The Heisenberg Cut as a Resource Boundary: An Operational Outlook from Coherence Maintenance Costs

Version 2: exact rate inheritance in the quasi-free class, and withdrawal of the v1 windowed-proxy evidence

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Abstract

We revisit the proposal that the Heisenberg cut is best understood operationally, as a *resource boundary*: a superposition counts as effectively classical for an agent once the power required to maintain its coherence against decoherence, $P_{\text{extra}} \geq k_{\text{B}}T\,\dot{C}_{\text{loss}}$, exceeds that agent's budget. Version 1 supported the key dynamical ingredient — the *rate-inheritance hypothesis* (Hypothesis 8.1 of v1), that decoherence rates transmitted through a gapped buffer of width ε are suppressed as $\text{poly}(\varepsilon)\,e^{-m\varepsilon}$ — with a windowed numerical proxy on a transverse-field Ising chain. Two things change in this revision. First, rate inheritance is now derived in an exactly solvable quasi-free model and verified against exact Liouvillian rapidities: for a probe mode of renormalized sub-gap frequency ω_b coupled through a Kitaev buffer to a Markovian loss, the exact Liouvillian rapidity obeys $\kappa(\varepsilon) \propto e^{-2q(\omega_b)\varepsilon}$ with $\cosh q(\omega) = (\mu^2 + 4 - \omega^2)/(4\mu)$, verified to four decimal places over ten decades of κ; the exponent is the *squared* (amplitude^2) one, resolving Remark 8.1 of v1, and it closes continuously at the band edge. Second, the central numerical evidence of v1 (its Fig. 1) is *withdrawn*: an exact critical-point control shows that the co-moving-window proxy $\kappa_{\text{int}}(\varepsilon)$ decays *faster* when the gap is removed, so what it measured was arrival kinematics, not gap physics. We identify the mechanism with photonic-band-gap suppression of emission and evanescent atom–photon bound states, restate carefully what is imported versus what is new, derive a logarithmic law for the resource cost of coherence lifetime, note that the protected object is a sub-gap *subalgebra* rather than a spatial region, and state the interacting-buffer conjecture that would take rate inheritance beyond the quasi-free class. The non-claims of v1 stand unchanged: nothing here derives the Born rule or selects single outcomes.

1 Introduction

Textbook quantum mechanics needs a cut: somewhere between the microscopic degrees of freedom described by a state vector and the apparatus whose readings are described classically, a line is drawn, and the theory is silent about where. Bohr insisted the cut is movable within a region of effective equivalence; von Neumann showed its position does not affect predictions; Bell [1] complained, with reason, that "for all practical purposes" (FAPP) was doing unacknowledged load-bearing work in all of this. The decoherence program [2, 3, 4] explains *why* the cut's position is unobservable in practice, but it quantifies the physics on the quantum side of the cut, not the resources of the agent on the classical side.

Version 1 of this paper proposed to make the cut's placement an *operational, quantitative* matter. The organizing idea is a maintenance inequality: holding the coherence $C(\rho)$ of a system steady against a decoherence channel of rate κ requires, from any agent respecting the second law, a power

$$P_{\text{extra}} \;\geq\; k_{\mathrm{B}}T\,\dot{C}_{\text{loss}} \;\simeq\; k_{\mathrm{B}}T\,\kappa\,C(\rho), \tag{1}$$

so that "effectively classical for agent A" can be given the sharp meaning: every isolation-and-maintenance strategy within A's power budget fails. The cut sits wherever the required maintenance power crosses the available one. For this to be more than rhetoric, one needs to know how κ depends on the *geometry* of isolation. The central dynamical claim of v1 (Hypothesis 8.1) was *rate inheritance*: if the only route from the system to the dissipative environment passes through a buffer of width ε made of gapped matter (gap Δ), then $\kappa(\varepsilon) \lesssim \text{poly}(\varepsilon)\,e^{-m\varepsilon}$, so isolation is exponentially cheap in ε and the resource boundary is sharply located.

This revision does three things to that claim, one constructive, one corrective, one prospective.

