Surface-Tension Gravity Calibrated by GPS Time Dilation

Christopher C. O'Neill

December 21, 2025

Abstract

We summarize a minimal "surface-tension gravity" model in which (i) a spherically symmetric compression factor $C(r)$ of a space-like medium controls an intrinsic wave speed $v(r)$ for light-like propagation, (ii) local clock rates adjust inversely with $v(r)$ so that measured light speed remains c, and (iii) compression implies density and pressure variations governed by a bulk modulus K. A single GPS gravitational time-dilation datum is used to fix the compression profile parameter α in $C(r) = 1 + \alpha/r$. A coupling constant σ is introduced to fit K so that the model reproduces $g \approx 9.8\,\mathrm{m/s^2}$ at Earth's surface. The resulting model predicts the correct $1/r^2$ scaling of $g(r)$ and yields a parameter-free prediction for the gravitational time-dilation offset at ISS altitude. In later work we promote $C(r)$ to an effective metric with reciprocal time/space scaling; this is required for correct leading-order light-propagation tests (deflection, Shapiro delay), while leaving the GPS clock-ratio calibration unchanged.

1 Most minimal summary (one-page version)

Empirical input

GPS satellites exhibit a well-known gravitational clock-rate offset of about $+45.7\,\mu\mathrm{s/day}$ relative to clocks on Earth's surface (separate from the special-relativistic velocity effect).[1, 2] Define the fractional gravitational rate difference

$$\epsilon_{\mathrm{GPS}} \equiv \frac{\Delta\dot{\tau}}{\dot{\tau}} \approx \frac{45.7 \times 10^{-6}}{86400} \approx 5.289 \times 10^{-10}. \tag{1}$$

Core postulates (minimal)

1. **Compression field:** space/ether has a dimensionless compression factor $C(r)$ with $C(\infty) = 1$.

2. **Grid-frequency law (with $\beta = 2$):** intrinsic propagation speed scales as

$$v(r) \propto C(r). \tag{2}$$

3. **Clock-speed reciprocity:** proper time rate scales inversely with $v(r)$,

$$\frac{d\tau(r)}{dt} \propto \frac{1}{v(r)} \propto \frac{1}{C(r)}. \tag{3}$$

   Hence for two radii r_1, r_2,

$$\frac{\tau(r_2)}{\tau(r_1)} = \frac{C(r_1)}{C(r_2)}. \tag{4}$$

4. **Isotropic compression:** density scales as $\rho \propto C^3$.

5. **Linear bulk modulus:** pressure changes obey $\Delta P \approx K\,\Delta\rho/\rho$.[3]

Compression profile and calibration

Assume the minimal spherically symmetric profile

$$C(r) = 1 + \frac{\alpha}{r}, \qquad C(\infty) = 1. \tag{5}$$

From (4), the GPS datum implies

$$\frac{C(R_E)}{C(r_{\mathrm{GPS}})} - 1 = \epsilon_{\mathrm{GPS}}. \tag{6}$$

Using $R_E = 6.378\,137 \times 10^6$ m and $r_{\mathrm{GPS}} \approx 2.66 \times 10^7$ m gives

$$\alpha \approx 4.44 \times 10^{-3}\,\mathrm{m}. \tag{7}$$

Gravity scaling (Option B)

Option B treats γ as a boundary property and $g(r)$ as a local field determined by the spatial variation of the compression profile. Since

$$\frac{\mathrm{d}C}{\mathrm{d}r} = -\frac{\alpha}{r^2}, \tag{8}$$

any local coupling of the form $g(r) \propto |dC/dr|$ yields

$$\boxed{g(r) = g(R_E)\left(\frac{R_E}{r}\right)^2}, \tag{9}$$

matching the familiar $1/r^2$ scaling.

