Both are consistent with the long proven infinitude of primes.

The density of twin prime templates at step n is given by the product of the eigenvalues of all previous steps:

$$Density(a)_n \propto \prod_{k=1}^{n}\left(1-\frac{2}{p_k}\right) \tag{32}$$

This product diverges to 0 very slowly, proving that while twin primes become rare (density → 0), the underlying structural template persists structurally.

The matrix proves that the "deletion" is never total. It is noteworthy that the transition probabilities for letters b and c are identical in W_p. This algebraic eigenvalue symmetry ($\lambda_b = \lambda_c$) provides the rigorous explanation for the empirical observation that the populations of 'left-sided' and 'right-sided' single prime templates remain balanced as $n \to \infty$. The system does not favor one side over the other.

The entry W_p (d) = 1 characterizes letter d as an absorbing state in the Markov process. The entropy of the system maximizes as the density of letter d approaches 1, yet the structured survivors letters a persist in the vanishingly small remainder.

This matrix formulation lifts the argument from empirical observation by pure logic to algebraic necessity. The persistence of the Twin Prime template is not a lucky statistical accident, but a property encoded in the non-zero leading eigenvalue of the structural transition matrix

This formulation identifies the system as a non-homogeneous Markov process, where letter d acts as an absorbing state (entropy), while letter a and thus the twin template remains a persistent transient state.

7.3 Spectral Derivation of the Fractal Dimension

The eigenvalues of the transition matrix allow us to calculate the theoretical fractal dimension of the generated set analytically. Following the definition of the similarity dimension D, where the scaling factor is p and the replication factor for all L symbols is the eigenvalue $\lambda = p-1$, we obtain:

$$D_{all} = \frac{\ln(\lambda_a)}{\ln(p)} = \frac{\ln(p-1)}{\ln(p)} \tag{33}$$

For small primes, this dimension reflects the strong "dust-like" filtering. As $p \to \infty$, the dimension converges:

$$\lim_{p\to\infty} \frac{\ln(p-1)}{\ln(p)} = 1 \quad with \quad \frac{\ln(p-1)}{\ln(p)} < 1 \,\forall\, p \tag{34}$$

Although the fractal dimension converges to 1 asymptotically, it remains strictly less than 1 for every finite step p. This ensures that the system retains its fractal, self-similar characteristics throughout the entire process, enforcing the self-similar distribution of the letters. And for the twin primes we can write:

$$\lim_{p \to \infty} \frac{\ln(p-2)}{\ln(p)} = 1 \quad with \quad \frac{\ln(p-2)}{\ln(p)} < 1 \, \forall \, p \tag{35}$$

7.4 Proposition: Formal Mapping and Distinctness

Let CP_n denote the state of the pattern at step n, and let CPn[k] denote the symbol at the relative index k ≥ 1 on the tape.

Definition (Mapping Φ)

We define a bijection Φ that maps a symbol at relative index k on the tape CP_n to the natural number m:

$$\Phi(k,n) = n + k \tag{36}$$

The letter 'a' is defined as the sequence of length 6:

a = <L_1, M_2, L_3, M_4, M_5, M_6>.

The 'twin' template is defined as the pair of symbols at local indices 1 and 3 of any instance of letter a.

If an instance of letter a is located at starting index k on the tape, its twin template corresponds to the pair of natural numbers:

$$\Phi(a_{\text{pos=k, n}}) = (n+k, n+k+2) \tag{37}$$

Proposition (Injectivity and Distinctness)

A Twin Prime Pair is considered "generated" by the automaton if and only if both L's of a twin template are sequentially encoded as <P> by register BP.

Specifically, for a template corresponding to (p, p + 2):

1. At step n* = p, the first symbol (L_1) is encoded in BP as <P>.
2. At step n = p + 2, the second symbol (L_3) is encoded in BP as <P>.

Since the update rule N ← N + 1 is strictly monotonic, the register BP processes every natural number exactly once. Therefore, the specific event sequence of encoding p and p + 2 as primes is unique to that specific value of n. Consequently no Twin

Prime Pair can be generated multiple times. The mapping from the automaton's output to the set of twin primes in ℕ is strictly injective.

Corollary to Lemma 4:

For any step n, the output of the register BP (<P> or <M>) is strictly equivalent to the number-theoretic primality of n. There are no "pseudo-primes" in the Cosmos Automaton.

7.5 Theorem: Infinitude of Twin Primes

Based on the preceding lemmas and propositions, we state:

Theorem: There are infinitely many twin primes within the CA model.

Proof

1. **Existence (Hydra)**: As shown in Section 6.3 (Equation 21) and Section 7.3, the population of twin prime templates 'twin' on the tape grows geometrically with factor (p – 2) and has a strictly positive fractal dimension. Thus, the set of templates is non-empty and unbounded for all n.

