Stress Testing the Rate Inheritance Principle: Spectral Decoherence Rates and an Operational Resource Horizon

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July 4, 2026 (v2; v1: December 2025) ai.viXra:2512.0070

Editorial status: companion note. This is the numerical stress-test companion of the coherence-maintenance series. The rigorous work theorem is ai.viXra:2512.0061 (v2) [7]; the exact rate-inheritance law and the windowed-proxy lessons are ai.viXra:2512.0064 (v2) [8]; the algebraic (Type III) kinematics is ai.viXra:2512.0060 (v2) [6]. All four papers, with their verification scripts and reference logs, are distributed in a single companion repository; this paper's scripts live in `verification/0070`.

Abstract

In gapped quantum many-body systems, static correlations decay exponentially with distance. A common heuristic expectation is that this geometric suppression carries over to dynamical decoherence rates induced by local environments; this expectation has been isolated as the Rate Inheritance Principle (RIP). We stress test RIP in a fully specified Davies-type Markovian setting: a gapped transverse-field Ising chain weakly coupled to a thermal bosonic bath through a strictly local operator. RIP is formulated operatorially through *two* spectral envelopes of the Dirichlet form on operators supported at distance ϵ from the coupling region: a ceiling $\kappa_{\sup}(\epsilon)$ (v1's envelope) and a floor $\kappa_{\perp}(\epsilon)$ (smallest nonzero rate, new in v2 — the quantity a maintenance no-go actually needs; v1's inference from ceiling saturation to a power floor was a non sequitur and is corrected here). Numerically, rate inheritance remains *conditional*: for energy-exchange-dominated coupling the ceiling decreases with separation, while for near-zero-Bohr-frequency coupling it saturates — and, more importantly for the no-go, the *projected* floor also persists in that regime. Version 2 adds the diagnostic that the series' methodology demands: the near-zero-frequency Bohr components of the local coupling are strongly delocalized at finite size ($\sim 90\%$ of their weight beyond the coupling site), so the saturation is established *within the Davies model class*, whose secular construction is nonlocal; the exactly solvable local-sink model of [8] shows genuine geometric suppression, and the two results bracket the physics. Combined with the corrected maintenance bounds of [7] (v2), a persistent projected spectral floor yields a quadratic-proxy resource horizon; the corresponding relative-entropy no-go is stated with its required uniform-$\dot{C}_{\mathrm{loss}}$ / MLSI-type hypothesis explicit.

1 Introduction

Static correlation decay is a hallmark of gapped quantum many-body systems: a spectral gap implies exponential clustering of equal-time correlators [4]. When such systems are coupled to an external environment, the relevant question becomes dynamical: how strongly does local noise affect degrees of freedom supported far away?

The Rate Inheritance Principle (RIP) asserts that dynamical decoherence rates inherit the same envelope class as static correlation bounds. RIP does not follow from static clustering alone and must be tested microscopically. In the quasi-free class it is now a *theorem* with a frequency-resolved exponent [8]; the present note stress tests it in a setting deliberately outside that class's comfort zone: a Davies generator whose bath couples through channels that include near-zero Bohr frequencies.

Version 1 of this note reported conditional inheritance: suppression of an effective rate envelope with distance for energy-exchange coupling, saturation for near-zero-frequency coupling. Version 2 keeps that finding but repairs the logic connecting it to resource conclusions, quantifies a construction-level caveat, and replaces v1's broken figure. Specifically: (i) v1's envelope is a *supremum* over operators — a ceiling; a maintenance no-go needs a *floor*, and ceilings and floors turn out to behave differently (Table 1); (ii) the "saturation" channel is carried by Bohr components of the coupling operator that are strongly *delocalized* at finite size, an intrinsic feature of the secular (Davies) construction whose physical reach must therefore be stated carefully (Section 7) — the same diagnose-the-tool lesson that [8] (v2) applied to its own v1 windowed protocol; (iii) the imported maintenance inequality is re-anchored to the corrected extra-power theorems of [7] (v2).

The point of this note is *not* to refute rate inheritance, but to separate two mechanisms: local-sink, sub-gap propagation, where RIP holds cleanly and exactly [8]; and secular-Davies near-zero-frequency dephasing, where the jump operators are delocalized by construction and the projected floor can persist. These are two distinct modelling limits that bracket the physics — not a contradiction.

