Stress Testing the Rate Inheritance Principle: Spectral Decoherence Rates and an Operational Resource Horizon

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This manuscript is a companion note in the coherence-maintenance program.

Role: Numerical companion / stress tests for effective rate envelopes.

Core technical reference: For the rigorous finite-dimensional proofs and witness diagnostics, see:

Finite-Dimensional Davies Interface Lemmas and TFIM Witness Tests for the Heisenberg Cut as a Resource Boundary

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Abstract

In gapped quantum many-body systems, static correlations decay exponentially with distance. A common heuristic expectation is that this geometric suppression carries over to dynamical decoherence rates induced by local environments. This expectation has been isolated as the *Rate Inheritance Principle* (RIP).

We perform a direct stress test of RIP in a fully specified Davies-type Markovian setting. We consider a one-dimensional gapped spin chain weakly coupled to a thermal bosonic bath through a strictly local system operator supported near a site j_0. To avoid ambiguities associated with state preparation, we formulate RIP operatorially: for operators supported at distance ϵ from the coupling region, we define an effective decay-rate envelope $\kappa(\epsilon)$ from the Heisenberg-picture Liouvillian.

Numerical results show that rate inheritance is conditional. In energy-exchange-dominated regimes, $\kappa(\epsilon)$ decreases with separation, consistent with geometric suppression. In contrast, in regimes dominated by near-zero-Bohr-frequency channels, $\kappa(\epsilon)$ saturates with distance despite static clustering. Combined with standard thermodynamic maintenance bounds, these results yield an operational resource horizon: whenever effective rate floors persist under increasing separation, sustained coherence becomes impossible under finite available power.

1 Introduction

Static correlation decay is a hallmark of gapped quantum many-body systems. In one dimension, a spectral gap implies exponential clustering of equal-time correlators [4]. When such systems are coupled to an external environment, however, the relevant question becomes dynamical: how strongly does local noise affect degrees of freedom supported far away?

The *Rate Inheritance Principle* (RIP) asserts that dynamical decoherence rates inherit the same envelope class as static correlation bounds. Importantly, RIP does not follow from static clustering alone and must be tested microscopically.

In this work we stress test RIP using an operatorial formulation based on a Davies generator [1, 2, 3]. Our goal is not to modify quantum dynamics or explain outcome selection, but to identify physical conditions under which sustained quantum coherence ceases to be operationally feasible under finite control resources.

2 Model and Davies generator

2.1 System

We consider a spin-1/2 chain of length N with open boundary conditions,

$$H_S = -J \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x - h \sum_{i=1}^{N} \sigma_i^z,$$

operated in the paramagnetic gapped phase $h > J$.

2.2 Bath and Davies construction

The environment is a bosonic thermal bath at inverse temperature β, with Ohmic spectral density

$$J(\nu) = \gamma_0 \nu e^{-\nu/\nu_c},$$

where γ_0 sets the coupling scale and ν_c is a soft ultraviolet cutoff. The system–bath coupling is strictly local,

$$H_{SB} = \lambda\, S_{j_0} \otimes B.$$

In the weak-coupling–secular (Davies) limit [1, 3], the reduced dynamics are generated by a Markovian Liouvillian $\mathcal{L}$ with thermal fixed point

$$\sigma := \frac{e^{-\beta H_S}}{\mathrm{Tr}(e^{-\beta H_S})}.$$

Writing $S_{j_0}(\omega)$ for the Bohr-frequency components of the coupling operator, the Davies rates are

$$\gamma(\omega) = \begin{cases} J(|\omega|)\big(1 + n_\beta(|\omega|)\big), & \omega > 0, \\ J(|\omega|)\, n_\beta(|\omega|), & \omega < 0, \end{cases} \qquad n_\beta(\nu) = \frac{1}{e^{\beta\nu} - 1},$$

satisfying the KMS relation $\gamma(-\omega) = e^{-\beta\omega}\gamma(\omega)$.

In numerical implementations we set

$$\gamma(0) := \lim_{\omega \to 0} \gamma(\omega) = \frac{\gamma_0}{\beta},$$

and impose a minimal frequency resolution $|\omega| \geq \omega_{\min}$ to avoid infrared divergences of $n_\beta(\omega)$. Bohr frequencies satisfying $|\omega - \omega'| \leq \Delta\omega$ are grouped into a single secular block, with $\Delta\omega$ chosen small compared to the spectral gap.

