Dimensional Sufficiency and the Geometric Origin of Black Hole Singularities

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Abstract

Classical general relativity predicts that gravitational collapse generically produces spacetime singularities, commonly interpreted as physical points of divergent curvature. This work proposes a complementary, purely geometric reinterpretation: singularities may signal *dimensional insufficiency* in the manifold used to represent extreme gravitational configurations. Motivated by a high-dimensional packing example in which objects satisfying local tangency constraints can exceed a global bounding volume, the singularity is interpreted as the point where the assumed geometric "container" fails to encode a finite interior configuration without divergence. A minimal toy model is then constructed in which gravitational compression is balanced by an effective *dimensional geometric pressure* associated with activating additional internal geometric capacity at high density. The balance yields an emergent finite core scale, naturally replacing a point singularity with a stabilized radius while leaving the conservative possibility that the exterior remains well described by standard general relativity.

I. INTRODUCTION

Under broad conditions, classical general relativity predicts the formation of incomplete geodesics and divergent curvature invariants in gravitational collapse [1–3]. The singularity theorems establish this outcome with remarkable generality, yet they do not determine the physical meaning of the resulting divergence. A standard reading is that a singularity marks missing microphysics and motivates an ultraviolet completion.

This paper explores a complementary, conservative viewpoint: a point-like singularity can be interpreted as an artifact of attempting to encode a finite mass-energy configuration within a geometric representation of insufficient effective dimensional capacity. In this language, the singularity is not treated as a physical object, but as a signal that the assumed spacetime "container" has been pushed beyond what it can faithfully represent.

The aim is intentionally narrow and PRD-style. No claim is made of a complete dynamical theory of dimensional flow, nor of an exact interior metric. Instead, a geometric principle is articulated, an explicit high-dimensional analogy is provided, and a minimal toy model is constructed to show how a finite core scale can emerge from a balance between gravitational compression and an effective geometric resistance.

II. GEOMETRIC MOTIVATION FROM HIGH-DIMENSIONAL PACKING

High-dimensional geometry exhibits cases where local constraints do not guarantee global containment. A concrete illustration places unit balls at the vertices of a hypercube and inscribes an inner ball tangent to all corner balls. In ten dimensions, a hypercube of side length 2 has $2^{10} = 1024$ vertices. With unit 10-balls centered at these vertices, the inner 10-ball tangent to all of them has radius

$$R_{10} = \sqrt{10} - 1 \approx 2.16, \tag{1}$$

which exceeds the half-width of the hypercube. Consequently, although the inner ball satisfies all local tangency constraints, it protrudes outside the bounding box that contains the corner unit balls [4].

The qualitative lesson used here is simple:

> Local geometric constraints can imply global geometric extents that exceed the bounds of an assumed container.

The working analogy is that collapse may impose local geometric constraints that cannot be accommodated within a fixed effective dimensionality without producing divergences.

III. SINGULARITIES AS FAILURES OF DIMENSIONAL CONTAINMENT

In gravitational collapse, increasing density and curvature act as progressively stronger constraints on the interior geometry. When represented on a smooth four-dimensional manifold with fixed effective dimensionality, these constraints can drive invariants to diverge, producing the classical singularity. The present interpretation is that the divergence indicates a failure of geometric containment: the configuration implied by collapse cannot be encoded within the assumed container without encountering divergence.

This proposal aligns naturally with broader research suggesting that effective spacetime dimension can be scale dependent in quantum-gravity-inspired settings, for example via spectral-dimension flows [5, 6]. The emphasis here is narrower: even without committing to a specific microscopic mechanism, allowing the interior to respond through additional geometric capacity leads to a finite core scale as a generic outcome.

IV. A MINIMAL TOY MODEL

A. Finite density and dimensional volume

Consider an interior region characterized by an effective spatial dimension d (with book-keeping spacetime dimension $D = d + 1$). Let total mass M be distributed uniformly inside an effective d-ball of radius R. The d-dimensional volume is

$$V_d(R) = \Omega_d R^d, \qquad \Omega_d \equiv \frac{\pi^{d/2}}{\Gamma\left(\frac{d}{2}+1\right)}. \tag{2}$$

The uniform density is therefore

$$\rho_d(R) = \frac{M}{V_d(R)} = \frac{M}{\Omega_d R^d}. \tag{3}$$

Assume that there exists a maximal admissible density $\rho_{\max}$ set by microphysics (Planckian, vacuum-stiffness, or analogous). The condition $\rho_d(R) \le \rho_{\max}$ implies a minimal radius

$$R \ge R_{\min}(d) \equiv \left(\frac{M}{\Omega_d \rho_{\max}}\right)^{1/d}. \tag{4}$$

Equation (4) already shows the basic dimensional-sufficiency effect: increasing d reduces the severity of compression required to remain below a fixed density ceiling, favoring a finite-radius interior rather than a point.