2 Model and Davies generator

2.1 System

We consider a spin-$\frac{1}{2}$ chain of length N with open boundary conditions,

$$H_S = -J \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x - h \sum_{i=1}^{N} \sigma_i^z, \tag{1}$$

operated in the paramagnetic gapped phase $h > J$.

2.2 Bath and Davies construction

The environment is a bosonic thermal bath at inverse temperature β, with Ohmic spectral density $J(\nu) = \gamma_0\, \nu\, e^{-\nu/\nu_c}$. The system–bath coupling is strictly local, $H_{SB} = \lambda\, S_{j_0} \otimes B$. In the weak-coupling–secular (Davies) limit [1, 3], the reduced dynamics are generated by a Markovian Liouvillian $\mathcal{L}$ with thermal fixed point $\sigma := e^{-\beta H_S} / \operatorname{Tr}(e^{-\beta H_S})$. Writing $S_{j_0}(\omega)$ for the Bohr-frequency components of the coupling operator, the Davies rates are

$$\gamma(\omega) = \begin{cases} J(|\omega|)\,[1 + n_\beta(|\omega|)], & \omega > 0, \\ J(|\omega|)\, n_\beta(|\omega|), & \omega < 0, \end{cases} \qquad n_\beta(\nu) = \frac{1}{e^{\beta\nu} - 1}, \tag{2}$$

with the KMS relation $\gamma(-\omega) = e^{-\beta\omega}\gamma(\omega)$, and $\gamma(0) := \gamma_0/\beta$. Bohr frequencies within $\Delta\omega$ are grouped into a single secular block, $\Delta\omega$ small compared to the gap. The Davies generator satisfies quantum detailed balance with respect to σ and is KMS-symmetric for the inner product below.

Remark 2.1 (The price of the secular limit). The components $S_{j_0}(\omega)$ are spectral projections of a local operator onto energy-difference shells; *they are not local operators.* At finite N this is not a small effect — Section 7 quantifies it — and the secular limit itself is only controlled on time scales exceeding the inverse minimal Bohr-frequency spacing, which grows with N. All statements of this note are therefore statements about the Davies model class as such.

3 Operatorial formulation: two envelopes

Fix the coupling site j_0, let $d(i,j) = |i-j|$, $R_\epsilon := \{i : d(i,j_0) \geq \epsilon\}$, and let $\mathcal{A}_\epsilon$ denote operators supported on R_ϵ.

Definition 3.1 (KMS inner product and Dirichlet form)**.** For operators A, B: $\langle A, B\rangle_\sigma := \mathrm{Tr}(\sigma^{1/2}A^\dagger\sigma^{1/2}B)$, $\|O\|_{2,\sigma}^2 := \langle O, O\rangle_\sigma$, and $\mathcal{E}_\sigma(O) := -\Re\langle O, \mathcal{L}^\dagger(O)\rangle_\sigma \geq 0$.

Definition 3.2 (Ceiling and floor envelopes)**.** For $\epsilon \geq 0$ define

$$\kappa_{\sup}(\epsilon) := \sup_{0\neq O\in\mathcal{A}_\epsilon} \frac{\mathcal{E}_\sigma(O)}{\|O\|_{2,\sigma}^2}, \qquad \kappa_\perp(\epsilon) := \inf_{\substack{O\in\mathcal{A}_\epsilon \\ O\perp_\sigma \ker}} \frac{\mathcal{E}_\sigma(O)}{\|O\|_{2,\sigma}^2}, \tag{3}$$

where the infimum runs over the orthogonal complement (in $\langle\cdot,\cdot\rangle_\sigma$, within the computational subspace) of the null directions of the form. $\kappa_{\sup}$ is v1's envelope; $\kappa_\perp$ is new.

