Koide's Formula as a Possible Signature of Anisotropy in a Space Medium

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Abstract

Koide's formula, which relates with remarkable precision the masses of the three charged leptons, remains without a convincing explanation within the framework of standard particle theories. In this article, we show that this relation admits a natural interpretation within an alternative paradigm in which space is described as a physical medium endowed with intrinsic mechanical properties. We propose that the three generations of charged leptons correspond to fundamental excitations associated with three directional rigidities of an anisotropic space-medium, subject to a global redistribution constraint. The trigonometric parametrization of Koide's formula then appears as the general solution of a simple constitutive law under constraint. Within this framework, the observed instability and decay hierarchy of the heavier charged leptons arise naturally as a relaxation of higher-energy vibrational modes toward lower-rigidity directions. The cosmological and conceptual implications of this interpretation are discussed, including a possible connection with a cyclic multiverse dynamics based on non-singular bounces.

Keywords

Koide formula; charged lepton masses; anisotropic space-medium; directional rigidities; constitutive laws; vibrational modes; lepton decay hierarchy; Big Bounce; cyclic cosmology; multiverse synchronization.

1 Introduction

The hierarchy of fermion masses, and in particular the existence of exactly three generations of charged leptons, remains one of the persistent mysteries of fundamental physics. Despite its remarkable phenomenological success, the Standard Model provides no principled explanation for either the number of generations or the numerical values of the lepton masses.

Among the known empirical relations, Koide's formula occupies a singular place. For the charged leptons electron (e), muon (μ), and tau (τ), it reads:

$$\frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}, \tag{1}$$

and is experimentally verified with exceptional precision.

Within conventional approaches, this relation is often regarded as accidental or numerological, due to the absence of a clear underlying dynamical principle. In contrast, the present work adopts a different viewpoint: Koide's formula is interpreted as the manifestation of an underlying physical structure of space itself. Beyond the mass relation itself, this approach also provides a natural explanation for the instability and decay hierarchy of charged leptons, a feature that remains unexplained in standard frameworks.

This article is part of a broader paradigm developed in several previous works, in which space is not treated as a mere geometric background but as a physical medium endowed with mechanical, energetic, and dynamical properties [3, 4, 5]. Within this framework, issues such as inertia, gravitation, nonlocality, cosmological expansion, and certain conceptual paradoxes of relativity and quantum mechanics can be addressed in a unified manner. The reader is encouraged to consult these references for a comprehensive overview.

Here, we deliberately focus on a specific aspect of this paradigm: the possible link between Koide's formula and a fundamental constitutive anisotropy of the space-medium.

2 Overview of the space-medium paradigm

The analysis developed in this article relies on an alternative physical picture in which space is not treated as a passive geometric background, but as a physical medium endowed with intrinsic mechanical and energetic properties.

Within this picture, matter particles are interpreted as localized excitations of the space-medium, while large-scale cosmological dynamics emerge from internal energy exchanges and constitutive responses of this medium. Cosmological evolution is not singular, but proceeds through successive phases of expansion, contraction, and non-singular bounce.

For the purposes of the present work, only a minimal subset of this framework is required. Specifically, we assume that:

- the space-medium can store energy in the form of metric deformation,
- internal degrees of freedom of the medium admit anisotropic constitutive responses,
- global constraints govern the redistribution of these internal degrees of freedom.

No detailed microphysical model of the medium is needed here. These general structural assumptions suffice to establish a direct link between Koide's formula and the constitutive properties of space.

3 The Space-Medium and Directional Rigidities

In the space-medium paradigm, space is described as a fundamental continuous medium capable of carrying energy and supporting localized excitations. Elementary particles are interpreted as localized vibrational or excitation states of this medium.

Departing from the usual assumption of isotropy, we postulate that the space-medium may possess, at a constitutive level, an intrinsic anisotropy characterized by three principal directions. To each of these directions is associated an effective directional density ρ_i and a corresponding rigidity R_i, with $i = 1, 2, 3$.

A central assumption is that these directional densities are not independent, but are constrained by a global redistribution law:

$$\rho_1 + \rho_2 + \rho_3 = 3\rho_0 = \text{constant}. \tag{2}$$

This relation expresses a global conservation of the medium, analogous to a trace incompressibility condition: anisotropy is permitted, but it can only arise through internal redistribution.