B. Effective pressures and stabilization

A density bound alone does not specify the mechanism that enforces it. The model therefore introduces an effective inward "gravitational pressure" P_{grav} associated with self-compression, and an effective outward "geometric pressure" P_{geom} associated with activating additional internal geometric capacity as the configuration approaches the regime where a fixed-dimensional description would otherwise diverge.

The following subsection defines minimal scaling forms and uses them to identify an emergent stabilization scale.

C. Dimensional geometric pressure balance

An instructive scaling for the inward effective pressure is obtained by dimensional analysis of a self-gravitating configuration in d spatial dimensions. Up to dimensionless coefficients,

$$P_{\rm grav}(R) \sim \frac{G\,M^2}{R^{d+1}}, \tag{5}$$

where G denotes an effective gravitational coupling relevant for the interior regime. This scaling captures the expected growth as R decreases.

For the outward geometric response, assume that once $\rho_d(R)$ approaches $\rho_{\rm max}$ the geometry develops an effective stiffness characterized by a dimensionless coefficient κ, such that

$$P_{\rm geom}(R) \sim \kappa\,\rho_{\rm max}\,R^{d-1}. \tag{6}$$

Equation (6) is a minimal phenomenological choice: it encodes that the resistance associated with geometric capacity increases and prevents unbounded compression, without committing to a specific ultraviolet completion.

Define the stabilized core scale by the equilibrium condition

$$P_{\rm grav}(R_{\rm core}) = P_{\rm geom}(R_{\rm core}). \tag{7}$$

Using Eqs. (5) and (6) yields

$$R_{\rm core} \sim \left(\frac{G\,M^2}{\kappa\,\rho_{\rm max}}\right)^{\frac{1}{2d}}. \tag{8}$$

Equation (8) is the central toy-model result: a finite core radius emerges as a geometric fixed point of pressure balance. In this interpretation, the classical singularity is replaced by a stabilized interior core characterized by an emergent length scale.

V. AN INTERNAL "DIMENSIONAL SCHWARZSCHILD" SCALE

The exterior Schwarzschild radius sets the classical horizon scale in the standard vacuum solution. The present framework introduces a distinct internal scale: the stabilized core radius $R_{\rm core}$. For conceptual clarity, define an internal effective gravitational scale by

$$r_s^{(\rm int)} \equiv R_{\rm core}, \tag{9}$$

which acts as a proxy for the radius below which a fixed-dimensional classical description becomes insufficient.

This separation of scales allows a conservative physical reading: the exterior geometry may remain well described by standard general relativity, while the interior reorganizes through an effective dimensional response that prevents divergence.

VI. DISCUSSION AND OUTLOOK

The proposed picture reframes the meaning of the classical singularity. Rather than a physical point of infinite density, the singularity is interpreted as a diagnostic of dimensional insufficiency: the interior configuration implied by collapse cannot be represented within a fixed effective dimensionality without loss of regularity. A high-dimensional packing example illustrates how local constraints can imply global extents that escape an assumed bounding volume [4]. The toy model then shows that once an effective geometric resistance is included, a finite stabilization scale arises as an equilibrium rather than as an imposed cutoff.

This viewpoint is compatible with broader indications of scale-dependent effective dimension in quantum-gravity-inspired settings [5, 6]. It also resonates with studies of regular black holes and finite-core structures [7, 8]. The present contribution is deliberately limited: it isolates a geometric principle (dimensional sufficiency) and demonstrates, at toy-model level, how it yields an internal stabilization scale without requiring a specific microscopic mechanism.

Future work may proceed by: (i) constructing explicit interior metrics consistent with the pressure-balance principle; (ii) modeling an effective dimensional profile $d(r)$ that interpolates between an external four-dimensional regime and an internal higher-capacity core; and (iii) exploring potential implications for black-hole thermodynamics and dynamical processes.

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