

Geometry, Membranes, and Life as a Resource Boundary: A Correct-by-Construction Operational Pipeline from Static Suppression to Maintenance Costs

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Abstract

We propose a logically explicit operational program connecting three themes: geometry as suppressibility of cross-region influence, membranes as engineered interfaces implementing that suppressibility, and life as sustained maintenance of internal organization under finite resources. The program composes: (i) a law-grade thermodynamic inequality relating incremental maintenance power to the instantaneous loss rate of an organization functional, (ii) a static geometric suppression layer in which cross-interface leakage admits an envelope $f(\epsilon) \sim \text{poly}(m\epsilon)e^{-m\epsilon}$ (with $K_\nu(m\epsilon)$ as a canonical representative in massive homogeneous models), and (iii) a dynamical hinge (Rate Inheritance Principle, RIP) connecting static suppression to separation-dependent effective dynamical rates. This version is referee-oriented: it distinguishes upper and lower rate envelopes $\kappa^\uparrow(\epsilon)$ and $\kappa^\downarrow(\epsilon)$ to avoid sign/quantifier errors, it separates a law-grade Δ -track (energy pinching) from a conditional biology-grade E -track (general conditional expectations), and it adds two interface anchors: a recoverability layer via conditional mutual information (CMI) and the Fawzi–Renner guarantee [1], and a minimal Davies interface lemma showing how correlator envelopes imply Davies-rate envelopes (supporting RIP-U microscopically in standard weak-coupling settings [2, 3]). A concrete electrical testbed using membrane-embedded spin probes is proposed to measure dephasing-rate envelopes and detect near-zero-frequency floors that create a resource horizon. A dependency and falsification matrix is included to make the logical structure audit-friendly.

Contents

1	Scope and non-claims	3
2	Two parallel tracks: law-grade Δ vs biology-grade E	3
2.1	The law-grade thermodynamic track (Δ -track)	3
2.2	The biology-grade interface track (E -track)	3
3	Operational framework	3
3.1	Thermal states and relative entropy	3
3.2	Uncontrolled dynamics	3
3.3	Loss rates	3
3.4	Control primitives and incremental power	4
4	Imported core law: incremental maintenance power lower bound	4
5	Static geometry as suppressibility: leakage envelopes	4
5.1	A canonical choice of leakage proxy (example)	4
6	Recoverability layer: from leakage to reconstruction via CMI (conditional)	5
6.1	Tripartition and a recoverability proxy	5
6.2	Remove the $-\log F$ caveat: an elementary inequality	5
6.3	Leakage-to-recoverability via conditional mutual information	5
7	A minimal microscopic route to RIP-U (Davies interface lemma)	6
7.1	Validated Davies regime (working definition)	7
8	The dynamical hinge: upper and lower rate envelopes and correct directions	7
8.1	Why one must distinguish κ^\uparrow and κ^\downarrow	7
8.2	Definitions: κ^\uparrow and κ^\downarrow on a specified family	7
8.3	RIP in upper-envelope and floor-robust forms	8
9	Pipeline theorem: hypotheses \Rightarrow valid implications	8
10	Resource horizon from rate floors	8
11	Dependency and falsification matrix (one-page)	9
12	Discussion and open problems	10
13	Conclusion	10

1 Scope and non-claims

This manuscript is an operational program. It does not propose modified quantum dynamics, does not derive the Born rule, and does not claim to solve the measurement problem. The central object is an operational feasibility boundary: maximal maintainable organization is constrained by minimal maintenance power implied by thermodynamic bookkeeping and uncontrolled dynamics.

2 Two parallel tracks: law-grade Δ vs biology-grade E

2.1 The law-grade thermodynamic track (Δ -track)

The cleanest proved thermodynamic statements use energy pinching Δ associated with a system Hamiltonian $H_S = \sum_n E_n \Pi_n$:

$$\Delta[\rho] := \sum_n \Pi_n \rho \Pi_n, \quad C_\Delta(\rho) := S(\rho \| \Delta[\rho]).$$

2.2 The biology-grade interface track (E -track)

To encode membrane-relevant coarse-grainings, introduce a conditional expectation E and

$$C_E(\rho) := S(\rho \| E[\rho]).$$

Extending Δ -track thermodynamic identities to general E typically requires a Pythagorean/sufficiency-like condition, closely related to Petz sufficiency [4] and approximate sufficiency/recovery [5]. All E -track power statements are explicitly conditional on such an extension.