Definition 3.3 (Projected numerical envelopes)**.** The envelopes above are defined over $\mathcal{A}_\epsilon$. The numerical stress test computes their *projections* to the finite operator subspace $\mathcal{B}_\epsilon \subset \mathcal{A}_\epsilon$ spanned by single-site Pauli operators and nearest-neighbour Pauli pairs on R_ϵ:

$$\kappa_{\sup}^{\mathcal{B}}(\epsilon) := \sup_{0\neq O\in\mathcal{B}_\epsilon} \frac{\mathcal{E}_\sigma(O)}{\|O\|_{2,\sigma}^2}, \qquad \kappa_\perp^{\mathcal{B}}(\epsilon) := \inf_{O\in\mathcal{B}_\epsilon,\ O\perp_\sigma\ker} \frac{\mathcal{E}_\sigma(O)}{\|O\|_{2,\sigma}^2}. \tag{4}$$

Trivially $\kappa_{\sup}^{\mathcal{B}} \leq \kappa_{\sup}$, while $\kappa_\perp^{\mathcal{B}} \geq \kappa_\perp$ means the true floor could in principle be carried by an untested operator; all numerical claims of this note concern the projected envelopes, and tables and figures write $\kappa_{\sup}, \kappa_\perp$ for them with this understanding.

Remark 3.4 (Why the floor is the operative quantity)*.* v1's Remark 5 argued: *if $\kappa(\epsilon)$ saturates, increasing separation does not reduce the minimal required maintenance power.* With $\kappa = \kappa_{\sup}$ that inference is invalid: a saturating *supremum* says only that *some* operator far away still decays fast; the controller pays for the modes its target actually populates, and those could in principle decay arbitrarily slowly. The statement that survives is floor-based: if $\kappa_\perp(\epsilon)$ stays bounded away from zero, then every coherent direction *within the tested operator subspace* at distance ϵ (orthogonal to conserved ones) decays at least at that rate, and the maintenance cost cannot be reduced by separation. Table 1 shows the two envelopes genuinely differ.

4 Mechanisms of inheritance and failure

Decay rates in Davies theory depend on the Bohr components of S_{j_0} and the bath weights $\gamma(\omega)$. When decay in $\mathcal{A}_\epsilon$ requires propagation of gapped excitations, geometric suppression is expected — and in the exactly solvable local-sink quasi-free model of [8] it is a theorem, $\kappa(\epsilon) = P(\epsilon)\, e^{-2q(\omega_b)\epsilon}$. Conversely, when near-zero-Bohr-frequency channels dominate, decoherence can proceed without gap-mediated transport: the $\omega \simeq 0$ component acts as dephasing in the energy eigenbasis, and energy eigenstates of a chain are global objects. In the TFIM, longitudinal coupling $S_{j_0} = \sigma_{j_0}^z$ concentrates spectral weight near $\omega \simeq 0$ in the high-field regime and serves as the proxy for this channel class.

5 Numerical stress test

Exact diagonalization for $N \leq 12$ (production runs below: $N = 6$, all 2^N eigenpairs, no truncation). For each ϵ we take the operator basis $\{O_a\} \subset \mathcal{A}_\epsilon$ of all single-site Pauli operators and all nearest-neighbour Pauli pairs on R_ϵ, form $G_{ab} = \langle O_a, O_b\rangle_\sigma$ and $M_{ab} = -\Re\langle O_a, \mathcal{L}^\dagger(O_b)\rangle_\sigma$, and

ϵ	$S_{j_0} = \sigma^x$ (energy exchange)		$S_{j_0} = \sigma^z$ (near-zero frequency)	
	$\kappa_{\sup}$	$\kappa_\perp$	$\kappa_{\sup}$	$\kappa_\perp$
1	3.68	0.036	1.161	0.173
2	3.04	0.069	1.156	0.177
3	2.58	0.137	1.134	0.180
4	2.35	0.293	1.100	0.112
5	0.89	0.257	0.530	0.347

Table 1: Ceiling and floor envelopes for the two coupling classes (TFIM $N = 6$, $h = 1.5$, gap 1.34; $\epsilon = 5$ edge-dominated). The σ^x ceiling decreases with separation while the σ^z ceiling saturates — v1's finding, reproduced. New in v2: the *floors* behave differently from the ceilings in both cases (Remark 3.4); in the σ^z channel the floor persists at ≈ 0.17, which is what the resource horizon of Section 8 actually requires.

