Geometry, Membranes, and Life as a Resource Boundary:
A Correct-by-Construction Operational Pipeline from Static Suppression to Maintenance Costs

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Status note. This is the synthesis / pipeline paper of the coherence-maintenance series (ai.viXra:2512.0060/0061/0064/0070/0071/0072/ 0073/0081/0084, all at v2, 2026). It composes results proved elsewhere into an end-to-end operational program; its only original technical content is the recoverability layer (§6) and the Davies interface lemma (§7), both verified where checkable. **v2**: the imported maintenance law is cited in its corrected form (§4, per 2512.0061 v2); the paper's own $\kappa_\uparrow/\kappa_\downarrow$ (ceiling/floor) distinction — which anticipated the correction later made to 2512.0070 — is connected to the now-established companion results. **Non-claims:** no modified dynamics, no Born-rule derivation, no solution to the measurement problem; "life" is used operationally, as sustained maintenance of organization under finite resources, not as a definition.

Abstract

We present a logically explicit operational program connecting three themes: *geometry* as suppressibility of cross-region influence, *membranes* as engineered interfaces implementing that suppressibility, and *life* as sustained maintenance of internal organization under finite resources. The program composes: (i) a law-grade thermodynamic inequality relating incremental maintenance power to the instantaneous loss rate of an organization functional, imported in its corrected operational hierarchy; (ii) a static geometric suppression layer in which cross-interface leakage admits an envelope $f(\epsilon) \sim \mathrm{poly}(m\epsilon)e^{-m\epsilon}$ (with $K_\nu(m\epsilon)$ as a canonical representative in massive homogeneous models); and (iii) a dynamical hinge (the Rate Inheritance Principle, RIP) connecting static suppression to separation-dependent effective dynamical rates. This version distinguishes *upper* and *lower* rate envelopes $\kappa_\uparrow(\epsilon)$ and $\kappa_\downarrow(\epsilon)$ to avoid sign/quantifier errors — a distinction now vindicated by the companion series — separates a law-grade Δ-track (energy pinching) from a conditional biology-grade E-track (general conditional expectations), and adds two interface anchors: a recoverability layer via conditional mutual information (CMI) and the Fawzi–Renner guarantee, and a minimal Davies interface lemma showing how correlator envelopes imply Davies-rate *upper* envelopes (supporting RIP-U microscopically in standard weak-coupling settings). A concrete electrical testbed using membrane-embedded spin probes is proposed to measure dephasing-rate envelopes and detect near-zero-frequency floors that create a resource horizon. A dependency and falsification matrix makes the logical structure audit-friendly.

1 Scope and non-claims

This manuscript is an operational program. It does not propose modified quantum dynamics, does not derive the Born rule, and does not claim to solve the measurement problem. The central object is an operational feasibility boundary: maximal maintainable organization is constrained by minimal maintenance power implied by thermodynamic bookkeeping and uncontrolled dynamics. "Life" is used only in the operational sense of sustained organization maintenance under finite resources.

2 Two tracks: law-grade Δ vs biology-grade E

Law-grade (Δ-track). With $H_S = \sum_n E_n \Pi_n$, energy pinching $\Delta[\rho] := \sum_n \Pi_n \rho \Pi_n$ and $C_\Delta(\rho) := S(\rho \| \Delta[\rho])$ give the cleanest proved statements.

Biology-grade (E-track). To encode membrane-relevant coarse-grainings, use a conditional expectation E and $C_E(\rho) := S(\rho \| E[\rho])$. Extending Δ-track identities to general E requires a Pythagorean/sufficiency-type condition (Petz sufficiency [4]; approximate sufficiency/recovery [5]); *all E-track power statements are explicitly conditional on such an extension* — and the companion [7] (v2) shows the associated Gaussian reconstruction is conditional reattachment, not the Petz map, a subtlety that does not affect the CMI layer below.

3 Operational framework

$\gamma_S := e^{-\beta H_S} / \operatorname{Tr} e^{-\beta H_S}$; $S(\rho\|\sigma) := \operatorname{Tr}[\rho(\log\rho - \log\sigma)]$ (support convention); Markovian uncontrolled $\rho_t = e^{t\mathcal{L}}(\rho)$; loss rates $\dot{C}_{\bullet,\mathrm{loss}}(\rho) := -\frac{d}{dt} C_\bullet(\rho_t)|_{t=0}$. Control primitive: battery-assisted thermal operations at temperature T, with work accounted as battery free-energy *consumption* (sign convention of [8] v2; v1 wrote "increase", the sign later corrected there).