Constructive. Section 3 derives rate inheritance in an exactly solvable quasi-free model, upgrading Hypothesis 8.1 from a conjecture supported by fits to a closed law with a derived, frequency-resolved exponent, $\kappa(\varepsilon) = P(\varepsilon)\,e^{-2q(\omega_b)\varepsilon}$, verified against exact Liouvillian spectra to four decimal places (Table 1, Fig. 1). The exponent answers the question left open in Remark 8.1 of v1 (rate exponent = twice the amplitude exponent), closes continuously as the probe frequency approaches the band edge, and disappears — as it must — for in-band probes and at criticality.

Corrective. Section 4 withdraws the numerical evidence that v1 offered for Hypothesis 8.1 (v1 Fig. 1). Re-running that protocol with exact density-matrix integration and adding the control v1 lacked — the same sweep at the critical point, where the gap vanishes — shows that the windowed proxy $\kappa_{\text{int}}(\varepsilon)$ decays *more steeply without a gap than with one.* The observed decay was arrival kinematics seen through a co-moving time window, not spectral suppression. The hypothesis survives (Section 3 proves a version of it); the v1 evidence for it does not, and Section 4 explains the general lesson for windowed proxies.

Prospective. Sections 5 and 6 place the mechanism where it belongs in the literature — it is the physics of photonic band gaps and evanescent atom–photon bound states, transplanted to the cut-placement question — and state the one version of rate inheritance that is *not* already implicit in known physics: the interacting gapped buffer, where a genuine theorem remains to be proven.

The scope discipline of v1 is retained. Every statement below is tagged, explicitly or by context, as *imported* (known results used as inputs), *interface* (identities connecting imported pieces to the framework), *result* (established here), or *conjecture.* And the non-claims stand: nothing in this framework derives the Born rule (the trace rule is presupposed by every object in it, so any internal derivation would be circular), and nothing selects single outcomes. What is on offer is an engineering-grade account of where the cut can be placed and what it costs to move it — Bell's FAPP, with exponents.

2 The cut as a resource boundary

2.1 Setup and the maintenance inequality

Fix a pointer basis $\{|i\rangle\}$ for the system of interest and let $\Delta[\rho] = \sum_i |i\rangle\langle i|\rho|i\rangle\langle i|$ be full dephasing in that basis. Coherence is quantified by the relative entropy of coherence [6, 7, 8]

$$C(\rho) \;=\; S(\rho\,\|\,\Delta[\rho]) \;=\; S(\Delta[\rho]) - S(\rho), \tag{2}$$

which is also the coherent excess of free energy in units of $k_{\mathrm{B}}T$: $F(\rho) - F(\Delta[\rho]) = k_{\mathrm{B}}T\,C(\rho)$ for states degenerate in the pointer basis, and more generally the work value of the coherent part of the state [9, 11, 10]. The maintenance inequality (1) is then second-law bookkeeping: an agent who keeps $C(\rho_t)$ constant while the dissipative dynamics destroys it at rate $\dot{C}_{\mathrm{loss}}$ must inject coherent free energy at least at rate $k_{\mathrm{B}}T\,\dot{C}_{\mathrm{loss}}$. *Status: interface, now with a proved battery-model instance.* The inequality assembles known elements of the resource theory and thermodynamics of coherence [9, 10, 11, 8]; v2 makes no novelty claim for its general form. In the explicit battery-assisted thermal-operation model of the companion work theorem [37] (v2), (1) *is* a theorem in exactly the setting used here: for pointer-basis pure dephasing the pinched target is stationary, the population-maintenance baseline is free, and $P_{\mathrm{extra}} = P_{\min} \geq k_{\mathrm{B}}T\,\dot{C}_{\mathrm{loss}}$ holds with no contraction or achievability assumptions (its "free-baseline" proposition); for general Davies dynamics the same bound holds against thermodynamically efficient population-stabilizing baselines, and the unconditional floor is $k_{\mathrm{B}}T$ times the entropy production rate. (v2 of [37] also corrects a work-sign error and replaces a vacuous pair-infimum definition of P_{extra} from its v1; the form used here is the corrected one.) Companion papers develop the work-theorem side [37, 39] and the algebraic (Type III) setting [36].

