The Rate Inheritance Principle: From Static Correlations to Dynamical Decoherence Rates

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Editorial status: companion note. Theoretical framing note (RIP) of the coherence-maintenance series. The exact rate-inheritance law is ai.viXra:2512.0064 (v2) [3]; the Davies-class stress test is ai.viXra:2512.0070 (v2) [4]; the work theorem is ai.viXra:2512.0061 (v2) [2]; the closure map is ai.viXra:2512.0071 (v2) [5]. All papers, verification scripts, and reference logs are distributed in a single companion repository; this note's figure script lives in `verification/0072`.

Abstract

In gapped open quantum systems with localized couplings, static correlations across an operational interface of width ϵ are exponentially suppressed by the mass gap. Independently, the energetic cost of maintaining quantum coherence is governed by the rate at which coherence is lost under uncontrolled dynamics. The Rate Inheritance Principle (RIP) is the hypothesis connecting the two: effective coherence-loss rates inherit the suppression envelope of static correlations across the interface. We distinguish a weak upper-envelope form — stated here as a short proved lemma under explicit integrability hypotheses — from a stronger envelope-class conjecture, and we record what is now known: the strong form is *derived, with a frequency-resolved exponent, in an exactly solvable quasi-free local-sink class*, where the rate inherits the *square* of the static amplitude envelope, $\kappa(\epsilon) \propto e^{-2q(\omega_b)\epsilon}$ — confirming the squared-envelope caveat anticipated in v1 — and it *fails through near-zero-frequency channels within the secular Davies model class*, whose delocalized jump operators sustain a persistent rate floor. Version 1's surrogate decay-curve evidence is withdrawn: it belonged to the uncontrolled proxy class whose failure mode was exposed by the critical-point control of the companion paper, and it is replaced here by the exact Liouvillian-rapidity evidence. Failure modes (now including secular delocalization), a proxy-validated falsification protocol, and the conditional operational consequences for the quantum–classical resource boundary are stated with explicit scope.

1 Preliminaries and definitions

We consider a finite-dimensional quantum system with Hamiltonian $H_S = \sum_n E_n \Pi_n$ undergoing uncontrolled Markovian evolution generated by a Lindbladian $\mathcal{L}_\epsilon$. Logarithms are natural (nats).

Energy pinching. $\Delta[\rho] := \sum_n \Pi_n \rho \Pi_n$.

Coherence functional. For faithful states ρ (or with the standard support convention), $C(\rho) := S(\rho \| \Delta[\rho])$, with $S(\rho\|\sigma) := \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$.

Coherence-loss rate. For $\rho_t = e^{t\mathcal{L}_\epsilon}(\rho)$, $\dot{C}_{\mathrm{loss}}(\rho) := -\frac{d}{dt} C(\rho_t)\big|_{t=0}$.

Weak-coherence regime. A family of states lies in the weak-coherence regime if for every $\delta > 0$ there exists $C_0(\delta) > 0$ such that $C(\rho) \le C_0(\delta)$ implies $|\dot{C}_{\mathrm{loss}}(\rho) - \kappa(\epsilon)\, C(\rho)| \le \delta\, C(\rho)$, uniformly over the family. (This is a *two-sided* rate-proportionality assumption: it asserts that the channel with rate $\kappa(\epsilon)$ is the one actually seen by the states of interest. The ceiling/floor distinction of [4] (v2) makes precise what can go wrong when it is dropped.)

2 The missing dynamical bridge

Static clustering in gapped systems and thermodynamic lower bounds on coherence maintenance are independently established. Static correlators across an interface of width ϵ obey bounds of the form

$$\text{correlations} \lesssim f(\epsilon), \qquad f(\epsilon) \sim \mathrm{poly}(m\epsilon)\, e^{-m\epsilon} \text{ or } K_\nu(m\epsilon), \tag{1}$$

with the lattice and continuum instances developed in [1]. Independently, maintenance costs satisfy $P_{\mathrm{extra}}(\rho) \geq k_{\mathrm{B}} T\, \dot{C}_{\mathrm{loss}}(\rho)$ under an explicit operational control model — in the corrected form of [2] (v2): unconditionally for the total power via the entropy production rate; against thermodynamically efficient population baselines for the extra power; and with no further assumptions when the pinched target is stationary (pure dephasing), the case relevant below. What remains is how the effective rate $\kappa(\epsilon)$ depends on ϵ. RIP isolates this step.

