Operational Coherence Maintenance: Proven Results, Conditional Interfaces, and Open Dynamical Gaps

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December 2025

Editorial Status: Companion Note

This manuscript is a companion note in the coherence-maintenance program.

Role: Architectural/closure overview (no new technical results).

Core technical reference: For the rigorous finite-dimensional proofs and witness diagnostics, see:

Finite-Dimensional Davies Interface Lemmas and TFIM Witness Tests for the Heisenberg Cut as a Resource Boundary

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Abstract

Maintaining quantum coherence against uncontrolled open-system dynamics is an operational control task with unavoidable thermodynamic cost. In finite dimensions, explicit lower bounds on the minimal power required to stabilize coherence can be derived under standard Markovian assumptions, independently of geometric or field-theoretic structure.

At the same time, many model-specific results indicate that static correlations in gapped systems are geometrically suppressed, raising the question of how such suppression might influence dynamical decoherence rates and, consequently, coherence-maintenance power. Bridging these two domains requires additional dynamical input that is not provided by static clustering alone.

This article does not introduce new technical results. Instead, it provides a logical closure of the coherence-maintenance program by explicitly separating: (i) results that are proven without additional structure, (ii) conditional interfaces that depend on standard open-system dynamics or geometric assumptions, and (iii) open dynamical hypotheses. In particular, we identify *rate inheritance*—the relation between static correlation envelopes and effective decoherence rates—as the unique unresolved hinge on which geometric scaling arguments depend.

By making this logical structure explicit, the framework remains robust under partial refutation: even if specific inheritance hypotheses fail, coherence maintenance remains a well-defined operational resource with unavoidable dynamical costs. The purpose of this paper is architectural rather than technical.

1 Introduction: why a closure note is useful

The loss of quantum coherence under uncontrolled dynamics is a central feature of open quantum systems. While decoherence theory explains passive suppression of interference at the level of reduced dynamics, it is largely silent on a complementary operational question: what control resources are required to prevent this loss and maintain a target coherent state in steady time?

Coherence maintenance can be formulated as a thermodynamic control task. Under explicit control models—such as battery-assisted thermal operations—one can derive lower bounds on the minimal power required to counteract uncontrolled dynamics and stabilize a desired state. These bounds depend on dynamical quantities such as the intrinsic coherence-loss rate, and do not rely on spatial structure, field theory, or geometric separation.

Independently, many results in quantum many-body physics and algebraic quantum field theory establish that static correlations in gapped systems are suppressed with distance, often exponentially (or with Bessel-type envelopes in continuum massive regimes). It is therefore natural to ask whether geometric suppression of static correlations constrains dynamical decoherence rates and the power required to maintain coherence across an operational interface.

A sequence of manuscripts addresses different parts of this question: operational maintenance-power bounds, model-specific geometric suppression, dynamical rate-domination properties, and explicit hypotheses connecting static and dynamical quantities. The purpose of the present note is to make the logical structure explicit: what is proved, what is conditional, and what remains open.

2 Layer I: proved core statements (finite-dimensional, model-independent)

Definition 1 (Relative entropy, energy pinching, and coherence-loss rate)**.** *For density operators* ρ, σ *with* $\mathrm{supp}(\rho) \subseteq \mathrm{supp}(\sigma)$*, the quantum relative entropy is*

$$S(\rho\|\sigma) := \mathrm{Tr}[\rho(\log \rho - \log \sigma)],$$

and is defined as $+\infty$ *otherwise.*

Let the system Hamiltonian have spectral decomposition $H = \sum_n E_n \Pi_n$*. The energy pinching map is*

$$\Delta[\rho] := \sum_n \Pi_n \rho \Pi_n.$$

For a Markovian semigroup $\rho_t = e^{t\mathcal{L}}(\rho)$*, define the coherence*