3 Operational framework

3.1 Thermal states and relative entropy

Definition 1 (Thermal state and relative entropy). *For finite-dimensional S with Hamiltonian H_S and inverse temperature $\beta = 1/(k_B T)$, define*

$$\gamma_S := \frac{e^{-\beta H_S}}{\text{Tr}(e^{-\beta H_S})}.$$

For density operators ρ, σ define

$$S(\rho \| \sigma) := \text{Tr}[\rho(\log \rho - \log \sigma)],$$

when $\text{supp}(\rho) \subseteq \text{supp}(\sigma)$ and $+\infty$ otherwise.

3.2 Uncontrolled dynamics

We assume Markovian uncontrolled evolution

$$\rho_t = e^{tL}(\rho),$$

with generator L and a temperature scale T , in a regime where the thermodynamic bookkeeping model is meaningful.

3.3 Loss rates

Definition 2 (Instantaneous organization-loss rates). *For uncontrolled evolution $\rho_t = e^{tL}(\rho)$ define the loss rates*

$$\dot{C}_{\Delta, \text{loss}}(\rho) := - \left. \frac{d}{dt} C_\Delta(\rho_t) \right|_{t=0}, \quad \dot{C}_{E, \text{loss}}(\rho) := - \left. \frac{d}{dt} C_E(\rho_t) \right|_{t=0}.$$

3.4 Control primitives and incremental power

We adopt a battery-assisted thermal-operations control model at temperature T , with work accounted as battery free-energy increase, and define incremental stabilization power P_{extra} as an infimum over pairs of strategies maintaining the full target and maintaining the coarse-grained target.

4 Imported core law: incremental maintenance power lower bound

Theorem 1 (Imported maintenance inequality, Δ -track). *Assume strategies exist that maintain ρ and that maintain $\Delta[\rho]$ under battery-assisted thermal operations at temperature T . Then*

$$P_{\text{extra}}(\rho; L, T) \geq k_{\text{B}}T \dot{C}_{\Delta, \text{loss}}(\rho).$$

Remark 1 (Citation pointer). *This bound is imported from [6, Theorem 4.12] (finite-dimensional, battery-assisted thermal operations).*

Assumption 1 (E -track extension condition (optional)). *When working with general E , assume a sufficiency/Pythagorean-type identity strong enough to upgrade the proof to yield*

$$P_{\text{extra}}(\rho; L, T) \geq k_{\text{B}}T \dot{C}_{E, \text{loss}}(\rho).$$

All E -track power statements are conditional on this assumption.

5 Static geometry as suppressibility: leakage envelopes

Definition 3 (Static leakage proxy (general)). *Let $\text{Leak}(\epsilon)$ be any nonnegative quantity measuring cross-interface dependence for an interface of effective width ϵ , such as a correlation norm, a mutual-information proxy in a weak-correlation regime, or a recoverability-relevant cross-correlation proxy.*

5.1 A canonical choice of leakage proxy (example)

Definition 4 (Trace-distance correlation leakage (example)). *Given the inside–collar–outside tripartition (A, B, C) , define the (unconditional) correlation leakage proxy*

$$\text{Leak}(\epsilon) := \frac{1}{2} \|\rho_{AC}(\epsilon) - \rho_A(\epsilon) \otimes \rho_C(\epsilon)\|_1,$$

where $\rho_{AC}(\epsilon) = \text{Tr}_B(\rho_{ABC}(\epsilon))$.

Remark 2 (Why this is a reasonable canonical proxy). *This choice is operational (it bounds optimal distinguishability between correlated and product hypotheses) and is basis-independent. In weak-correlation regimes, one expects quadratic relations between information quantities and small correlation norms; accordingly we treat the leakage–CMI identification as an explicit, family-qualified assumption rather than a theorem.*

Remark 3 (Measurement routes). *In practice, $\|\cdot\|_1$ is hard to access directly at scale. For experiments one may replace Definition 4 by a calibrated proxy such as:*

- a Hilbert–Schmidt proxy $\text{Leak}_2(\epsilon) := \frac{1}{2} \|\rho_{AC} - \rho_A \otimes \rho_C\|_2$ estimated via randomized measurements,
- a Gaussian/covariance proxy in quasi-free regimes, where cross-covariance norms can be extracted from second moments,

- or directly estimate $I(A : C|B)$ using small-subsystem tomography or shadow estimators, then invoke the CMI layer.

