Operational Coherence Maintenance: Proven Results, Conditional Interfaces, and Open Dynamical Gaps

Lluis Eriksson
Independent Researcher
lluiseriksson@gmail.com

July 4, 2026 (v2; v1: December 2025) ai.viXra:2512.0071

Editorial status: companion note. Architectural/closure overview of the coherence-maintenance series; no new technical results. The series consists of ai.viXra:2512.0060, 0061, 0064, 0070, and the RIP framing note 2512.0072, all at v2 (2026), each with verification material; papers, scripts, and reference logs are distributed in a single companion repository. This note is the map.

Abstract

Maintaining quantum coherence against uncontrolled open-system dynamics is an operational control task with unavoidable thermodynamic cost. In finite dimensions, explicit lower bounds on the minimal power required to stabilize coherence can be derived under standard Markovian assumptions, independently of geometric or field-theoretic structure. At the same time, static correlations in gapped systems are geometrically suppressed, raising the question of how such suppression influences dynamical decoherence rates and, consequently, coherence-maintenance power. Bridging the two domains requires dynamical input that static clustering alone does not provide.

This note introduces no new technical results. It provides a logical closure of the program by separating (i) results proven without additional structure, (ii) conditional interfaces, and (iii) dynamical hypotheses — and, new in this version, it *updates the status of the central hinge*. In v1, rate inheritance — the relation between static correlation envelopes and effective decoherence rates — was identified as the unique unresolved hinge. Since then it has been *partially resolved* in both directions anticipated by v1's scenario analysis: it is now a derived, frequency-resolved law in an exactly solvable quasi-free local-sink class (2512.0064 v2), and the failure scenario through near-zero-frequency channels has been realized within the Davies model class, with the persistent floor computed and the secular nonlocality of that construction quantified (2512.0070 v2). The imported maintenance bound is restated in its corrected v2 form (2512.0061 v2), whose v1 formulation was vacuous. The framework's design goal — robustness under partial refutation — has thus been exercised in practice, twice.

1 Introduction: why a closure note is useful

The loss of quantum coherence under uncontrolled dynamics is a central feature of open quantum systems. Decoherence theory explains passive suppression of interference at the level of reduced dynamics, but is largely silent on a complementary operational question: what control resources are required to *prevent* this loss and maintain a target coherent state in steady time?

Coherence maintenance can be formulated as a thermodynamic control task. Under explicit control models — battery-assisted thermal operations — one derives lower bounds on the minimal power required to counteract uncontrolled dynamics [6]. These bounds depend on dynamical quantities such as the intrinsic coherence-loss rate, and do not rely on spatial structure, field theory, or geometric separation. Independently, static correlations in gapped systems are suppressed with distance, often exponentially [4], with Bessel-type envelopes in continuum massive regimes [5]. It is natural to ask whether geometric suppression of static correlations constrains dynamical decoherence rates and the power required to maintain coherence across an operational interface.

Five manuscripts address different parts of this question: algebraic kinematics and collar-suppressed correlations [5], operational maintenance-power bounds [6], exact dynamical rate inheritance and its proxy pathologies [7], a Davies-class stress test with ceiling/floor envelopes [8], and the formal statement of the rate-inheritance principle itself, with its proxy-validated falsification protocol [9]. The purpose of this note is to make the logical structure explicit — what is proved, what is conditional, what remains open — and, in this version, to record how the open hinge of v1 has since moved.

Remark 1.1 (What changed since v1, in one paragraph). Every paper in the series was revised in 2026 after an exact-verification campaign, and this note inherits those corrections: the reconstruction theorem of [5] is now stated for conditional reattachment (not the Petz map, which fails marginal preservation for correlated references); the work theorem of [6] corrected a sign error and replaced a vacuous extra-power definition; the dynamical evidence of [7] v1 was withdrawn and replaced by an exact law; and the envelope of [8] v1 was split into a ceiling and the floor that a no-go actually needs. The architecture of v1 of this note survived all four revisions unchanged — which is what it was for — but its statements of the imported results and of the status of the hinge did not, and are updated below.

2 Layer I: proved core statements (finite-dimensional, model-independent)

Definition 2.1 (Relative entropy, energy pinching, coherence-loss rate)**.** For density operators ρ, σ with $\mathrm{supp}(\rho) \subseteq \mathrm{supp}(\sigma)$, $S(\rho\|\sigma) := \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$ ($+\infty$ otherwise). With $H = \sum_n E_n \Pi_n$, the energy pinching is $\Delta[\rho] := \sum_n \Pi_n \rho \Pi_n$, the coherence is $C(\rho) := S(\rho\|\Delta[\rho])$, and for a Markovian semigroup $\rho_t = e^{t\mathcal{L}}(\rho)$ the loss rate is $\dot{C}_{\mathrm{loss}}(\rho) := -\frac{d}{dt} C(\rho_t)|_{t=0}$.