Fit of K (requires a coupling constant)

To fix an absolute scale, introduce a universal coupling σ (dimensions m^2/kg) such that the surface gravity is generated by a characteristic pressure scale:

$$g(R_E) = \sigma\,\Delta P_{\mathrm{GPS}}. \tag{10}$$

With isotropic compression and linear bulk modulus,

$$\Delta P_{\mathrm{GPS}} \approx K\,(3\epsilon_{\mathrm{GPS}}), \tag{11}$$

so

$$\boxed{K = \frac{g(R_E)}{3\epsilon_{\mathrm{GPS}}\,\sigma}.} \tag{12}$$

For $\sigma = 1\,\mathrm{m}^2/\mathrm{kg}$ and $g(R_E) = 9.806\,65\,\mathrm{m/s}^2$,

$$\boxed{K \approx 6.18 \times 10^9\,\mathrm{Pa}.} \tag{13}$$

ISS prediction (parameter-free once α is fixed)

At ISS altitude $h \approx 400\,\mathrm{km}$ (so $r_{\mathrm{ISS}} = R_E + h$), the model predicts

$$\epsilon_{\mathrm{ISS}} = \frac{C(R_E)}{C(r_{\mathrm{ISS}})} - 1 \approx 4.106 \times 10^{-11} \tag{14}$$

corresponding to about

$$\Delta\tau_{\mathrm{ISS}} \approx \epsilon_{\mathrm{ISS}} \times 86400\ \mathrm{s} \approx 3.55\,\mathrm{\mu s/day} \tag{15}$$

(gravitational component only).

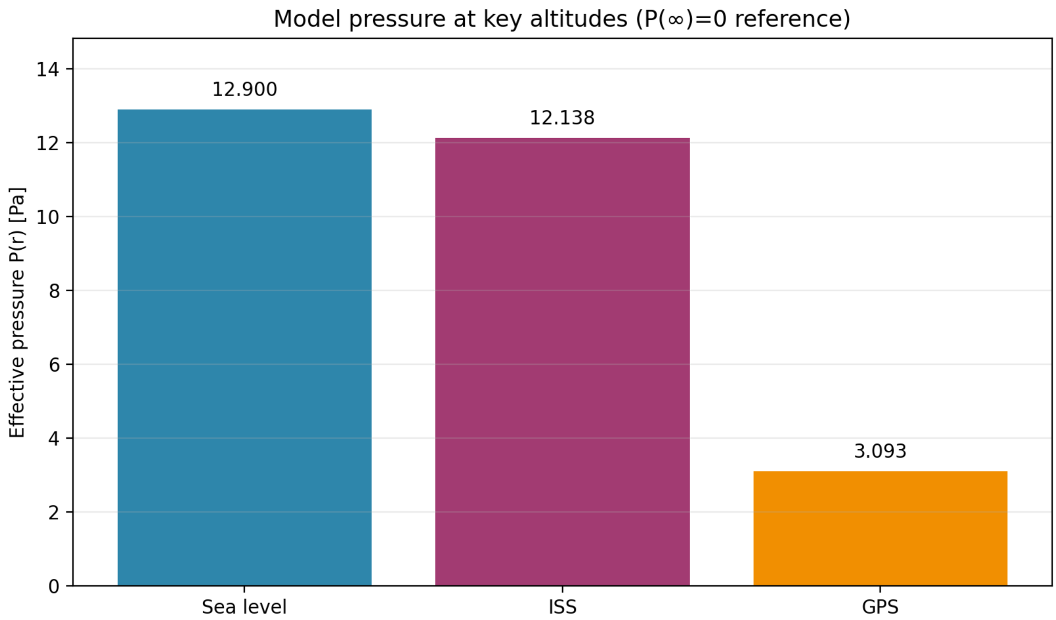

Figure 1: Absolute pressure bar chart.

Boundary surface tension (computed once)

Using Young–Laplace for a spherical interface, $\Delta P = 2\gamma/R_E$,[4, 5] choose a boundary condition $\Delta P_{\text{boundary}} \approx P(R_E)$ with $P(\infty) = 0$. From the bulk-modulus integration,

$$P(r) = 3K \ln C(r), \tag{16}$$

so $P(R_E) \approx 12.90\,\text{Pa}$ and

$$\gamma = \frac{R_E}{2} P(R_E) \approx 4.11 \times 10^7\,\text{N/m}. \tag{17}$$

The corresponding circumference force is $F = \gamma(2\pi R_E) \approx 1.65 \times 10^{15}\,\text{N}$.