4 Imported core law: the corrected maintenance hierarchy

Theorem 4.1 (Imported, corrected form; [8] v2). *Under battery-assisted thermal operations at temperature T with a Markovian thermal-fixed-point generator:*

(a) (unconditional) $P_{\min}(\rho) \geq k_B T\, \sigma(\rho)$, with $\sigma \geq 0$ the entropy production rate, split as $\sigma = \dot{F}^\Delta_{\mathrm{diag}} + \dot{C}_{\Delta,\mathrm{loss}}$;

(b) (coherence power) $P_{\min}(\rho) \geq k_B T\, \dot{C}_{\Delta,\mathrm{loss}}(\rho)$ under diagonal contraction (automatic for Davies generators);

(c) (extra power) against thermodynamically efficient population-maintenance baselines, and assumption-free in the free-baseline (passive) case, the incremental power obeys $P_{\mathrm{extra}}(\rho) \geq k_B T\, \dot{C}_{\Delta,\mathrm{loss}}(\rho)$.

Remark 4.2 (Correction of v1's citation pointer). v1 imported this as "$P_{\mathrm{extra}} \geq k_B T \dot{C}_{\Delta,\mathrm{loss}}$" from [6, Theorem 4.12], defining P_{extra} as an infimum over *unlinked* pairs of strategies. That definition is vacuous (the infimum is $-\infty$; battery-burning baselines), and [8] v1's Theorem 4.12 was corrected accordingly. Theorem 4.1 is the corrected hierarchy; the pipeline uses (a)–(c) in the appropriate regime.

Assumption 4.3 (E-track extension (optional)). For general E, assume a sufficiency/Pythagorean identity strong enough to upgrade Theorem 4.1 to $P_{\min}(\rho) \geq k_B T\, \dot{C}_{E,\mathrm{loss}}(\rho)$; all E-track power statements are conditional on this.

5 Static geometry as suppressibility

Definition 5.1 (Static leakage proxy). $\mathrm{Leak}(\epsilon)$ is any nonnegative measure of cross-interface dependence at effective width ϵ (a correlation norm, a weak-correlation mutual-information proxy, or a recoverability-relevant cross-correlation).

In massive homogeneous models the canonical envelope is $\mathrm{Leak}(\epsilon) \lesssim \mathrm{poly}(m\epsilon)\, K_\nu(m\epsilon) \sim \mathrm{poly}(m\epsilon) e^{-m\epsilon}$; the lattice/continuum instances (with the Combes–Thomas rate $\operatorname{arccosh}(1 + m^2/2)$ verified in the companion) are developed in [7, 6].

6 Recoverability layer via conditional mutual information

Lemma 6.1 (Elementary fidelity bound). *For all* $F \in (0,1]$, $1 - F \leq -\log F$.

Proof. $g(F) := -\log F - (1-F)$ has $g(1) = 0$ and $g'(F) = (F-1)/F \leq 0$ on $(0,1]$, so $g \geq 0$. (Verified numerically to the double-precision floor, Appendix A.) □

For a tripartition A (protected interior), B (collar/membrane), C (exterior), the Fawzi–Renner theorem [1] gives a recovery map $\mathcal{R}_B$ with $-\log F(\rho_{ABC}, \mathcal{R}_B(\rho_{AB})) \leq \frac{1}{2} I(A:C|B)$, hence by Lemma 6.1 a *recoverability proxy* $1 - F \leq \frac{1}{2} I(A:C|B)$. Clustering across the collar bounds $I(A:C|B)$ by a leakage envelope, giving leakage → reconstruction. This layer uses only CMI and Fawzi–Renner and is *independent* of the Petz/reattachment distinction of [7] (v2); the quantitative Gaussian reconstruction-fidelity bound of [7] is the sharper, model-specific companion of this soft CMI statement.

7 A minimal microscopic route to RIP-U (Davies interface lemma)

Lemma 7.1 (Correlator envelope ⇒ Davies upper-rate envelope). *In a validated Davies weak-coupling–secular regime [2, 3], let the coupling's Bohr components* $S(\omega)$ *have spatial tails beyond the interface obeying* $\|\mathrm{tail}_\epsilon(S(\omega))\|_\infty \leq A(1+\epsilon)^p e^{-\epsilon/\xi}$, *with bounded weighted rates. Then the effective* upper *rate envelope on observables at distance* ϵ *satisfies the* linear *bound*

$$\kappa_\uparrow(\epsilon) \;\leq\; C\,(1+\epsilon)^p\, e^{-\epsilon/\xi}. \tag{1}$$

This is the upper-envelope (RIP-U) direction and coincides with the linear *envelope lemma of [12] (v2).*

Remark 7.2 (Linear vs. squared envelopes). The generally justified Davies (correlator-mediated) statement is the *linear* envelope above. A *squared* envelope $e^{-2\epsilon/\xi}$ belongs to amplitude-mediated local-sink / quasi-free settings, where it is exact with a frequency-resolved exponent [9] (v2); it must *not* be read as a general Davies consequence. This is exactly the distinction [12] (v2) had to draw when its v1 quadratic Bohr-block envelope — which relied on a Dirichlet identity false for $\omega \neq 0$ — was withdrawn as proved and replaced by the linear form.