3 The Rate Inheritance Principle

3.1 Effective coherence-loss rate

We define the effective rate as an envelope over microscopic Davies rates,

$$\kappa(\epsilon) := \max_{\omega} \|\Gamma(\omega;\epsilon)\|, \tag{2}$$

where $\Gamma(\omega;\epsilon) = [\Gamma_{\alpha\beta}(\omega;\epsilon)]$ is the Hermitian Kossakowski matrix at Bohr frequency ω.

Remark 3.1 (This is a ceiling). $\kappa(\epsilon)$ bounds the fastest channel. Upper-envelope statements (Lemma 3.2) are therefore statements about κ; conclusions about maintenance *costs* additionally need the two-sided weak-coherence regime above, or a floor envelope in the sense of [4] (v2). v1 did not draw this distinction; the series now does, and this note inherits it.

3.2 Weak form (upper envelope): a lemma

Lemma 3.2 (Rate inheritance, weak form). *Let the Davies rates be $\Gamma_{\alpha\beta}(\omega;\epsilon) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \langle B_\alpha(t;\epsilon) B_\beta(0;\epsilon)\rangle$, where $B_\alpha(\cdot;\epsilon)$ enters a system–bath coupling whose support is separated from the system by at least ϵ. If*

$$|\langle B_\alpha(x,t) B_\beta(y,0)\rangle| \leq g_{\alpha\beta}(t)\, f(\epsilon), \qquad |x-y| \geq \epsilon, \tag{3}$$

with $\int dt\, |g_{\alpha\beta}(t)| < \infty$ uniformly over the regime of interest, let $G_{\alpha\beta} := \int dt\, |g_{\alpha\beta}(t)|$. Then, in the Markovian regime,

$$\|\Gamma(\omega;\epsilon)\| \;\leq\; \|G\|\, f(\epsilon) \quad \textit{for every } \omega, \qquad \textit{hence} \qquad \kappa(\epsilon) \;\leq\; \|G\|\, f(\epsilon). \tag{4}$$

Proof. Entrywise, $|\Gamma_{\alpha\beta}(\omega;\epsilon)| \leq G_{\alpha\beta}\, f(\epsilon)$ for every ω. The operator norm of a matrix is bounded by the operator norm of any entrywise majorant (for the Hermitian matrices at hand, via $\|A\| \leq \||A|\|_{\mathrm{entrywise}}$ applied through the Schur test), so $\|\Gamma(\omega;\epsilon)\| \leq \|G\|\, f(\epsilon)$ uniformly in ω; take the maximum over ω. $\square$

Remark 3.3. The content is in the hypotheses, not the arithmetic: uniform time integrability fails for critical baths (Section 5), and the secular construction itself can decouple ϵ from the operative channel (Section 5, item 5). For gapped baths, $|\langle B(x)B(y)\rangle| \leq A\,\mathrm{poly}(m|x-y|)\, e^{-m|x-y|}$ [1] supplies f; the Markovian validity must itself hold uniformly in ϵ, which we state as an explicit standing assumption.

3.3 Strong form (envelope class): status updated

Conjecture 3.4 (Rate inheritance, strong form)**.** *If a single decoherence channel dominates and no symmetry-induced cancellations occur, $\kappa(\epsilon) \asymp f(\epsilon)$ up to polynomial prefactors — with the amplitude-mediated variant $\kappa(\epsilon) \asymp f(\epsilon)^2$ when ϵ separates the system from the dissipator through a coherent buffer (so that ϵ suppresses coupling* amplitudes *rather than bath* correlators*).*

Remark 3.5 (What is now known (July 2026)).