2.1 Single-law lower bounds on maintenance power

Theorem 2.2 (Imported, corrected form; proved in [6] (v2))**.** *Consider a finite-dimensional system at temperature T undergoing uncontrolled Markovian dynamics with thermal stationary state, controlled by stroboscopic battery-assisted thermal operations. Then:*

*(a) (*Unconditional*) The minimal total maintenance power obeys $P_{\min}(\rho) \geq k_B T\, \sigma(\rho)$, where $\sigma(\rho) \geq 0$ is the entropy production rate of the target, which splits as $\sigma = \dot{F}^{\Delta}_{\mathrm{diag}} + \dot{C}_{\mathrm{loss}}$.*

*(b) (*Coherence power*) Under diagonal contraction (automatic for Davies generators), $P_{\min}(\rho) \geq k_B T\, \dot{C}_{\mathrm{loss}}(\rho)$.*

*(c) (*Extra power*) Against thermodynamically ε-efficient population-maintenance baselines, under population autonomy, $P(s) - P(s_{\mathrm{diag}}) \geq k_B T\, \dot{C}_{\mathrm{loss}}(\rho) - \varepsilon$; in the free-baseline case of [6] — e.g. pure dephasing, where both $\mathcal{L}(\Delta[\rho]) = 0$ and $\Delta[\mathcal{L}(\rho)] = 0$ — one has $P_{\mathrm{extra}}(\rho) = P_{\min}(\rho) \geq k_B T\, \dot{C}_{\mathrm{loss}}(\rho)$ with no further assumptions.*

Remark 2.3 (Correction relative to v1 of this note). v1 imported the bound as: "assume control strategies exist for ρ and for $\Delta[\rho]$; then $P_{\mathrm{extra}}(\rho) \geq k_B T\, \dot{C}_{\mathrm{loss}}(\rho)$." In that generality the statement is *false*: with P_{extra} defined as an infimum over unlinked strategy pairs (as in [6] v1), the infimum is $-\infty$ (battery-burning baselines). The corrected hierarchy is Theorem 2.2(a)–(c); see [6] (v2), whose v2 also corrected the sign convention of work (battery consumption, not increase).

2.2 What these bounds do and do not require

The bounds are purely operational: they depend on a control model and dynamical functionals, not on geometric structure. They remain valid even if all geometric scaling hypotheses fail. Geometry enters only when one asks how $\dot{C}_{\text{loss}}$ or effective rates depend on a separation parameter.

3 Layer II: standard dynamical inputs

In weak-coupling limits with secular approximation, generators take the Davies form [1, 3]; coherence sectors decouple and decay at rates fixed by the generator; detailed balance yields the classical H-theorem for populations (diagonal contraction) [2]. These facts support rate domination in controlled regimes, but do not by themselves imply any dependence on geometric separation — and, as [8] (v2) quantifies, the secular construction has a price: at finite size its jump operators are delocalized, so "local coupling" statements within the Davies class must be interpreted with care.

4 Layer III: the hinge, updated from open to partially resolved

Hypothesis 4.1 (Rate inheritance principle (RIP), v1 formulation)**.** Let $\kappa(\epsilon)$ denote an effective coherence-loss rate associated with an operational separation parameter ϵ in a validated Markovian regime. Rate inheritance holds if there exist $A, \beta > 0$ with $\kappa(\epsilon) \le A\,\mathrm{poly}(m\epsilon)\,e^{-\beta m\epsilon}$ uniformly over the regime of interest.

Remark 4.2 (Status, July 2026)*.* v1 identified Hypothesis 4.1 as the unique unresolved hinge. Its status is now:

- **Derived in an exactly solvable quasi-free local-sink class and verified against exact Liouvillian spectra** [7]: for a sub-gap probe coupled through a gapped quasi-free buffer to a strictly local Markovian sink, the exact Liouvillian rapidity obeys $\kappa(\epsilon) = P(\epsilon)\,e^{-2q(\omega_b)\epsilon}$ with the frequency-resolved Combes–Thomas exponent $\cosh q(\omega) = (\mu^2 + 4 - \omega^2)/(4\mu)$, verified against exact spectra over ten decades; the exponent closes at the band edge and vanishes in-band and at criticality.
- **Failure scenario realized, Davies class** [8]: with near-zero-Bohr-frequency channels, the projected floor envelope persists under separation — v1's "$\omega = 0$ floors" scenario — *within* the secular model class, whose jump operators are delocalized at finite size ($\sim 90\%$ of the zero-frequency component's weight beyond the coupling site); the two results bracket the physics rather than contradicting each other.
- **Open**: the interacting gapped buffer with strictly local dissipation — the one version not implicit in known physics — stated as an explicit conjecture in [7].

Remark 4.3 (Proxy discipline (new in v2))*.* v1 allowed $\kappa(\epsilon)$ to be "any operationally extracted rate proxy consistent with the Markovian description." The series has since learned, at its own expense, that this is too permissive: the windowed proxy of [7] v1 measured arrival kinematics, not gap physics (its evidence was withdrawn after a critical-point control), and the ceiling envelope of [8] v1 could not feed a maintenance no-go (a floor is required). Valid extractions include Liouvillian rapidities, fixed-absolute-time exponential fits with gap-removed controls and dissipation-free baselines, and projected Dirichlet-form floors with declared operator subspaces [7, 8]; the protocol-level codification of this discipline is [9] (v2).