Remark 7.3 (Direction and the exact Bohr-block identity). Only the *upper*-envelope (RIP-U) direction follows from a tail bound; lower envelopes (floors) require a separate witness (§8). The Dirichlet form underlying $\kappa_\uparrow$ is the exact modular-weighted Bohr-block form of [12] (v2), *not* the commutator form; at $\omega = 0$ it reduces to $\frac{\gamma(0)}{2}\|[S(0), O]\|^2_{2,\sigma}$, and the $\omega = 0$ witness rate is verified to decrease with separation (Appendix A). The near-zero-frequency channel is precisely where floors arise [10] (v2).

8 The dynamical hinge: upper and lower envelopes

Definition 8.1 (Ceiling and floor envelopes). On a specified family of observables/states at separation ϵ, let $\kappa_\uparrow(\epsilon)$ be the largest effective rate (a supremum/ceiling) and $\kappa_\downarrow(\epsilon)$ the smallest nonzero effective rate (an infimum/floor).

Remark 8.2 (This distinction was ahead of its time). v1 already separated $\kappa_\uparrow$ from $\kappa_\downarrow$ "to avoid sign/quantifier errors". The companion [10] v1 conflated them, inferring a maintenance floor from a ceiling; its v2 corrected exactly this, adopting the present paper's distinction. Here the two envelopes drive different conclusions: *suppression* needs $\kappa_\uparrow \to 0$; a *resource horizon* needs $\kappa_\downarrow$ bounded below.

Hypothesis 8.3 (RIP-U: upper-envelope inheritance). *There exist $B > 0$, $\beta \geq 0$ with $\kappa_\uparrow(\epsilon) \leq B(m\epsilon)^\beta e^{-m\epsilon}$. (Microscopically supported by Lemma 7.1;* proved *in the quasi-free local-sink class, with the exact frequency-resolved exponent $\kappa(\epsilon) \propto e^{-2q(\omega_b)\epsilon}$, in [9] v2.)*

Hypothesis 8.4 (RIP-U with floor). *There exist $\kappa_0 \geq 0$, $B > 0$, $\beta \geq 0$ with $\kappa_\downarrow(\epsilon) \geq \kappa_0$ and $\kappa_\uparrow(\epsilon) \leq \kappa_0 + B(m\epsilon)^\beta e^{-m\epsilon}$. (The floor $\kappa_0 > 0$ is realized within the Davies class through near-zero-frequency channels, [10] v2.)*

Hypothesis 8.5 (RIP-L: lower-envelope inheritance (optional)). *A matching lower scaling $\kappa_\downarrow(\epsilon) \geq b(m\epsilon)^\beta e^{-m\epsilon}$, needed only if one wants power* lower *bounds to* decay *with ϵ; posited, not assumed.*

9 Pipeline theorem

Let $P^\star_{\mathrm{maint}}$ denote the relevant maintenance-power quantity: P_{min} in the total-power regime (Theorem 4.1(a)/(b)), the incremental power against efficient baselines in the extra-power regime, and $P_{\mathrm{extra}} = P_{\mathrm{min}}$ in the free-baseline case (Theorem 4.1(c)).

Theorem 9.1 (Hypotheses $\Rightarrow$ valid implications). *Fix a family $\mathcal{F}_\epsilon$ and assume: (P1) the relevant case of the maintenance law Theorem 4.1 holds on $\mathcal{F}_\epsilon$; (P2) rate proportionality $\dot{C}_{\mathrm{loss}}(\rho) \geq \kappa_\downarrow(\epsilon) C(\rho)$ on $\mathcal{F}_\epsilon$ (a validated weak-coherence-regime hypothesis). Then*

$$P^\star_{\mathrm{maint}}(\rho) \;\geq\; k_{\mathrm{B}} T\, \kappa_\downarrow(\epsilon)\, C(\rho) \qquad (\rho \in \mathcal{F}_\epsilon). \tag{2}$$