We work in the Heisenberg picture with the adjoint generator $\mathcal{L}^\dagger$. The Davies generator satisfies quantum detailed balance with respect to σ and is therefore KMS-symmetric with respect to the inner product defined below.

3 Operatorial formulation of an effective rate

Fix the coupling site j_0 and define the graph distance $d(i,j) = |i-j|$. Let

$$R_\epsilon := \{\, i : \; d(i, j_0) \geq \epsilon \,\},$$

and let $\mathcal{A}_\epsilon$ denote the subspace of operators supported on R_ϵ.

Definition 1 (KMS inner product and Dirichlet form)**.** *For operators A, B, define the KMS (GNS) inner product with respect to the thermal state σ by*

$$\langle A, B \rangle_\sigma := \mathrm{Tr}\Big(\sigma^{1/2} A^\dagger \sigma^{1/2} B\Big).$$

The associated norm and Dirichlet form are

$$\|O\|_{2,\sigma}^2 := \langle O, O \rangle_\sigma, \qquad \mathcal{E}_\sigma(O) := -\Re \,\langle O, \mathcal{L}^\dagger(O) \rangle_\sigma.$$

Definition 2 (Effective decay-rate envelope)**.** *For $\epsilon \geq 0$, define*

$$\kappa(\epsilon) := \sup_{\substack{O \in \mathcal{A}_\epsilon \\ O \neq 0}} \frac{\mathcal{E}_\sigma(O)}{\|O\|_{2,\sigma}^2}.$$

Remark 1. *For Davies generators satisfying quantum detailed balance, the Dirichlet form $\mathcal{E}_\sigma$ is nonnegative and provides a variational characterization of a maximal instantaneous decay-rate envelope for operators supported at distance ϵ.*

4 Mechanisms of inheritance and failure

Decay rates in Davies theory depend on Bohr-frequency components of S_{j_0} and bath spectral weights $\gamma(\omega)$. When decay in $\mathcal{A}_\epsilon$ requires propagation of gapped excitations, geometric suppression of $\kappa(\epsilon)$ is expected.

Conversely, when near-zero-Bohr-frequency channels dominate, decoherence can proceed without gap-mediated transport, and $\kappa(\epsilon)$ need not decrease with ϵ despite static clustering. In the TFIM with $J \neq 0$, longitudinal coupling $S_{j_0} = \sigma^z_{j_0}$ does not commute exactly with H_S; it is employed here as a proxy for a coupling whose spectral weight can be concentrated near $\omega \simeq 0$ in the high-field regime.

5 Numerical stress test

We perform exact diagonalization for $N \leq 12$ and estimate $\kappa(\epsilon)$ via a variational operator probe method. For each ϵ, we choose a finite operator basis $\{O_a\} \subset \mathcal{A}_\epsilon$ of Pauli strings supported on R_ϵ.

Defining

$$G_{ab} := \langle O_a, O_b \rangle_\sigma, \qquad M_{ab} := -\Re \,\langle O_a, \mathcal{L}^\dagger(O_b) \rangle_\sigma,$$

we estimate $\kappa(\epsilon)$ as the largest generalized eigenvalue of

$$Mv = \kappa \, Gv.$$

Convergence is checked by enlarging the basis and varying N.