2.2 An exact rate identity

The loss rate in (1) admits a closed expression, which v1 used only implicitly.

Lemma 1 (coherence-loss rate)**.** *Let $\dot{\rho} = \mathcal{L}(\rho)$ be any trace-preserving generator and ρ full rank. Then*

$$\frac{d}{dt}C(\rho_t) \;=\; \mathrm{Tr}\big[\mathcal{L}(\rho)\,\big(\log\rho - \log\Delta[\rho]\big)\big], \qquad \dot{C}_{\mathrm{loss}} \;\leq\; \big\|\mathcal{L}(\rho)\big\|_1\,\big\|\log\rho - \log\Delta[\rho]\big\|_\infty. \tag{3}$$

Proof. $\frac{d}{dt}S(\rho) = -\mathrm{Tr}[\dot{\rho}\log\rho]$ since $\mathrm{Tr}\dot{\rho} = 0$. For the dephased part, $\frac{d}{dt}S(\Delta\rho) = -\mathrm{Tr}[(\Delta\dot{\rho})\log\Delta\rho] = -\mathrm{Tr}[\dot{\rho}\,\Delta(\log\Delta\rho)] = -\mathrm{Tr}[\dot{\rho}\log\Delta\rho]$, using that Δ is self-adjoint and $\log\Delta\rho$ is diagonal. Subtracting gives (3); the bound is Hölder's inequality. □

Status: interface. Identity (3) is elementary; its consequence — that a rate-domination bound $\dot{C}_{\mathrm{loss}} \leq \kappa\,C$ holds whenever the dissipative semigroup satisfies a modified logarithmic Sobolev inequality with constant α (then $C(\rho_t)$ decays exponentially at rate set by α) — belongs to the functional-inequalities literature [12, 13]. This identifies the κ appearing in the resource criterion with an MLSI-type constant of the effective generator seen by the system, and turns Hypothesis 8.1 into a statement about the *geometric suppression of that constant* — which Section 3 now computes exactly in a solvable class.

The single-qubit worked example of v1 is retained unchanged, since it is correct: for pure dephasing $\dot{c} = -\Gamma_\phi c$ of $\rho = \frac{1}{2}(\mathbb{1} + z\sigma^z) + (c\,\sigma^+ + \mathrm{h.c.})$ one finds, with $r = \sqrt{z^2 + 4|c|^2}$,

$$\dot{C}_{\mathrm{loss}} \;=\; \frac{2\Gamma_\phi|c|^2}{r}\,\log\frac{1+r}{1-r} \;\xrightarrow[|c|\to 0]{}\; 2\Gamma_\phi\,C(\rho), \tag{4}$$

the small-coherence limit quoted in v1 (Prototype 8.1 and Lemma 10.1).

3 Exact rate inheritance across a quasi-free gapped buffer

3.1 Model

The clean question is: a system whose relevant Bohr frequency lies *below* the gap of a buffer, coupled to a Markovian sink only through that buffer — how does its induced decay rate scale with buffer width? We answer it in a model where the full Liouvillian spectrum is exactly computable.

Take a fermionic probe mode d of bare frequency ω_0, a Kitaev chain [25] of ε sites as the buffer, and a Markovian loss on the far site:

$$H = \omega_0\, d^\dagger d \;+\; g\,(d^\dagger c_1 + c_1^\dagger d) \;+\; \sum_{i=1}^{\varepsilon-1} \Big[-t\,(c_i^\dagger c_{i+1} + \text{h.c.}) + \Delta_p\,(c_i c_{i+1} + \text{h.c.}) \Big] \;-\; \mu \sum_{i=1}^{\varepsilon} c_i^\dagger c_i,$$
$$L = \sqrt{\gamma}\; c_\varepsilon\,, \tag{5}$$