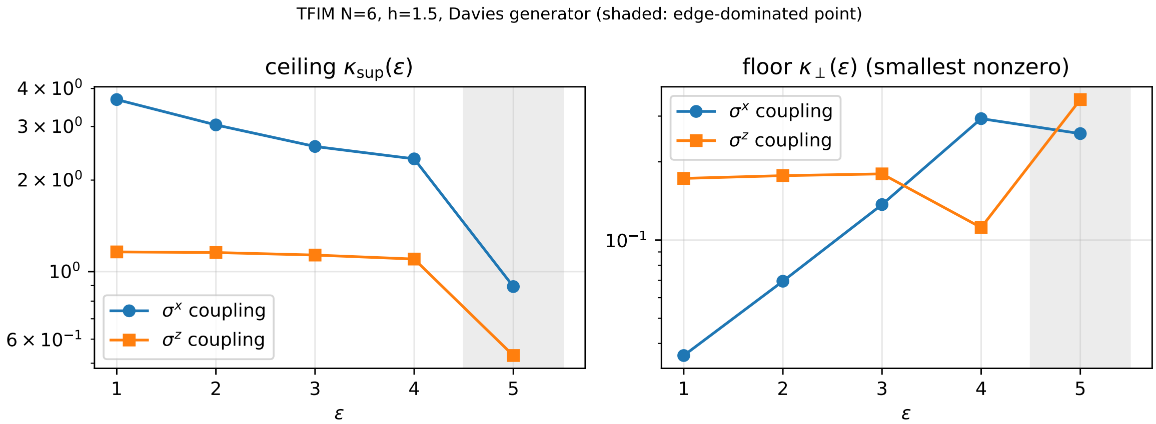

Figure 1: Ceiling (left) and floor (right) envelopes versus separation, from exact diagonalization (data of Table 1; shaded region: edge-dominated point). Generated by `verification/0070/verify_0070.py`; this figure replaces v1's Fig. 1, which shipped as an unresolved placeholder (v1 Remark 2).

obtain $\kappa_{\sup}$ and $\kappa_\perp$ as the largest and smallest-nonzero generalized eigenvalues of $Mv = \kappa Gv$ (kernel directions projected out at relative threshold 10^{-8}). Parameters: $J = 1$, $h = 1.5$, $\beta = 1$, $\gamma_0 = 1$, $\nu_c = 10$, $\Delta\omega = 0.05$, $j_0 = 1$. The script (`verification/0070/verify_0070.py`) and its reference log are distributed with this paper; the last point $\epsilon = N-1$ is edge-dominated (a single site remains) and is shaded in Fig. 1.

Remark 5.1 (Sensitivity; not a thermodynamic-limit claim). The $N = 6$ data are a finite-size diagnostic, not a thermodynamic-limit claim. Robustness checks (distributed with the script): at $N = 5$ the bulk σ^z floor persists at $\kappa_\perp^{\mathcal{B}} \approx 0.19$; doubling the secular grouping to $\Delta\omega = 0.1$ changes the $N = 6$ values by less than 10^{-2} (e.g., $0.1727 \to 0.1734$ at $\epsilon = 1$); relaxing the kernel threshold from 10^{-10} to 10^{-6} changes them by less than 10^{-4}. Proving or falsifying persistence of the floor uniformly in N is left open.

6 Results

For energy-exchange-dominated coupling (σ^x), $\kappa_{\sup}(\epsilon)$ decreases with separation, consistent with geometric suppression (modestly so at $N = 6$; the clean exponential law belongs to the local-sink model of [8], where it is exact). For near-zero-frequency coupling (σ^z), $\kappa_{\sup}(\epsilon)$ saturates — v1's central finding, confirmed. The new floor data sharpen both sides: the σ^z floor persists at $\kappa_\perp \approx 0.17$ across the bulk, upgrading the saturation from a statement about the fastest tested mode to one about every non-conserved mode *within the projected Pauli/nearest-neighbour subspace* (Definition 3.3), which is what the quadratic-proxy no-go consumes; the σ^x floor is small near the coupling and grows with ϵ at this size — the slowest modes live near the sink — illustrating concretely that ceilings and floors need not co-vary, i.e., v1's ceiling-based inference was structurally unsafe even where its conclusion happened to be right.