2. **Transport (Streaming)**: The automaton operates as a streaming mechanism. The Resch operation cyclically moves the pattern. This ensures that every template twin existing on the tape at position k will eventually decrease its index as n increases, until it reaches the first square (k = 1). The argument that templates might be 'left behind' by exponential tape growth applies to abstractly growing data structures. However, the Cosmos Automaton operates as a strictly sequential, first-square-driven process. The Resch operation guarantees that every symbol not yet processed deterministically shifts one position closer to the first square (k=1) at each step n. While the tape length expands exponentially (factor p), this expansion occurs at the last-square of the tape and does not impede the deterministic forward progression of unprocessed elements towards the first square. Thus, every surviving template is guaranteed to eventually reach position k=1 where its primality is encoded in BP. And while the fractal dimension converges to 1 asymptotically, it remains strictly less than 1 for every finite step p.

3. **Validation (Stability Zone)**: When the first L of a template twin reaches index k = 1, it corresponds to the number m = n + 1. Since k = 1 is the start of the Stability Zone (Lemma 4), this 'L' is guaranteed to be a prime. Two steps later (at n = n* +2), the second 'L' of the same template reaches index k = 1. Since it is also in the Stability Zone, it is also guaranteed to be prime.

4. **Conclusion**: Since the supply of templates 'twin' is infinite (Step 1) and the mechanism guarantees their sequential evaluation as primes (Steps 2 & 3), the Cosmos Automaton generates an infinite sequence of distinct twin prime pairs.

q.e.d.

7.6 Asymptotic Density and the Hardy-Littlewood Connection

A heuristic argument for **consistency** with **existing theory**. Using the eigenvalues from the transition matrix M_p, we can calculate the density D_n of the template letter 'a' on the tape at step n.

The density is defined as the ratio of the surviving population (Hydra growth) to the total length of the pattern (Primorial growth):

$$D_n = \frac{|a|_n}{p_n\#} = \frac{\prod_{p \le p_n} (p-2)}{\prod_{p \le p_n} p} = \prod_{p \le p_n} \left(1 - \frac{2}{p}\right) \quad (38)$$

This product is well-known in analytic number theory. It matches the Hardy-Littlewood (1923) heuristic, which predicts that the density of twin primes decays asymptotically as:

$$D_n \approx \frac{C}{(\ln p_n)^2} \quad (39)$$

where C is a constant.

7.7 Quasi-Equidistribution and Bounded Gaps

Internal Mechanism

The Bet operation ensures that the pattern of survivors is periodically repeated and distributed across the new tape length $P_n\#$. The Mem operation then eliminates candidates modulo p_n. By the Chinese Remainder Theorem, these eliminations are distributed uniformly across the periods. It is structurally impossible for survivors to be confined to a segment that permanently evades the first-square of the tape (k = 1).

Consistency with Analytic Results

This constructive behavior aligns with the breakthrough results by Zhang (2014) and Maynard (2015) on Bounded Gaps. They proved that the gaps between primes do not grow indefinitely relative to their position but cluster within a fixed bound

(K=246). Since the Cosmos Automaton models the exact sieving process that produces these primes, its output must reflect this clustering. A scenario where gaps grow faster than the linear expansion of the Stability Zone would contradict the proven existence of Bounded Gaps.

8 Conclusion

This paper presents the Cosmos Automaton (CA), a fully deterministic framework for generating prime numbers through symbolic substitution and recursive refinement. By encoding the sieving process into a local, periodically expanding pattern CP, the CA operates as a true Streaming Algorithm (O'Neill 2009), overcoming the upper-bound limitations inherent in the classical Sieve of Eratosthenes. The automaton demonstrates that the distribution of primes is not a result of random stochasticity, but of high-complexity deterministic interference ("beats") between additive pulse trains.

The CA bridges the gap between constructive algorithms, automata theory and analytic number theory. The symbolic tape acts as a constructive generator for the Fractal Strings described analytically by Lapidus (2006) and realizes the deterministic obstruction schedule proposed topologically by Bilokon (2025). The computed fractal dimension of the residue set ($D \approx 0.9$) aligns with established statistical models, confirming that the automaton faithfully reproduces the asymptotic properties of the prime number distribution.

We lay a foundation for the formerly axiomatic prime number definition.

We applied this framework to the Twin Prime Conjecture. Through a population dynamics analysis based on the Chinese Remainder Theorem, we demonstrated the Structural Persistence of the Twin Prime template. The "Hydra Effect" — where the geometric expansion of the search space (p_n) systematically outpaces the linear elimination of candidates ($p_n - 2$) — ensures that the population of twin prime candidates grows geometrically. The deterministic conveyor belt of the CA necessitates that these surviving templates eventually enter the stability zone, confirming the infinitude of twin primes as a structural inevitability of the automaton's design.

The CA redefines the prime number landscape from a static field of values to a dynamic process of pattern evolution. By shifting the perspective from "what are the primes?" to "how does the pattern grow?", we reveal that the apparent chaos of the primes is simply the shadow of a deeply ordered, infinite fractal machine.

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12 Author

Ms. Birke Heeren has a Master of Science in Oceanography from Oregon State University (Corvallis, Oregon, USA) and a Diploma in Biology from Hamburg University (Hamburg, Germany), and is a software engineer (University of Applied Sciences, Trier, Germany).