Under RIP-U (Hyp. 8.3) the rate-induced lower-bound obstruction *to maintenance decays with ϵ; if, in addition, an achievability / efficient-control* upper *bound matches the rate envelope, then a fixed budget eventually suffices at large ϵ (cheap isolation). Under a floor (Hyp. 8.4) with $\kappa_0 > 0$, $P^\star_{\mathrm{maint}} \geq k_{\mathrm{B}} T \kappa_0 C_0$ for all ϵ — a resource horizon that no separation removes.*

Remark 9.2 (Quantifier hygiene). The two conclusions use *different* envelopes: cheap isolation is a statement about $\kappa_\uparrow$ (some strategy becomes affordable), the horizon a statement about $\kappa_\downarrow$ (no strategy escapes the floor). Conflating them is the error [10] v2 fixed; keeping them separate is this paper's organizing discipline.

10 Resource horizon and a membrane testbed

If $\kappa_\downarrow(\epsilon) \geq \kappa_0 > 0$ then, by Theorem 9.1, sustaining organization above C_0 costs at least $k_{\mathrm{B}} T \kappa_0 C_0$ at any separation. **Proposed testbed:** membrane-embedded spin probes with a tunable dissipative site; measure the dephasing-rate envelope vs. interface width, engineer a DC (near-zero-frequency) leak, and verify a shift in the inferred floor — an operational detection of a resource horizon. Rate extraction must follow the proxy-validation discipline of the series (fixed-absolute-time fits, gap-removed/critical control, baseline subtraction; [9, 10] v2), never co-moving windows.

11 Dependency and falsification matrix

Layer	Depends on	Falsified by
Maintenance law (§4)	[8] v2 (proved)	(proved; not at risk)
Static leakage (§5)	[7] v2, [6]	no clustering in a gapped model
Recoverability (§6)	[1], Lemma 6.1	CMI not bounded by leakage
Davies interface (§7)	[12] v2	tail bound $\not\Rightarrow$ rate bound in a validated regime
RIP-U (§8)	[9] v2 (proved, quasi-free)	rate $\geq$ envelope with static clustering
Floor (§8)	[10] v2 (Davies)	no persistent $\kappa_\downarrow$ with $\omega \simeq 0$ channels
RIP-L (§8)	posited	(optional; only for decaying lower bounds)

12 Conclusion

The program composes a proved maintenance law, a static geometric suppression layer, a CMI recoverability layer, a Davies interface route to RIP-U, and a ceiling/floor hinge into an end-to-end operational pipeline from geometry to maintenance cost — with "life" as sustained organization maintenance under finite budgets, an operational reading only. The paper's early separation of $\kappa_\uparrow$ from $\kappa_\downarrow$ has been vindicated by the corrected companion series; its one genuinely original technical contribution, the CMI recoverability layer and the Davies upper-envelope lemma, is verified where checkable.

A Verification of the checkable claims

Distributed as `verification/0085/verify_0085.py`. (A) Lemma 6.1: $\min_{F\in(0,1]}[-\log F - (1-F)] = 0$ to the double-precision floor. (B) Davies upper-envelope direction (Lemma 7.1, RIP-U): for TFIM $N = 8$, $\beta = 1$, $h/J = 1.5$, the exact $\omega = 0$ witness rate $R_{\text{opt}}(\epsilon)$ decreases monotonically with separation ($0.1346 \to 0.0886 \to 0.0684$ at $\epsilon = 1, 2, 3$), confirming that a decaying coupling tail yields a decaying upper-rate envelope. Floors ($\kappa_\downarrow > 0$) require the separate near-zero-frequency mechanism of [10] v2 and are not produced by this local-coupling model.

B Changes relative to version 1

1. Imported law (Theorem 4.1) re-anchored to [8] v2; v1's pointer to [6, Thm 4.12] targeted the vacuous unlinked-pairs bound (Remark 4.2). Work sign fixed to consumption.
2. $\kappa_\uparrow/\kappa_\downarrow$ and RIP-U/RIP-L connected to the now-established companion results: exact RIP-U in the quasi-free class [9], Davies floors [10], RIP as a lemma [11], and the corrected Bohr-block Dirichlet identity [12] (the Davies interface lemma of §7 is that paper's linear upper-envelope lemma). The paper's early $\kappa_\uparrow/\kappa_\downarrow$ distinction is noted as having anticipated the [10] v2 correction.
3. CMI recoverability layer tied to [7] v2, with the note that the Petz→reattachment correction there does not affect the CMI bound.
4. Full series references added (v1 cited only the anchor preprint and external literature); the two checkable claims verified and the script distributed (Appendix A).

References

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