7 The Davies-locality caveat

The series' own methodology (cf. the withdrawal analysis in [8], v2) requires diagnosing the tool before trusting the physics. Here the tool is the secular construction. We measured the spatial delocalization of the Bohr components of $\sigma^z_{j_0}$: for a Bohr component $S(\omega)$, define its relative weight beyond distance d by

$$w_{>d}(\omega) := \frac{\left\|S(\omega) - \mathbb{E}_{[1,d]}[S(\omega)] \otimes \mathbf{1}_{>d}\right\|_{HS}}{\|S(\omega)\|_{HS}}, \tag{5}$$

where $\mathbb{E}_{[1,d]}$ is the normalized partial trace onto the first d sites. Measured values:

	$d=1$	$d=2$	$d=3$	$d=4$
$\omega \simeq 0$ block	0.90	0.84	0.75	0.64
$\lvert\omega\rvert \simeq 1$ block	0.99	0.94	0.87	0.66

i.e., the Lindblad operators of the finite-size Davies generator are *global*: 90% of the zero-frequency channel's weight lies beyond the site it nominally couples to. Consequently the saturation of Table 1 is established *within the Davies model class*, whose jump operators are delocalized by construction; whether a microscopically local environment reproduces it is exactly the question the secular limit cannot answer at finite N (Remark 2.1). The two poles are now both on record: the exact, strictly-local-sink quasi-free model of [8] obeys clean geometric rate inheritance; the secular Davies model with a near-zero-frequency channel does not. The physical question — which pole a given experimental platform sits near — is a statement about the validity time of the secular approximation for that platform, and we flag it as open rather than resolved.

8 Operational resource horizon (no-go consequence)

Remark 8.1 (Coherence functional). We use energy pinching $\Delta[\rho] := \sum_n \Pi_n \rho \Pi_n$ in the eigenbasis of H_S and $C(\rho) := S(\rho \| \Delta[\rho])$.

Remark 8.2 (Imported maintenance inequality, corrected anchoring). In the battery-assisted thermal-operations framework at temperature T, the extra power of coherence stabilization obeys $P_{\mathrm{extra}}(\rho) \geq k_{\mathrm{B}} T\, \dot{C}_{\mathrm{loss}}(\rho)$, where $\dot{C}_{\mathrm{loss}}(\rho) := -\frac{d}{dt} C(e^{t\mathcal{L}}\rho)|_{t=0}$. v1 imported this from the v1 of [7], whose "assumption-free" formulation was vacuous (unlinked-pair infimum $= -\infty$); the form used here is the corrected v2 statement: against thermodynamically efficient population-maintenance baselines under population autonomy, or unconditionally when the pinched target is stationary; the unconditional total-power floor is $k_{\mathrm{B}} T$ times the entropy production rate.

Theorem 8.3 (Resource-cut no-go under a uniform rate floor)**.** *Fix $C_0 > 0$ and a nonempty family $\mathcal{F}_{C_0}$ of faithful states with $C(\rho) \geq C_0$. If $\inf_{\rho \in \mathcal{F}_{C_0}} \dot{C}_{\mathrm{loss}}(\rho) \geq \dot{C}(C_0) > 0$, then any controller maintaining any $\rho \in \mathcal{F}_{C_0}$ requires $P_{\mathrm{extra}}(\rho) \geq k_{\mathrm{B}} T\, \dot{C}(C_0)$; if $P_{\max} < k_{\mathrm{B}} T\, \dot{C}(C_0)$, sustained maintenance is operationally impossible.*

Proof. Immediate from Remark 8.2 and the hypothesis. □

Proposition 8.4 (Projected spectral floor to quadratic decay)**.** *Let $O = O^\dagger \in \mathcal{B}_\epsilon$ be a traceless coherent perturbation direction, orthogonal to the null space in the KMS inner product and normalized by $\|O\|_{2,\sigma} = 1$. For sufficiently small real u, define*

$$\rho_u := \frac{\sigma^{1/2}(\mathbf{1} + uO)\sigma^{1/2}}{\mathrm{Tr}[\sigma^{1/2}(\mathbf{1} + uO)\sigma^{1/2}]}, \tag{6}$$

and let $C_2(\rho_u)$ denote the KMS-χ^2 coherence proxy: the squared $\|\cdot\|_{2,\sigma}$-norm of the off-diagonal (in the energy eigenbasis) component of the relative perturbation. Then, at $t = 0$,

$$-\frac{d}{dt} C_2\big(e^{t\mathcal{L}}\rho_u\big)\Big|_{t=0} \;\geq\; 2\,\kappa_\perp^{\mathcal{B}}(\epsilon)\, C_2(\rho_u) + O(u^3), \tag{7}$$

provided the coherent component lies in $\mathcal{B}_\epsilon \ominus \ker \mathcal{E}_\sigma$.