- *Derived, quasi-free local-sink class* [3]: for a sub-gap probe coupled through a gapped quasi-free buffer of width ϵ to a strictly local Markovian sink, the exact Liouvillian rapidity obeys $\kappa(\epsilon) = P(\epsilon)\, e^{-2q(\omega_b)\epsilon}$ with $\cosh q(\omega) = (\mu^2+4-\omega^2)/(4\mu)$, verified against exact spectra to four decimal places over ten decades (Fig. 1). This *is* the amplitude-mediated variant of Conjecture 3.4: the rate inherits the square of the evanescent amplitude, precisely the caveat v1 recorded. The exponent is frequency-resolved, closes at the band edge, and vanishes in-band and at criticality.

- *Failure realized, Davies model class* [4]: with near-zero-Bohr-frequency channels, a projected Dirichlet-form rate floor persists under separation. The finite-size secular construction is, however, nonlocal (the $\omega \simeq 0$ jump operators carry $\sim 90\%$ of their weight beyond the coupling site), so this realizes the failure *within* the Davies model class, bracketing the physics against the local-sink derivation rather than contradicting it.

- *Open*: the interacting gapped buffer with strictly local dissipation, stated as an explicit conjecture in [3].

4 Exact evidence (replacing v1's surrogate)

Remark 4.1 (Withdrawal of v1's surrogate evidence). v1 supported RIP with a decay-curve proxy $\kappa_{\text{eff}}(\epsilon)$ extracted from quantum-trajectory simulations of a TFIM buffer chain (its Table 1 and a figure that, moreover, shipped as an unresolved placeholder). That extraction belongs to the proxy class whose failure mode was exposed by the critical-point control of [3] (v2): decay-curve and windowed proxies can decay with ϵ for purely kinematic reasons, and v1 ran no gap-removed control. The surrogate evidence is therefore withdrawn — not because it is known wrong, but because it is not known right — and replaced by the exact evidence below, which needs no extraction protocol at all: the rates are Liouvillian eigenvalues.

5 Failure modes

RIP is not universal. Explicit failure modes:

1. Critical baths with algebraic correlations $\langle B(x)B(y)\rangle \sim |x-y|^{-\alpha}$, yielding $\kappa(\epsilon) \sim \epsilon^{-\alpha}$ (and failing the uniform integrability of Lemma 3.2).
2. Non-Markovian regimes (the Davies form itself is invalid).
3. Symmetry-protected or cancellation-dominated channels.
4. Nonlocal couplings.
5. *Secular delocalization (new in v2, realized in [4]):* even for strictly local couplings, the finite-size Davies construction produces delocalized jump operators; near-zero-frequency channels then sustain rate floors that no geometric separation reduces — within that model class. Whether a microscopically local environment reproduces this is exactly the question the secular limit cannot answer at finite size.