Remark 4.4 (Falsification criterion, sharpened). Hypothesis 4.1 is falsified in a given regime if static correlations decay exponentially while no envelope bound on a *validly extracted* $\kappa(\epsilon)$ (Remark 4.3) holds there. All claims of geometric scaling of coherence-maintenance power depend on this hinge, now with its validity domain partially charted.

5 Scenario analysis (exercised, not merely stated)

- **RIP holds in a regime** (now actual for the quasi-free local-sink class): geometric suppression of effective rates combines with Theorem 2.2 to give collar-dependent power envelopes, e.g. $P_{\text{extra}} \gtrsim k_B T\, K_\nu(m\epsilon)\, C^\Delta(\rho)$ under the two-sided interface assumptions of [6] (v2), and a logarithmic isolation cost for coherence lifetime [7].
- **RIP fails through $\omega = 0$ floors** (now realized within the Davies class): maintenance remains costly at any separation; one obtains an operational resource horizon rather than geometric protection [8].
- **Markovianity fails**: the Davies picture is invalid; the maintenance bounds of Theorem 2.2 remain operational but the dynamical model requires revalidation.

The design goal of v1 — robustness under partial refutation — has been exercised twice in practice: the withdrawal of [7] v1's numerical evidence and the vacuity of [6] v1's extra-power definition both forced repairs, and in both cases the operational core (Layer I) survived unchanged while the affected layer was rebuilt with stronger statements.

6 What this program does not claim

This framework does not propose modified dynamics, collapse mechanisms, derivations of the Born rule, or ontological interpretations of the quantum–classical transition. Any notion of a "cut" is operational, defined solely by control feasibility under explicit resource constraints [7].

7 Conclusion

We provided an architectural closure of the coherence-maintenance program, separating proved operational power bounds from conditional dynamical/geometric interfaces and open hypotheses. Coherence maintenance is a well-defined operational resource with unavoidable dynamical cost, independent of geometry. Rate inheritance — v1's unique unresolved hinge — is now derived in an exactly solvable class and verified against exact Liouvillian spectra, realized in its failure scenario within the Davies class, and open precisely for interacting gapped buffers with local dissipation; the proxy-validation discipline that the series acquired along the way is recorded here as part of the program's method.

A Changes relative to version 1

1. Theorem 2.2 restates the imported maintenance bound in the corrected form of [6] (v2); v1's import reproduced the vacuous unlinked-pairs formulation (Remark 2.3).
2. The hinge status is updated from "open" to "partially resolved" (Remark 4.2), with the quasi-free proof and the Davies-class floor realization; the missing case (interacting buffers) is delegated to the conjecture of [7].

3. New proxy-discipline remark (Remark 4.3) distilling the windowed-proxy withdrawal of [7] and the ceiling/floor correction of [8].

4. References updated to the v2 series with companion-repository paths; [8] added (absent from v1); the "core technical reference" author-profile pointer of v1 is replaced by the repository.

5. The RIP framing note [9] (v2) is folded into the series map (editorial box, Section 1, Remark 4.3): it states the weak form of rate inheritance as a lemma under explicit hypotheses and codifies the falsification protocol referenced here.

References

[1] E. B. Davies, *Markovian master equations*, Commun. Math. Phys. **39** (1974), 91–110.

[2] H. Spohn, *Entropy production for quantum dynamical semigroups*, J. Math. Phys. **19** (1978), 1227–1230.

[3] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*, Oxford University Press (2002).

[4] M. B. Hastings and T. Koma, *Spectral gap and exponential decay of correlations*, Commun. Math. Phys. **265** (2006), 781–804.

[5] L. Eriksson, *Clustering, Recovery, and Locality in Algebraic Quantum Field Theory: Quantitative Bounds via Split Inclusions and Modular Theory*, ai.viXra:2512.0060, v2 (2026; v1: 2025). Companion repository, `papers/0060-clustering-recovery`.

[6] L. Eriksson, *The Conditional Maintenance Work Theorem: Operational Power Lower Bounds from Energy Pinching and a Split-Inclusion Blueprint for Type III AQFT*, ai.viXra:2512.0061, v2 (2026; v1: 2025). Companion repository, `papers/0061-maintenance-work`.

[7] L. Eriksson, *The Heisenberg Cut as a Resource Boundary: An Operational Outlook from Coherence Maintenance Costs*, ai.viXra:2512.0064, v2 (2026; v1: 2025). Companion repository, `papers/0064-heisenberg-cut`.

[8] L. Eriksson, *Stress Testing the Rate Inheritance Principle: Spectral Decoherence Rates and an Operational Resource Horizon*, ai.viXra:2512.0070, v2 (2026; v1: 2025). Companion repository, `papers/0070-rate-inheritance`.

[9] L. Eriksson, *The Rate Inheritance Principle: From Static Correlations to Dynamical Decoherence Rates*, ai.viXra:2512.0072, v2 (2026; v1: 2025). Companion repository, `papers/0072-rip-principle`.