$$C(\rho) := S(\rho\|\Delta[\rho]),$$

and the instantaneous coherence-loss rate

$$\dot{C}_{\mathrm{loss}}(\rho) := -\left.\frac{d}{dt} C(\rho_t)\right|_{t=0}.$$

2.1 Single-law lower bound on incremental maintenance power

Theorem 1 (Imported result: coherence maintenance power bound)**.** *Consider a finite-dimensional system at temperature* T *undergoing uncontrolled Markovian dynamics* $\rho_t = e^{t\mathcal{L}}(\rho)$ *with a thermal stationary state. Assume that (i) battery-assisted thermal-operation strategies exist that maintain* ρ*, and (ii) strategies exist that maintain* $\Delta[\rho]$*. Then the incremental power required to maintain coherence satisfies*

$$P_{\mathrm{extra}}(\rho) \geq k_{\mathrm{B}} T\, \dot{C}_{\mathrm{loss}}(\rho).$$

Remark 1. *A complete finite-dimensional proof is given in [6].*

2.2 What this bound does and does not require

The bound in Theorem 1 is purely operational: it depends on a control model and a dynamical functional $\dot{C}_{\mathrm{loss}}(\rho)$, but does not require any geometric structure. Consequently, it remains valid even if all geometric scaling hypotheses fail. Geometry only enters when one tries to understand how $\dot{C}_{\mathrm{loss}}(\rho)$ or effective rates depend on a separation parameter.

3 Layer II: standard dynamical inputs (Davies, detailed balance, decoupling)

In weak-coupling limits with secular approximation, generators take the Davies form [1, 2, 3]. Coherence sectors decouple and decay exponentially at rates fixed by the generator. For generators satisfying quantum detailed balance, populations evolve independently of coherences and satisfy a classical detailed-balance H-theorem (diagonal contraction).

These facts support *rate domination* in controlled regimes (e.g. weak coherence), but do not by themselves imply any dependence on geometric separation.

4 Layer III: the unique unresolved hinge (rate inheritance)

Hypothesis 1 (Rate inheritance principle (RIP))**.** *Let $\kappa(\epsilon)$ denote an effective coherence-loss rate associated with an operational separation parameter ϵ in a validated Markovian regime. In Davies-type settings one may take*

$$\kappa(\epsilon) := \max_{\omega} \|\Gamma(\omega;\epsilon)\|,$$

where $\Gamma(\omega;\epsilon)$ is the Hermitian Kossakowski rate matrix at Bohr frequency ω. More generally, $\kappa(\epsilon)$ may be any operationally extracted rate proxy consistent with the Markovian description.

Rate inheritance holds if there exist constants $A, \beta > 0$ such that

$$\kappa(\epsilon) \leq A\,\mathrm{poly}(m\epsilon)\,e^{-\beta m\epsilon}$$

uniformly over the regime of interest.

Remark 2 (Falsification criterion)**.** *Hypothesis 1 is falsified if static correlations decay exponentially while no such envelope bound on $\kappa(\epsilon)$ holds in a validated Markovian regime. All claims of geometric scaling of coherence-maintenance power depend on this hinge.*

5 Scenario analysis (what survives under partial refutation)

Depending on the validity (or failure) of Hypothesis 1, the program yields:

- **If RIP holds in a regime:** geometric suppression of effective rates can be combined with maintenance bounds to yield collar-dependent power envelopes.
- **If RIP fails (e.g. $\omega = 0$ floors):** maintenance remains costly even at large separation; one obtains an operational resource horizon rather than geometric protection.
- **If Markovianity fails:** the Davies picture may be invalid; the maintenance inequality remains operational but requires revalidation of the dynamical model.

6 What this program does not claim

This framework does not propose modified dynamics, collapse mechanisms, derivations of the Born rule, or ontological interpretations of the quantum–classical transition. Any notion of a "cut" is operational, defined solely by control feasibility under explicit resource constraints.

7 Conclusion

We provided an architectural closure of the coherence-maintenance program by separating proved operational power bounds from conditional dynamical/geometric interfaces and open hypotheses. Coherence maintenance emerges as a well-defined operational resource with unavoidable dynamical cost, independent of geometry. Rate inheritance is isolated as the unique unresolved hinge connecting static correlation suppression to dynamical decoherence-rate and power scaling.

References

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