The directional rigidities are assumed to be proportional to the corresponding densities,

$$R_i = \alpha \rho_i, \tag{3}$$

where α is a universal constitutive constant of the space-medium.

4 Fundamental Excitations and Lepton Masses

Within this framework, the three charged leptons are interpreted as the fundamental excitations of the three eigenmodes associated with the directional rigidities of the space-medium. The characteristic energy of a fundamental mode is assumed to scale with the square root of the corresponding rigidity, leading naturally to the identification

$$\sqrt{m_i} \propto \sqrt{R_i} \propto \sqrt{\rho_i}. \tag{4}$$

One may therefore write

$$\rho_i = \rho_0 \left(1 + \delta_i\right), \qquad \sum_{i=1}^{3} \delta_i = 0. \tag{5}$$

The problem then reduces to determining the most general form of the fluctuations δ_i consistent with this constraint.

5 Natural Emergence of Koide's Parametrization

If the redistribution dynamics of the medium is assumed to be smooth and controlled by a single global parameter—as expected for a constrained system with one effective degree of freedom—the general solution of the constraint $\sum \delta_i = 0$ can be written as

$$\delta_i = \sqrt{2}\,\sin\left(\theta + \frac{2\pi}{3}(i-1)\right). \tag{6}$$

This immediately yields

$$\sqrt{m_i} = M\left(1 + \sqrt{2}\,\sin\left(\theta + \frac{2\pi}{3}(i-1)\right)\right), \tag{7}$$

where M is proportional to the average density ρ_0 of the space-medium.

This expression is precisely the standard trigonometric parametrization of Koide's formula. The relation therefore appears not as a numerical coincidence, but as the direct consequence of a simple constitutive law under a global redistribution constraint.

In this interpretation, the phase parameter appearing in Koide's trigonometric parametrization is not a free mathematical angle. It acquires a physical meaning as a slowly evolving collective phase associated with the large-scale dynamics of the space-medium.

The observed charged-lepton masses therefore correspond to a snapshot of this collective state at a given cosmological phase, rather than to fixed fundamental constants.

6 Mechanical Interpretation: Three Coupled Spring-Like Axes

A useful physical picture of this parametrization consists in interpreting the three directional rigidities as three fundamental spring-like degrees of freedom associated with anisotropic principal axes of the space-medium. Each axis is characterized by an effective rigidity R_i, and the corresponding fundamental excitation is identified with one of the charged leptons.

These three spring-like degrees of freedom are not independent. They are coupled through the constitutive constraint

$$\rho_1 + \rho_2 + \rho_3 = \text{constant}, \tag{8}$$

which enforces a permanent redistribution of directional density within the medium. Any increase in rigidity along one axis necessarily occurs at the expense of the other two.

As a result, the system possesses only one independent dynamical degree of freedom. Geometrically, the allowed states of the system are confined to a circle in the space (ρ_1, ρ_2, ρ_3). The general solution corresponds to a uniform circular motion parametrized by a global phase θ, with the three components phase-shifted by $120°$:

$$\rho_i = \rho_0 \left(1 + \sqrt{2}\,\sin\left(\theta + \frac{2\pi}{3}(i-1)\right)\right). \tag{9}$$

In this interpretation, the square roots of the leptonic masses can be viewed as instantaneous projections of this collective circular motion in the space of directional rigidities. The phase θ acts as a normalized internal time parameter for the system.

6.1 Instability of heavier leptons and decay hierarchy

Within the space-medium paradigm, the instability of the muon and the tau lepton follows naturally from the same constitutive principles that give rise to the Koide mass relation. The three charged leptons do not correspond to distinct particle species in a fundamental sense, but rather to three realizations of the same type of localized excitation of the space-medium, projected onto different directional rigidity modes.

Although their internal dynamical geometry is identical, these excitations are supported by anisotropic directional rigidities of the space-medium. Modes associated with higher effective rigidities correspond to higher excitation energies. In a medium governed by energy redistribution and dissipation, such modes are intrinsically unstable and tend to relax toward lower-energy configurations.

As a consequence, excitations supported by the most rigid directions spontaneously degenerate into modes associated with less rigid directions. This provides a natural explanation for the observed decay hierarchy

$$\tau \to \mu \to e,$$

without invoking ad hoc mechanisms. The electron corresponds to the lowest-energy stable mode of excitation and therefore does not undergo further decay.

In this framework, lepton decays are interpreted not as transmutations between fundamentally different particles, but as relaxation processes between anisotropic vibrational modes of a single underlying medium.