The falsification logic in Section 11 only requires internal consistency of the chosen proxy within the stated regime.

Assumption 2 (Static envelope class). *There exist constants $A > 0$, $\alpha \geq 0$, and a scale parameter $m > 0$ such that*

$$\text{Leak}(\epsilon) \leq A(m\epsilon)^\alpha e^{-m\epsilon}$$

on the regime of interest. In models with known massive envelopes, one may replace the right-hand side by $A(m\epsilon)^\alpha K_\nu(m\epsilon)$ with the same large- $m\epsilon$ asymptotics, where K_ν is the modified Bessel function of the second kind [7].

Remark 4 (Distance from suppressibility). *Given an envelope $f(\epsilon)$, an induced distance-like quantity is*

$$d_{\text{eff}}(\epsilon) := -\frac{1}{m} \log f(\epsilon).$$

If $f(\epsilon) \sim \text{poly}(m\epsilon)e^{-m\epsilon}$, then $d_{\text{eff}}(\epsilon) = \epsilon + O(\log \epsilon)$, formalizing “geometry as log-suppressibility”.

6 Recoverability layer: from leakage to reconstruction via CMI (conditional)

This section provides a formal slot for the claim that “distance is reconstructibility” in a finite-dimensional regularized posture, without importing Type III machinery.

6.1 Tripartition and a recoverability proxy

Definition 5 (Tripartition and recoverability error proxy). *Fix a regularized inside–collar–outside tripartition (A, B, C) induced by an interface of effective width ϵ . For a faithful state ρ_{ABC} define the recoverability error proxy*

$$R(\epsilon) := 1 - \sup_{\mathcal{R}_{B \rightarrow BC}} F\left(\rho_{ABC}, (\text{id}_A \otimes \mathcal{R}_{B \rightarrow BC})(\rho_{AB})\right),$$

where the supremum ranges over a specified recovery class (e.g. all CPTP maps), and F is the Uhlmann fidelity.

6.2 Remove the $-\log F$ caveat: an elementary inequality

Lemma 1 (Elementary fidelity bound). *For all $F \in (0, 1]$,*

$$1 - F \leq -\log F.$$

Proof. Define $g(F) := -\log F - (1 - F)$. Then $g(1) = 0$ and $g'(F) = -1/F + 1 = (F - 1)/F \leq 0$ on $(0, 1]$. Hence $g(F) \geq 0$ and the claim follows. \square

6.3 Leakage-to-recoverability via conditional mutual information

Assumption 3 (Leakage–CMI identification on a specified family). *Fix a target family \mathcal{G}_ϵ of tripartite states $\rho_{ABC}(\epsilon)$ and a smallness threshold $\text{Leak}_0 > 0$. Assume $\text{Leak}(\epsilon) \leq \text{Leak}_0$ holds uniformly on \mathcal{G}_ϵ , and there exist constants $c_1, c_2 > 0$ independent of ϵ and uniform on \mathcal{G}_ϵ such that*

$$c_1 \text{Leak}(\epsilon)^2 \leq I_\rho(A : C|B) \leq c_2 \text{Leak}(\epsilon)^2.$$

Proposition 1 (Leakage-to-recoverability bridge (conditional)). *Assume the finite-dimensional Fawzi–Renner recovery guarantee [1]: there exists a recovery map $\mathcal{R}_{B \rightarrow BC}$ such that*

$$-\log F(\rho_{ABC}, (\text{id}_A \otimes \mathcal{R}_{B \rightarrow BC})(\rho_{AB})) \leq \frac{1}{2} I_\rho(A : C|B).$$

Then

$$R(\epsilon) \leq \frac{1}{2} I_\rho(A : C|B) \lesssim \text{Leak}(\epsilon)^2,$$

with constants depending only on the leakage–CMI identification constants in Assumption 3.