with $t = \Delta_p = 1$. The bulk quasiparticle dispersion is $E(k)^2 = \mu^2 + 4 + 4\mu\cos k$, a band $[\,|\mu-2|,\;\mu+2\,]$ with gap $\Delta = |\mu-2|$. Under Jordan–Wigner this is the transverse-field Ising chain with $J = t$, $h = \mu/2$: our production value $\mu = 3$ is exactly the $h = 1.5$, $J = 1$ paramagnet of v1's Prototype 2, with band $[1,5]$ and gap 1. Since H is quadratic and L linear in fermions, the Lindbladian is quasi-free and exactly solvable by third quantization [26, 27]: the $2(\varepsilon{+}1) \times 2(\varepsilon{+}1)$ Majorana covariance matrix obeys a closed Lyapunov equation $\dot\Gamma = X\Gamma + \Gamma X^{\mathsf T} + Y$ (Appendix A), and all relaxation rates are sums of pairs of eigenvalues ("rapidities") of X. The probe's decay rate is $\kappa(\varepsilon) = -2\,\mathrm{Re}\,\lambda_{\text{probe}}$, with λ_{probe} the eigenvalue of X whose eigenvector has maximal weight on the probe Majoranas; its imaginary part gives the *renormalized* bound-state frequency $\omega_b = |\mathrm{Im}\,\lambda_{\text{probe}}|$, shifted from ω_0 by the level's hybridization self-energy. The implementation was validated against brute-force Lindblad integration of the full 2^n-dimensional density matrix on small instances: operator-identity error 1.1×10^{-16}, dynamics error 2.6×10^{-15} (Appendix A).

3.2 The law

For a sub-gap frequency $\omega < \Delta$ the dispersion has no real solution; analytic continuation $k = \pi + iq$ gives the evanescent branch

$$\cosh q(\omega) \;=\; \frac{\mu^2 + 4 - \omega^2}{4\mu}\,, \qquad \xi(\omega) = 1/q(\omega), \tag{6}$$

the decay constant of the buffer's Green's function at that frequency (a lattice Combes–Thomas bound [20]). A bound state at ω_b leaks into the buffer with amplitude $\sim e^{-q(\omega_b)x}$; a loss of strength γ at distance ε then removes probability at a rate proportional to the *squared* amplitude there. This predicts

$$\boxed{\kappa(\varepsilon) \;=\; P(\varepsilon)\, e^{-2\,q(\omega_b)\,\varepsilon}} \tag{7}$$

with P subexponential, and three sharp consequences: the exponent is frequency-resolved and *closes* as $\omega_b \to \Delta$ (indeed $\cosh q \to 1$ there); it disappears for in-band frequencies; and it disappears at criticality ($\mu = 2$), where there is no sub-gap window at all.

Result 1 (quasi-free rate inheritance)**.** *In model* (5) *the exact probe rapidity follows the law* (7) *with the exponent* (6) *evaluated at the renormalized frequency ω_b. The agreement between the fitted tail slope of* $\ln\kappa$ *vs.* ε *and* $2q(\omega_b)$ *is at the* 10^{-4} *level across ten decades of* κ *(Table 1, Fig. 1a). Removing the gap at fixed probe frequency removes the suppression entirely (Fig. 1b), as does moving the probe frequency into the band.*

Three remarks on Result 1. *(i)* It settles Remark 8.1 of v1: the rate inherits the *square* of the evanescent amplitude, exponent $2/\xi(\omega_b)$, not $1/\xi$; in TFIM variables, $q(0) = \ln(h/J)$, so for the v1 parameters $h/J = 1.5$ the zero-frequency slope is $2\ln 1.5 \approx 0.81$ per site — exactly what the exact rapidities give. *(ii)* The identification of ω_b (rather than ω_0) as the frequency entering (6) is itself a falsifiable refinement: at $g = 0.3$ the shift is small but the agreement improves from the third to the fourth decimal once it is included, and the shift closes as $g \to 0$ (Table 1, last two rows). *(iii)* The result is about the *asymptotic* Liouvillian spectrum; no windows, no transients, no fitting choices beyond the trivial tail fit.