7 Cosmic Expansion and Multiverse Synchronization

Although not the central focus of this article, a brief summary of the cosmic expansion mechanism within the space-medium paradigm is necessary to motivate the multiverse interpretation discussed below.

A key ingredient of this dynamics is the role played by neutrinos, which are produced abundantly throughout cosmic history. Rather than acting solely as passive relics, they constitute a diffuse energy reservoir capable of contributing to metric expansion under appropriate conditions.

This contribution does not rely on modifications of the Standard Model neutrino sector. It is a phenomenological mechanism whereby neutrino energy can be converted into large-scale gravitational potential energy of the space-medium, provided local gravitational fields remain sufficiently weak.

An independent order-of-magnitude estimate further supports this picture. A comparison between the total gravitational potential energy associated with cosmic expansion and the cumulative energy carried by all neutrinos produced over cosmic history shows a quantitative compatibility within uncertainties. Although approximate, this result suggests that neutrinos provide a physically plausible energy reservoir capable of driving metric expansion in the space-medium. The detailed calculation is presented in [6].

In this framework, cosmic expansion is not treated as a purely kinematic or geometric effect, but as a dynamical process based on energy exchanges within the space-medium. Neutrinos, produced in large quantities throughout cosmic history, play a central role as an energy reservoir. Their energy can be converted into local metric expansion of the medium, resulting in an increase of the gravitational potential energy of all masses immersed in the corresponding field.

This conversion is effective only in regions where gravitational fields are sufficiently weak. As expansion proceeds, large-scale gravitational fields become weaker between massive structures, favoring the condensation of low-energy neutrinos produced since the last cosmological bounce. This mechanism naturally leads to an acceleration of cosmic expansion. However, neutrino production is intrinsically finite, since stellar nucleosynthesis cannot proceed beyond iron. Once this neutrino-driven "fuel" is exhausted, gravitational forces dominate again, initiating a phase of global contraction. The contraction phase is dynamically faster, as it lacks a compensating mechanism analogous to neutrino-driven expansion.

This introduces a fundamental asymmetry between expansion and contraction phases. In a multiverse scenario where many bubble-universes share the same space-medium, each bubble undergoes a globally isotropic expansion or contraction characterized by a single scalar phase parameter. Mechanical interactions through the shared medium impose collective constraints on the synchronization of these phases.

A simple biphasic pattern, in which neighboring universes alternate between expansion and contraction, is dynamically stable only if the two phases are symmetric. The intrinsic asymmetry described above renders such a configuration unstable over long timescales. In contrast, a triphasic configuration—where universes are distributed among three phases separated by 120°—allows a smoother redistribution of mechanical stresses and phase velocities. Such triphasic synchronization is well known in dissipative oscillatory systems and supports long-term coherence despite asymmetric dynamics.

It is remarkable that this triphasic structure is exactly the one that emerges in the trigonometric parametrization of Koide's formula. This suggests that the observed lepton mass pattern may be the present-day manifestation of a deeper cosmological synchronization process within an eternal, cyclic multiverse.

It is worth emphasizing that, as the phase θ evolves continuously, the equality of two directional rigidities is an unavoidable outcome of the dynamics. This equality does not, by itself, imply any instability of matter. In the present framework, stability is governed by the energy stored in the deformations of the space-medium rather than by their geometric form. Anisotropic metrics therefore remain physically indistinguishable for internal observers, and no loss of coherence of matter is expected when two rigidities coincide.

8 Discussion and Conclusion

We have shown that Koide's formula admits a natural interpretation as the manifestation of a constitutive anisotropy of the space-medium, characterized by three directional rigidities subject to a global redistribution constraint. In this picture, the three generations of charged leptons arise as fundamental excitations associated with these rigidities.

A direct consequence of this interpretation is that the instability of the muon and tau naturally follows from their identification as higher-energy excitations of anisotropic vibrational modes. Excitations associated with stiffer directions of the space-medium are metastable and relax spontaneously toward lower-energy modes, providing a simple physical origin for the observed decay hierarchy $\tau \to \mu \to e$.

Although speculative, this paradigm offers a unified conceptual description linking particle mass hierarchies, constitutive properties of space, and large-scale cosmological dynamics. The remarkable precision of Koide's relation may thus reflect not an accidental numerical coincidence, but a deep structural feature of the space-medium shaped by cyclic cosmological evolution.

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