2 Full pipeline (detailed and explicit)

2.1 Definitions and parameters

Let r denote radius from Earth's center. Define:

- $C(r)$: dimensionless compression factor of the medium.
- $v(r)$: intrinsic propagation speed in the medium.
- $\tau(r)$: proper time measured by a local clock.
- K: bulk modulus of the medium (assumed constant here).
- $\rho(r)$: effective density of the medium.
- $P(r)$: effective pressure in the medium.
- R_E: Earth reference radius; take $R_E = 6.378\,137 \times 10^6\,\text{m}$.
- r_{GPS}: GPS orbital radius; take $r_{\text{GPS}} \approx 2.66 \times 10^7\,\text{m}$.[1, 2]
- $h_{\text{ISS}} \approx 400\,\text{km}$ so $r_{\text{ISS}} = R_E + h_{\text{ISS}}$.[6]

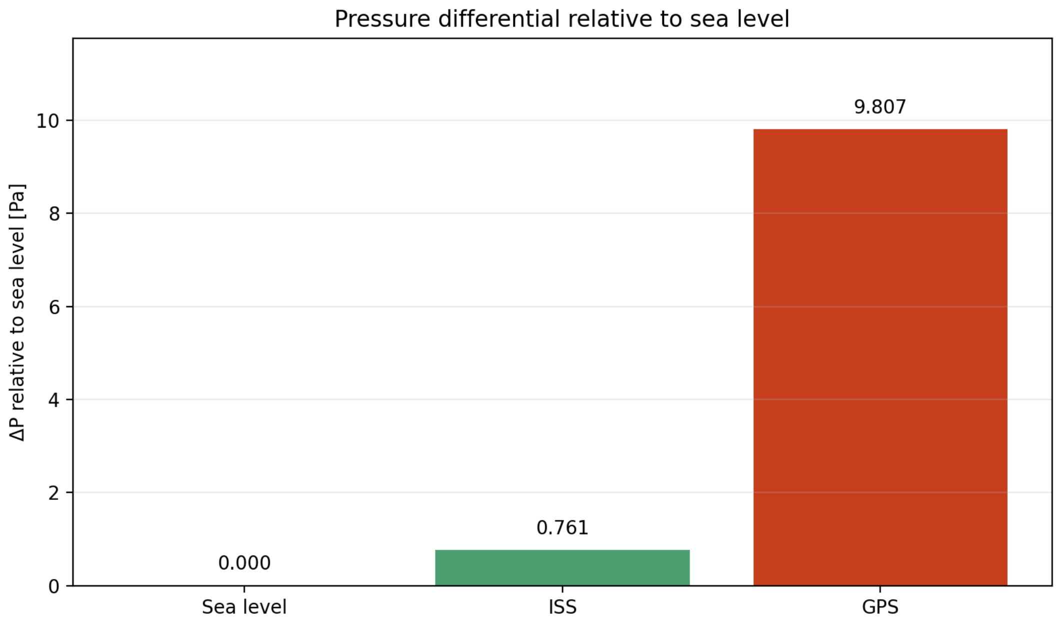

Figure 2: Pressure differential relative to sea level.

2.2 Step 1: Grid-frequency law ($\beta = 2$) and intrinsic speed

Motivating picture: a light-like mode alternates energy between neighboring cells; compressing the cell size increases oscillation frequency. A generalized law is $f \propto C^{\beta}$. Combined with a characteristic step length scaling as $a \propto 1/C$, the intrinsic speed scales as $v \sim af \propto C^{\beta-1}$. Choosing $\beta = 2$ yields Eq. (2).

2.3 Step 2: Clock-speed reciprocity

Assume local clock rates adjust inversely with the intrinsic propagation speed so that observed light speed is fixed:

$$\frac{d\tau(r)}{dt} = \mathcal{N}\,\frac{1}{C(r)}, \tag{18}$$

with a constant normalization $\mathcal{N}$ that cancels in ratios. This yields Eq. (4).