Proof. In the KMS representation the perturbation evolves by the (KMS-symmetric) generator, and $\frac{d}{dt}\|O_t\|_{2,\sigma}^2\big|_{t=0} = -2\mathcal{E}_\sigma(O) \leq -2\kappa_\perp^{\mathcal{B}}(\epsilon)\|O\|_{2,\sigma}^2$ by Definition 3.3 for O in the stated subspace; the diagonal component is orthogonal to the off-diagonal one in $\langle\cdot,\cdot\rangle_\sigma$, so the inequality passes to the off-diagonal proxy at $t = 0$. The $O(u^3)$ term collects the normalization and higher-order corrections of the state family. □

Remark 8.5 (Entropic version). Upgrading Proposition 8.4 from the quadratic proxy to the relative-entropy coherence $C(\rho)$ is the standard passage from spectral gap to modified logarithmic Sobolev inequality; positivity of an MLSI constant for the restricted dynamics would give $\dot{C}_{\mathrm{loss}} \geq \alpha\, C$ directly [5]. We do not claim that step here; the no-go (Theorem 8.3) is stated with its hypothesis explicit.

Remark 8.6 (Connection to the envelopes, corrected). When the projected floor persists (as in the σ^z channel of Table 1, within the Davies model class and subject to Section 7), increasing separation does not reduce the minimal required maintenance power in the quadratic sense of Proposition 8.4: a finite resource horizon. When the floor decays (local-sink models, [8]), the horizon recedes exponentially and isolation is cheap. v1 drew this contrast from the ceiling κ_{sup}; the floor makes it sound.

9 Discussion and conclusion

The quantum–classical boundary emerges here as a resource horizon, not a dynamical or ontological transition. Static clustering alone does not guarantee dynamical protection: geometric suppression of decoherence rates occurs when decay requires gap-respecting energy exchange — exactly, in the quasi-free local-sink class [8] — and can fail through near-zero-frequency channels, with the caveat that in the Davies model class those channels are carried by delocalized jump operators whose physical status at finite size is a property of the secular limit (Section 7). Whenever effective rate *floors* persist, finite available power imposes an operational cut beyond which sustained quantum coherence cannot be maintained [7]; the Type III kinematics for formulating such cuts as split inclusions is developed in [6]. This yields an interpretation-free, thermodynamically grounded account of when and why quantum descriptions cease to be physically sustainable in extended systems — with the load-bearing quantities now the right ones.

A Changes relative to version 1

1. **Ceiling vs floor.** v1's $\kappa(\epsilon)$ (a supremum) is renamed κ_{sup}; the floor $\kappa_\perp$ is defined and computed (Definition 3.2, Table 1). v1 Remark 5 inferred a maintenance-power floor

from ceiling saturation; that inference is a non sequitur (Remark 3.4) and is replaced by Proposition 8.4. Empirically the ceilings and floors do behave differently (σ^x: ceiling falls, floor rises), and the σ^z floor persists — so the corrected no-go survives, now on sound footing, within the model class.

2. **Davies-locality caveat.** New Section 7: the near-zero-frequency Bohr components of the local coupling carry $\sim 90\%$ of their weight beyond the coupling site at $N = 6$; the saturation finding is therefore a statement about the (nonlocal) Davies model class, to be contrasted with the exact local-sink suppression of ai.viXra:2512.0064 (v2). This is the same diagnose-the-tool discipline that led 2512.0064 (v2) to withdraw its own v1 figure.

3. **Broken figure fixed.** v1's Fig. 1 shipped as a compile-time placeholder (v1 Remark 2 asked the reader to supply `nedladdning2.png`); replaced by Table 1 and Fig. 1, generated by the distributed script.

4. **Imported inequality re-anchored** to the corrected v2 of ai.viXra:2512.0061 (Remark 8.2); the "core technical reference" pointer of v1 (an author-profile URL) is replaced by the series repository.

5. **Numerical parameters fully declared** (N, h, β, γ_0, ν_c, $\Delta\omega$, basis, thresholds), with script and log distributed; edge-dominated data points marked.

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