Proof. Let \mathcal{R}_\star be the recovery map provided by [1]. Then

$$R(\epsilon) = 1 - \sup_{\mathcal{R}} F(\rho_{ABC}, (\text{id}_A \otimes \mathcal{R})(\rho_{AB})) \leq 1 - F(\rho_{ABC}, (\text{id}_A \otimes \mathcal{R}_\star)(\rho_{AB})).$$

By Lemma 1, $1 - F \leq -\log F$, and by the Fawzi–Renner guarantee, $-\log F \leq \frac{1}{2} I(A : C|B)$. The final inequality uses Assumption 3. \square

Remark 5 (Role in the geometry narrative). *This layer justifies a clean chain: static suppression $\text{Leak}(\epsilon) \downarrow$ implies $I(A : C|B) \downarrow$ (assumption), which implies improved recoverability (Fawzi–Renner). Thus ϵ has operational meaning as a control parameter for reconstruction.*

7 A minimal microscopic route to RIP-U (Davies interface lemma)

We record a simple lemma explaining why, under standard weak-coupling hypotheses, a spatial envelope on bath correlators propagates to an envelope on Davies rates [2, 3].

Lemma 2 (Correlator envelope implies Davies-rate envelope). *Assume Davies-type weak-coupling dynamics with bath operators $B_\alpha(\epsilon)$ and rate matrix elements*

$$\Gamma_{\alpha\beta}(\omega; \epsilon) = \int_{-\infty}^{\infty} dt e^{i\omega t} \langle B_\alpha(t; \epsilon) B_\beta(0; \epsilon) \rangle.$$

If there exists a nonnegative function $f(\epsilon)$ and functions $g_{\alpha\beta}(t)$ such that

$$|\langle B_\alpha(t; \epsilon) B_\beta(0; \epsilon) \rangle| \leq g_{\alpha\beta}(t) f(\epsilon), \quad \int_{-\infty}^{\infty} dt |g_{\alpha\beta}(t)| < \infty,$$

then for all ω ,

$$|\Gamma_{\alpha\beta}(\omega; \epsilon)| \leq \left(\int_{-\infty}^{\infty} dt |g_{\alpha\beta}(t)| \right) f(\epsilon).$$

Proof. By triangle inequality,

$$|\Gamma_{\alpha\beta}(\omega; \epsilon)| \leq \int_{-\infty}^{\infty} dt |\langle B_\alpha(t; \epsilon) B_\beta(0; \epsilon) \rangle| \leq \int_{-\infty}^{\infty} dt |g_{\alpha\beta}(t)| f(\epsilon).$$

\square

Remark 6 (How floors arise). *A rate floor can arise if either (i) the bound does not factor as $g(t)f(\epsilon)$ uniformly, (ii) an effective near- $\omega \simeq 0$ channel does not inherit the same spatial envelope, or (iii) the Davies approximation is not valid uniformly in ϵ .*

7.1 Validated Davies regime (working definition)

Definition 6 (Validated Davies regime (working)). *We say the Davies (weak-coupling–secular) description is validated on a parameter range if:*

- *the system–bath coupling is weak enough for a Markovian approximation on the timescales of interest,*
- *bath correlation functions decay sufficiently fast in time to justify a finite correlation time and the relevant Fourier transforms,*
- *a secular (Bohr-frequency) resolution scale is chosen such that near-resonant frequencies are treated consistently,*
- *and the resulting generator has the intended stationary state (e.g. γ_S) within experimental error.*

8 The dynamical hinge: upper and lower rate envelopes and correct directions

8.1 Why one must distinguish κ^\uparrow and κ^\downarrow

The maintenance law is a lower bound:

$$P_{\text{extra}} \geq k_B T \dot{C}_{\text{loss}}.$$

Therefore:

- An upper bound $\dot{C}_{\text{loss}} \leq \kappa^\uparrow(\epsilon) C$ does not imply a power lower envelope.
- A lower bound $\dot{C}_{\text{loss}} \geq \kappa^\downarrow(\epsilon) C$ does imply a power lower envelope.

8.2 Definitions: κ^\uparrow and κ^\downarrow on a specified family

Definition 7 (Upper and lower rate envelopes on a family). *Fix a family of targets \mathcal{F}_ϵ for each ϵ , and a functional C . Define the upper envelope*

$$\kappa^\uparrow(\epsilon) := \sup_{\rho \in \mathcal{F}_\epsilon, C(\rho) > 0} \frac{\dot{C}_{\text{loss}}(\rho)}{C(\rho)},$$

and the lower envelope

$$\kappa^\downarrow(\epsilon) := \inf_{\rho \in \mathcal{F}_\epsilon, C(\rho) > 0} \frac{\dot{C}_{\text{loss}}(\rho)}{C(\rho)}.$$

Remark 7 (Local proportionality and nontrivial κ^\downarrow). *On restricted weak-coherence families bounded away from extremal diagonals, one often has local proportionality*

$$\dot{C}_{\text{loss}}(\rho) = \kappa(\epsilon) C(\rho) + o(C(\rho)),$$

which yields $\kappa^\downarrow(\epsilon) \approx \kappa^\uparrow(\epsilon) \approx \kappa(\epsilon)$ on that family.