ω_0	g	ω_b	fitted slope	$2q(\omega_b)$	$2q(\omega_0)$ (bare)
0.0	0.3	0.0296	0.8105	0.8106	0.8109
0.5	0.3	0.5238	0.6920	0.6921	0.7035
0.9	0.3	0.9189	0.3217	0.3217	0.3554
0.5	0.1	0.5027	0.7022	0.7022	0.7035
0.9	0.1	0.9021	0.3518	0.3518	0.3554

Table 1: Measured decay-rate exponents versus the analytic law (6), for the gapped buffer $\mu = 3$ ($\Delta = 1$, band $[1,5]$), loss $\gamma = 0.4$. Tail slopes are fitted over the last six resolvable points of $\ln \kappa(\varepsilon)$. The law holds at the *renormalized* bound-state frequency ω_b (read off the exact rapidity); the residual mismatch against the bare $2q(\omega_0)$ is entirely the hybridization shift, and shrinks with g as expected.

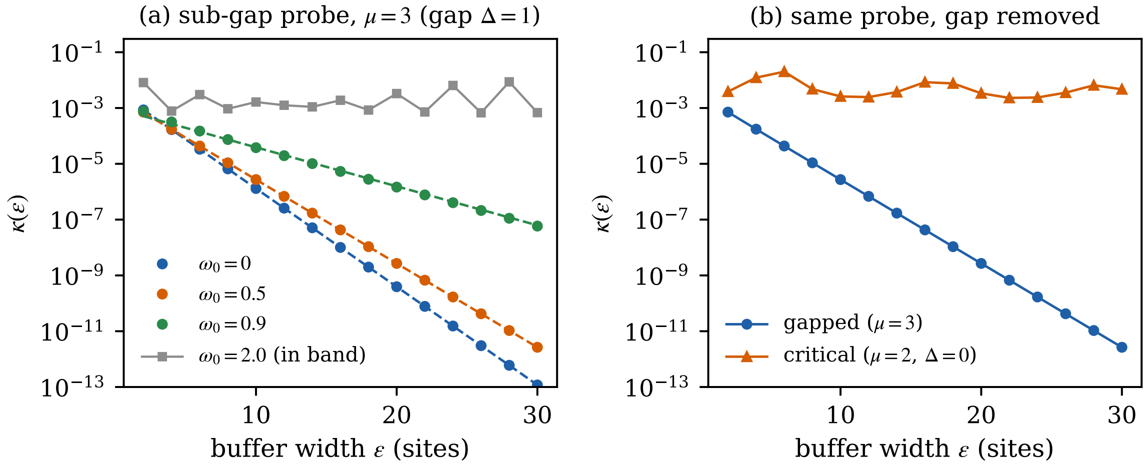

Figure 1: Exact probe decay rate $\kappa(\varepsilon)$ from the Liouvillian rapidities of model (5). (a) Sub-gap probes in the gapped buffer ($\mu = 3$): points are exact values; dashed lines have the *analytic* slopes $2q(\omega_b)$ of Eq. (6), not fits. The suppression softens as ω_0 approaches the band edge $\Delta = 1$ and is absent for the in-band probe $\omega_0 = 2$ (grey). (b) The same probe ($\omega_0 = 0.5$) with the gap removed ($\mu = 2$, critical): no suppression. The gap is necessary, not incidental.

3.3 Corollaries for the resource boundary

Logarithmic cost of coherence lifetime. Combining (7) with the maintenance criterion: a superposition on the probe survives unassisted for a time $T \sim \kappa(\varepsilon)^{-1}$, so sustaining coherence for time T requires a buffer of width only

$$\varepsilon^*(T) \simeq \frac{\xi(\omega_b)}{2} \ln\big(\kappa_0 T\big), \tag{8}$$

i.e. the isolation cost grows *logarithmically* in the ambition. At the v1 parameters each additional $\ln 2/(2q) \approx 0.85$ sites of buffer doubles the sustainable coherence time. Conversely, for an agent with maximum affordable buffer $\varepsilon_{\max}$ and power budget $P_{\max}$, the resource boundary of Eq. (1) sits at a computable place, with κ given by (7) rather than fitted. This is the quantitative content promised by the title: Bell's FAPP with exponents.