2.4 Step 3: Extract a compression profile from GPS

Use the GPS gravitational clock offset +45.7 µs/day (gravitational component).[1, 2] This implies

$$\frac{\tau(r_{\text{GPS}})}{\tau(R_E)} \approx 1 + \epsilon_{\text{GPS}}, \qquad \epsilon_{\text{GPS}} \approx 5.289 \times 10^{-10}. \tag{19}$$

By Eq. (4),

$$\frac{C(R_E)}{C(r_{\text{GPS}})} \approx 1 + \epsilon_{\text{GPS}}. \tag{20}$$

Assume the minimal one-parameter spherically symmetric profile Eq. (5). Solve

$$\frac{1 + \alpha/R_E}{1 + \alpha/r_{\text{GPS}}} - 1 = \epsilon_{\text{GPS}} \tag{21}$$

to obtain

$$\alpha \approx 4.44 \times 10^{-3}\,\text{m}. \tag{22}$$

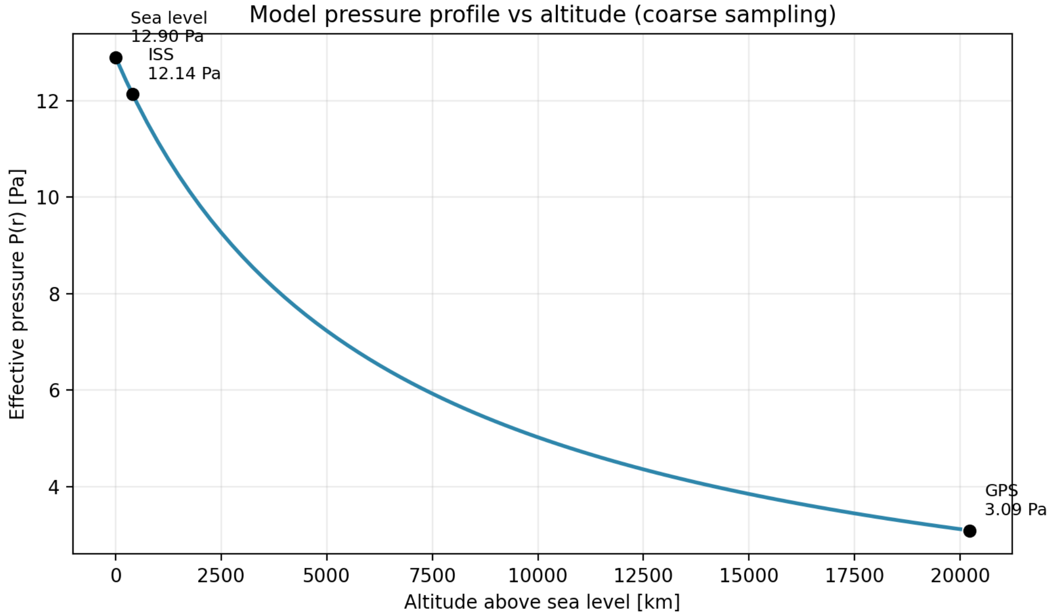

Figure 3: Pressure profile vs altitude from 0 to GPS altitude.

2.5 Step 4: Predict time dilation at a new altitude (ISS)

Given α and Eq. (5),

$$\epsilon(r) \equiv \frac{\tau(r)}{\tau(R_E)} - 1 = \frac{C(R_E)}{C(r)} - 1. \tag{23}$$

At $r = r_{\text{ISS}} = R_E + 400\,\text{km}$,

$$\epsilon_{\text{ISS}} \approx 4.106 \times 10^{-11} \quad \Rightarrow \quad \Delta\tau_{\text{ISS}} \approx \epsilon_{\text{ISS}} \times 86400 \text{ s} \approx 3.55\,\mu\text{s/day}. \tag{24}$$

2.6 Step 5: Compression → density

Assume isotropic compression in three spatial dimensions:

$$\rho(r) = \rho_0\, C(r)^3. \tag{25}$$

For small differences between two radii, $\Delta\rho/\rho \approx 3\,\Delta C/C$.

2.7 Step 6: Density → pressure via bulk modulus

Using the linear bulk-modulus relation for small changes,[3]

$$\Delta P \approx K\,\frac{\Delta\rho}{\rho}. \tag{26}$$

Differentially, $dP = K\,d\rho/\rho = 3K\,dC/C$, which integrates to Eq. (16) with $P(\infty) = 0$.