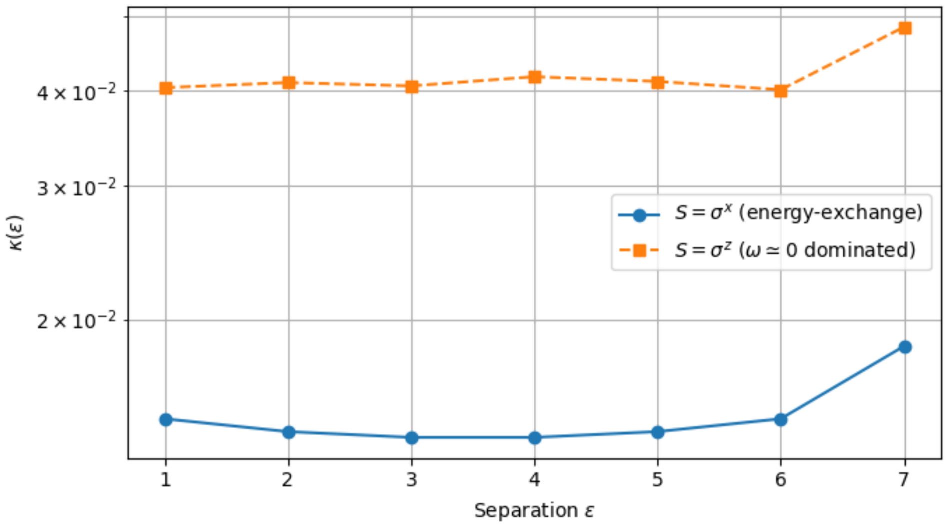

Figure 1: Effective decay-rate envelope $\kappa(\epsilon)$ versus separation ϵ. Energy-exchange-dominated coupling exhibits suppression with distance, whereas near-zero-Bohr-frequency-dominated coupling can saturate.

6 Results

Remark 2. *This manuscript expects the file* `nedladdning2.png` *to be available at compile time; otherwise, remove the figure environment or replace it with a placeholder.*

Figure 1 shows extracted decay-rate envelopes $\kappa(\epsilon)$. For energy-exchange-dominated coupling ($S_{j_0} = \sigma^x_{j_0}$), $\kappa(\epsilon)$ decreases with separation. In contrast, for near-zero-frequency-dominated coupling ($S_{j_0} = \sigma^z_{j_0}$), $\kappa(\epsilon)$ can saturate with distance.

7 Operational resource horizon (no-go consequence)

Remark 3 (Coherence functional used in the no-go consequence)**.** *When invoking thermodynamic maintenance bounds, we use energy pinching*

$$\Delta[\rho] := \sum_n \Pi_n \rho \Pi_n$$

in the eigenbasis of H_S and the relative-entropy coherence

$$C(\rho) := S(\rho \| \Delta[\rho]).$$

Remark 4 (Imported maintenance inequality)**.** *In the battery-assisted thermal-operations framework at temperature T,*

$$P_{\text{extra}}(\rho) \geq k_B T\, \dot{C}_{\text{loss}}(\rho), \qquad \dot{C}_{\text{loss}}(\rho) := -\frac{d}{dt} C(e^{t\mathcal{L}}(\rho))\Big|_{t=0},$$

see [6].

Theorem 1 (Resource-cut no-go under a uniform rate floor)**.** *Fix $C_0 > 0$ and a nonempty family $\mathcal{F}_{C_0}$ with $C(\rho) \geq C_0$. If*

$$\inf_{\rho \in \mathcal{F}_{C_0}} \dot{C}_{\text{loss}}(\rho) \geq \underline{\dot{C}}(C_0) > 0,$$

then any controller maintaining any $\rho \in \mathcal{F}_{C_0}$ requires

$$P_{\text{extra}}(\rho) \geq k_{\text{B}}T\,\underline{\dot{C}}(C_0).$$

If $P_{\max} < k_{\text{B}}T\,\underline{\dot{C}}(C_0)$, sustained maintenance is operationally impossible.

Proof. By the imported maintenance inequality, $P_{\text{extra}}(\rho) \geq k_{\text{B}}T\,\dot{C}_{\text{loss}}(\rho)$. Taking the infimum over $\rho \in \mathcal{F}_{C_0}$ yields the stated uniform lower bound, and the infeasibility condition follows. □

Remark 5 (Connection to $\kappa(\epsilon)$)**.** *When $\kappa(\epsilon)$ saturates, increasing separation does not reduce the minimal required maintenance power, yielding a finite resource horizon. When $\kappa(\epsilon)$ decays, the horizon can be pushed outward, in principle allowing reduced costs.*

8 Discussion and conclusion

The quantum–classical boundary emerges here as a resource horizon, not a dynamical or ontological transition. Static clustering alone does not guarantee dynamical protection: geometric suppression of decoherence rates occurs only when decay requires gap-respecting energy exchange.

Whenever effective rate floors persist, finite available power imposes an operational cut beyond which sustained quantum coherence cannot be maintained. This yields an interpretation-free, thermodynamically grounded account of when and why quantum descriptions cease to be physically sustainable in extended systems.

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