8.3 RIP in upper-envelope and floor-robust forms

Hypothesis 1 (RIP-U: upper-envelope inheritance). *There exist $B > 0$, $\beta \geq 0$ such that*

$$\kappa^\uparrow(\epsilon) \leq B(m\epsilon)^\beta e^{-m\epsilon}$$

on the regime of interest.

Hypothesis 2 (RIP-U with floor). *There exist $\kappa_0 \geq 0$, $B > 0$, $\beta \geq 0$ such that*

$$\kappa^\uparrow(\epsilon) \leq \kappa_0 + B(m\epsilon)^\beta e^{-m\epsilon}.$$

Hypothesis 3 (RIP-L: lower-envelope inheritance (optional)). *If one wants a lower scaling prediction for required power, assume there exist $\underline{B} > 0$, $\underline{\beta} \geq 0$ such that*

$$\kappa^\downarrow(\epsilon) \geq \underline{B}(m\epsilon)^{\underline{\beta}} e^{-m\epsilon},$$

or, in floor form, $\kappa^\downarrow(\epsilon) \geq \underline{\kappa}_0$.

9 Pipeline theorem: hypotheses \Rightarrow valid implications

Theorem 2 (Pipeline theorem (correct directions)). *Fix a track (Δ or E), a functional $C \in \{C_\Delta, C_E\}$, and a target family \mathcal{F}_ϵ . Assume:*

(P1) (Maintenance law) *For all $\rho \in \mathcal{F}_\epsilon$, $P_{\text{extra}}(\rho; L, T) \geq k_B T \dot{C}_{\text{loss}}(\rho)$, where \dot{C}_{loss} is the loss rate of C .*

(P2) (Static suppression) *There exists a leakage proxy $\text{Leak}(\epsilon)$ satisfying $\text{Leak}(\epsilon) \leq A(m\epsilon)^\alpha e^{-m\epsilon}$ on the regime.*

(P3) (Leakage-to-recoverability) *On a specified family, $R(\epsilon) \lesssim \text{Leak}(\epsilon)^2$ (Proposition 1).*

(P4) (Rate envelopes) *Upper and lower envelopes $\kappa^\uparrow(\epsilon)$ and $\kappa^\downarrow(\epsilon)$ are defined on \mathcal{F}_ϵ .*

Then:

(Q1) (Power lower envelope uses κ^\downarrow) *For all $\rho \in \mathcal{F}_\epsilon$,*

$$P_{\text{extra}}(\rho; L, T) \geq k_B T \kappa^\downarrow(\epsilon) C(\rho).$$

(Q2) (Recoverability improves with ϵ under static suppression) *If $\text{Leak}(\epsilon)$ decays with ϵ , then $R(\epsilon)$ decays at least quadratically in the same regime, hence reconstruction improves with separation.*

(Q3) (RIP-U is a testable dynamical interface) *RIP-U is an additional hypothesis linking $\kappa^\uparrow(\epsilon)$ to a static envelope class; it is supported microscopically by Lemma 2 when its preconditions hold in a validated Davies regime (Definition 7.1).*

10 Resource horizon from rate floors

Corollary 1 (Resource horizon under a nonzero lower-rate floor). *Fix a family \mathcal{F}_ϵ with $C(\rho) \geq C_0 > 0$ for all $\rho \in \mathcal{F}_\epsilon$. If there exists $\underline{\kappa}_0 > 0$ such that*

$$\kappa^\downarrow(\epsilon) \geq \underline{\kappa}_0$$

for all ϵ in the regime, then any controller maintaining any $\rho \in \mathcal{F}_\epsilon$ must satisfy

$$P_{\text{extra}}(\rho; L, T) \geq k_B T \underline{\kappa}_0 C_0.$$

In particular, increasing ϵ cannot reduce the minimal required power below this floor.