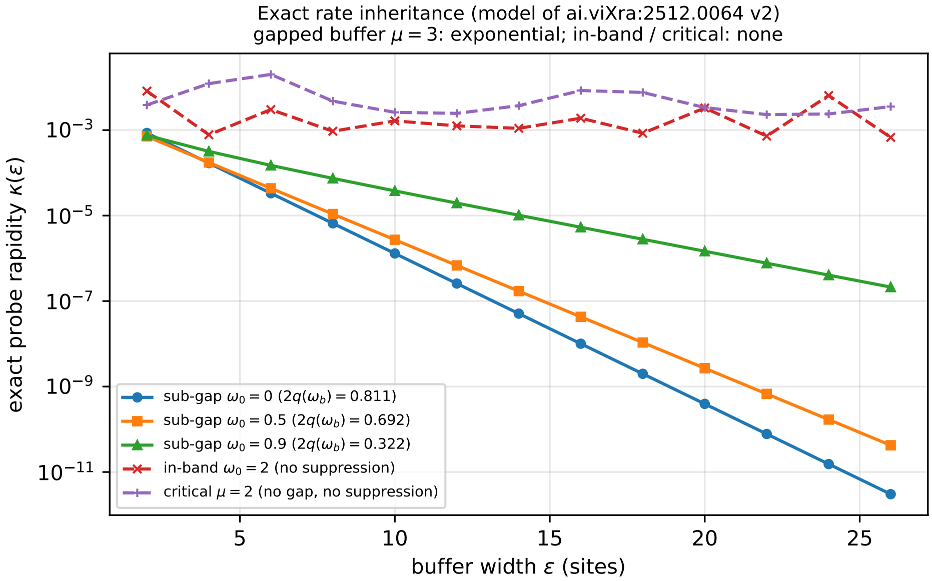

Figure 1: Exact rate inheritance and its boundaries, from the solvable model of [3] (v2) (probe–Kitaev-buffer–local-sink; exact Liouvillian rapidities). Sub-gap probes: exponential suppression with the analytic frequency-resolved slopes $2q(\omega_b)$ shown in the legend. In-band probe and critical buffer: no suppression. The high-precision four-decimal fitted-vs-analytic agreement is reported in [3], Table 1, with its tail-fit protocol; the curves here are the raw exact rapidities. Generated by `verification/0072/make_rip_figure.py` from the `verification/0064` data.

6 Operational consequences (conditional)

Assuming the weak-coherence regime (two-sided rate proportionality) and the maintenance inequality of [2] (v2) — which for pure dephasing holds with no baseline or contraction assumptions — one obtains for any $\delta > 0$:

$$C(\rho) \le C_0(\delta) \;\Rightarrow\; P_{\text{extra}}(\rho) \ge k_B T\,(\kappa(\epsilon) - \delta)\,C(\rho). \tag{5}$$

When rate inheritance holds (quasi-free local-sink class), isolation is exponentially cheap in ϵ and the coherence-lifetime cost is logarithmic [3]; when it fails (persistent floors), one obtains an operational resource horizon [4]. The architectural reading is [5] (v2).

7 Falsification protocol (proxy-validated)

RIP (weak form, as an empirical claim about a platform) is falsified if static correlations decay exponentially while no constants $A, \beta > 0$ exist such that $\kappa(\epsilon) \le A\,\mathrm{poly}(m\epsilon)\,e^{-\beta m\epsilon}$ holds uniformly in a validated gapped Markovian regime — *provided the rate is extracted by a validated protocol*: Liouvillian spectra or fixed-absolute-time exponential fits, with a gap-removed (critical) control and a dissipation-free baseline, never co-moving windows [3]; and with declared operator subspaces if spectral envelopes are used [4]. v1's protocol lacked the proxy-validation clause; the series' own history shows it is not optional.

8 Conclusion

RIP isolates the dynamical bridge between static clustering and decoherence rates. Its weak form is a lemma under explicit hypotheses; its strong form is now derived — with the squaredamplitude, frequency-resolved exponent — in an exactly solvable quasi-free local-sink class, fails through near-zero-frequency channels within the secular Davies class, and remains open precisely for interacting gapped buffers with local dissipation. Combined with the corrected maintenance-power bounds, RIP where valid makes the quantum–classical transition an engineering statement: isolation is exponentially cheap in buffer width, and where RIP fails, coherence has an unavoidable power floor — a resource boundary either way.

A Changes relative to version 1

1. **Surrogate evidence withdrawn** (Remark 4.1): v1's decay-curve proxy table and its broken figure placeholder (`nedladdning.png`, v1's compile-time remark) are replaced by exact Liouvillian-rapidity evidence (Fig. 1) generated by a distributed script from the [3] verification data.

2. **Weak form promoted to Lemma 3.2** with its hypotheses explicit; the ceiling nature of $\kappa(\epsilon)$ is flagged (Remark 3.1).

3. **Strong form status updated** (Remark 3.5): derived in the quasi-free local-sink class with exponent $2q(\omega_b)$ — v1's squared-envelope caveat, confirmed — and failure realized within the Davies class; the interacting-buffer case is delegated to the conjecture of [3].

4. **New failure mode** (secular delocalization, Section 5) and **proxy-validation clause** in the falsification protocol.

5. **Maintenance inequality re-anchored** to [2] (v2) (v1 cited the v1 form, whose extra-power definition was vacuous); references updated to the v2 series with repository paths; [4] and [5] added (absent from v1); the author-profile pointer replaced by the companion repository.

References

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