2.8 Step 7: Fit K using $g(R_E)$ (introducing σ)

To relate the medium pressure scale to observed surface gravity, introduce a universal coupling parameter σ with dimensions m^2/kg:

$$g(R_E) = \sigma\,\Delta P_{\text{GPS}}, \tag{27}$$

where ΔP_{GPS} is the pressure difference implied between R_E and r_{GPS}. Using the small-change relation with isotropic compression,

$$\Delta P_{\mathrm{GPS}} \approx K\,(3\epsilon_{\mathrm{GPS}}). \tag{28}$$

Substituting into Eq. (27) yields Eq. (12). For $\sigma = 1\,\mathrm{m^2/kg}$,

$$K \approx 6.18 \times 10^9\,\mathrm{Pa}. \tag{29}$$

2.9 Step 8: Boundary surface tension (Young–Laplace) and force

Apply the Young–Laplace relation for a spherical interface,[4, 5]

$$\Delta P_{\mathrm{boundary}} = \frac{2\gamma}{R_E}. \tag{30}$$

A minimal boundary condition is to take the pressure inside the massive body as the reference ("shielded") level $P_{\mathrm{in}} \approx 0$ in the gauge where $P(\infty) = 0$, so $\Delta P_{\mathrm{boundary}} \approx P(R_E) = 3K \ln C(R_E)$. Then

$$\gamma = \frac{R_E}{2} P(R_E) \approx 4.11 \times 10^7\,\mathrm{N/m}. \tag{31}$$

The corresponding force around the boundary (circumference $L = 2\pi R_E$) is

$$F = \gamma L = \gamma\,(2\pi R_E) \approx 1.65 \times 10^{15}\,\mathrm{N}. \tag{32}$$

2.10 Step 9 (Option B field scaling): recovering $1/r^2$

With Eq. (5),

$$\frac{\mathrm{d}C}{\mathrm{d}r} = -\frac{\alpha}{r^2}. \tag{33}$$

If gravitational acceleration is locally proportional to the gradient magnitude, $g(r) = \kappa\,|dC/dr|$, then

$$g(r) = g(R_E)\left(\frac{R_E}{r}\right)^2, \tag{34}$$

where κ cancels after fixing $g(R_E)$. At ISS altitude this gives $g_{\mathrm{ISS}} \approx 8.68\,\mathrm{m/s^2}$.

3 Notes, limitations, and next tests

- The pipeline above is deliberately minimal: one GPS datum fixes α; one coupling choice (σ) fixes K; Option B yields the correct $1/r^2$ scaling.
- The boundary condition for $\Delta P_{\mathrm{boundary}}$ controls the absolute magnitude of γ and F but does not affect altitude scaling predictions for time dilation.
- Next empirical checks: apply the same $C(r)$ to other satellite altitudes (LEO, GEO) and compare predicted gravitational clock offsets; compare predicted $g(r)$ scaling to measured g; and test whether a single σ can remain universal across materials.

References

[1] Kreier, "Relative time for GPS satellites," `https://kreier.github.io/gps-relativity/` (accessed 2025-12-21). https://www.gpsworld.com/inside-the-box-gps-and-relativity/

[2] P. Misra, "Inside the box: GPS and relativity," *GPS World* (Oct 9, 2023). `https://www.gpsworld.com/inside-the-box-gps-and-relativity/` (accessed 2025-12-21).

[3] Engineering ToolBox, “Bulk Modulus and Fluid Elasticities,” https://www.engineeringtoolbox.com/bulk-modulus-elasticity-d_585.html (accessed 2025-12-21).

[4] Wikipedia contributors, “Young–Laplace equation,” https://en.wikipedia.org/wiki/Young%E2%80%93Laplace_equation (accessed 2025-12-21).

[5] L. M. Siqveland and S. M. Skjæveland, “Derivations of the Young–Laplace equation,” *Capillarity* 4(2):23–30 (2021). https://userpages.umbc.edu/~rostamia/2024-01-math423/2021-Derivations_of_the_Young-Laplace_equation.pdf (accessed 2025-12-21).

[6] E. Howell, “International Space Station: Everything you need to know,” *Space.com* (last updated May 14, 2025). https://www.space.com/16748-international-space-station.html (accessed 2025-12-21).