11 Dependency and falsification matrix (one-page)

This section lists each assumption/interface, what it buys in the pipeline, and how to falsify it experimentally or numerically. Items marked “imported” are used as inputs; all others are explicitly conditional and testable.

Assumption / In-terface	Used for	Implication in this paper	How to falsify (experiment or numerics)
Maintenance law (imported)	Core lower bound on power	For chosen track, $P_{\text{extra}} \geq k_{BT} \dot{C}_{\text{loss}}$	Show persistent $P_{\text{extra}} < k_{BT} \dot{C}_{\text{loss}}$ after calibration (population leakage, battery vs wall-plug accounting)
Static leakage envelope	Geometry layer	$\text{Leak}(\epsilon) \leq A(m\epsilon)^\alpha e^{-m\epsilon}$ on the regime	Leakage proxy vs ϵ admits no fit to any gapped envelope class over validated regime
Leakage-CMI identification	Recoverability layer	On a family, $I(A : C B) \asymp \text{Leak}(\epsilon)^2 \Rightarrow R(\epsilon) \lesssim \text{Leak}(\epsilon)^2$	Tripartite estimates show $I(A : C B)$ does not scale quadratically with the leakage proxy on the chosen family
Fawzi–Renner guarantee	Recoverability inequality	$-\log F \leq \frac{1}{2}I(A : C B)$ in finite dimensions [?]	Document model mismatch (non-regularized setting) or incompatible F vs inferred CMI
Davies lemma preconditions	RIP-U plausibility	Factorized envelope with $g \in L^1 \Rightarrow$ rate envelope (Lemma 2)	Correlators show non-integrable $g(t)$ or no uniform factorization; near- $\omega \simeq 0$ channel violates envelope
Validated regime	Davies Model validity	Weak coupling + secular resolution + stationarity (Def. 7.1)	Direct diagnostics show non-Markovianity, breakdown of secular approximation, or wrong stationary state
RIP-U (upper envelope)	Rate–geometry link (upper)	$\kappa_U(\epsilon) \leq B(m\epsilon)^\beta e^{-m\epsilon}$ or floor form	Extracted rate proxy vs ϵ admits no envelope (or no compatible floor+envelope) fit
RIP-L (lower envelope)	Power lower-envelope scaling	If posited, $\kappa_L(\epsilon)$ yields $P_{\text{extra}} \geq k_{BT} \kappa_L(\epsilon) C$	On a specified family, \dot{C}_{loss}/C has no compatible lower envelope (or smaller floor)
Local proportionality	Turning envelopes into numbers	Plateau of $\dot{C}_{\text{loss}}/C \approx \kappa(\epsilon)$	Construct weak-coherence families and show no plateau after stability checks
E -track sufficiency	Extending law beyond Δ	If identity holds, $P_{\text{extra}} \geq k_{BT} \dot{C}_{\text{loss},E}$	Finite-dimensional counterexample where identity fails and bound becomes invalid/vacuous
Floor ($\omega \simeq 0$)	mechanism Resource horizon	If $\kappa_L(\epsilon) \geq \underline{\kappa}_0 > 0$, then $P_{\text{extra}} \geq k_{BT} \underline{\kappa}_0 C_0$	Engineer DC leak and verify shift in inferred floor; or show no plateau despite strong static suppression

12 Discussion and open problems

- Replace the working definitions of experimental proxies by platform-specific estimators (e.g. shadow tomography for small subsystems, randomized measurements for Hilbert–Schmidt proxies).
- Derive RIP-U and possible floor mechanisms microscopically for membrane-relevant baths and couplings, clarifying whether rates scale as $e^{-m\epsilon}$ or $e^{-2m\epsilon}$.
- Identify conditions ensuring a nontrivial lower envelope $\kappa^\perp(\epsilon)$ on a relevant state family (beyond upper-envelope statements).
- Extend the thermodynamic maintenance inequality from Δ to biology-motivated E by proving or validating the required sufficiency identity in experimentally relevant regimes.

13 Conclusion

We presented a correct-by-construction pipeline connecting static geometric suppression to dynamical maintenance costs without sign errors. Two referee-facing anchors were added: a recoverability layer via CMI and Fawzi–Renner [1], and a minimal Davies interface lemma [2, 3] providing a microscopic route to RIP-U. The dependency/falsification matrix makes explicit what is imported, what is assumed, what is testable, and what survives if RIP fails.

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