The cut is frequency-selective, not spatial. An unexpected sharpening: the buffer does not protect "the system"; it protects the *sub-gap spectral content* of the system. Observables of S whose Bohr frequencies lie in the buffer's band decohere essentially unimpeded (Fig. 1a, grey); those below the gap are exponentially protected. The object stabilized by a resource boundary is therefore a *subalgebra* — the one generated by sub-gap-frequency operators — rather than a spatial region. This matches the algebraic intuition behind the split-property formulation of the cut developed in the companion papers [36, 37], where a cut is a choice of type-I interpolating subalgebra, and it explains *which* interpolating algebra the physics selects.

Floors. Two mechanisms bound the criterion at large ε and belong in any honest budget: thermal activation of the buffer contributes a rate floor $\sim e^{-\Delta/k_B T}$ independent of ε, and any direct (bypass) coupling of strength g_b contributes $\sim g_b^2$. Beyond the crossover ε where these dominate, thickening the buffer buys nothing; the resource boundary saturates. This is relevant to the superdecoherence scaling discussed around Conjecture 10.1 of v1, which we leave otherwise untouched.

4 Correction: the v1 windowed protocol measured kinematics, not gap physics

4.1 The v1 protocol and its exact re-examination

Version 1 supported Hypothesis 8.1 with the following protocol (v1 Fig. 1, Prototype 2): a transverse-field Ising chain $H = -J\sum \sigma_i^x \sigma_{i+1}^x - h\sum \sigma_i^z$ ($J = 1$, $h = 1.5$), system site S at one end, an amplitude-damping dissipator $\sqrt{\gamma}\,\sigma^-$ at site $\varepsilon+1$, and an integrated coherence-loss proxy for the relative-entropy coherence $C_x(t)$ of S in the σ^x basis,

$$\kappa_{\text{int}}(\varepsilon) \;=\; \frac{1}{\tau}\ln\frac{C_x(t_0)}{C_x(t_0+\tau)}\,, \qquad t_0 = 0.8\,\varepsilon/v, \quad v = 2J, \tag{9}$$

evaluated from trajectory sampling at $N_{\text{tot}} = 12$. The window co-moves with ε by construction. The observed decay of κ_{int} with ε was offered as evidence of gap-induced rate suppression.

We re-ran this protocol family with *exact* integration of the full Lindblad density matrix (no sampling noise; $N = 10$, $\gamma = 0.8$, $\tau = 2$, all-spins-up initial product state, which carries maximal σ^x-basis coherence at every site; details and robustness remarks in Appendix A), together with the three controls v1 lacked: (i) the identical sweep at $\gamma = 0$; (ii) a fixed absolute window common to all ε; and (iii) — decisively — the identical sweep at the critical point $h = J$, where the gap vanishes and any genuinely gap-induced suppression must disappear.

4.2 Findings and diagnosis

The results (Fig. 2) are unambiguous. The gapped sweep reproduces the v1-style decay: κ_{int} falls from 1.02 to 0.23 over $\varepsilon = 1, \ldots, 6$ (raw slope 0.31/site; dissipative excess slope 0.28/site). But the *critical* sweep, with the gap removed, falls from 1.59 to 0.04 — raw slope 0.69/site, excess slope 0.57/site — i.e. *steeper without the gap than with it.* And replacing the co-moving window by a fixed absolute one destroys the exponential altogether: $\kappa_{\text{int}}(\varepsilon)$ becomes non-monotonic, rising to a peak near $\varepsilon \approx 4$ (where the dissipator's influence arrives mid-window) and falling beyond. No reading of these controls is compatible with "the decay of κ_{int} measures gap-induced rate suppression." **The evidence of v1 Fig. 1 is therefore withdrawn.**

The diagnosis is structural and worth recording, because the failure mode threatens any windowed proxy of this type. Three effects conspire. First, the dissipator's influence on S arrives ballistically at $t \approx \varepsilon/v$; the window (9) starts at $0.8\,\varepsilon/v$ and so contains a shrinking post-arrival exposure $\tau - 0.2\,\varepsilon/v$. Second, the disturbance disperses as it propagates, so the amplitude reaching S within the window decreases with ε on kinematic grounds alone — in *any* phase, gapped or not. Third, the co-moving window slides along the intrinsic (unitary) relaxation curve of C_x, sampling ever flatter portions of it. All three produce decay of κ_{int} with

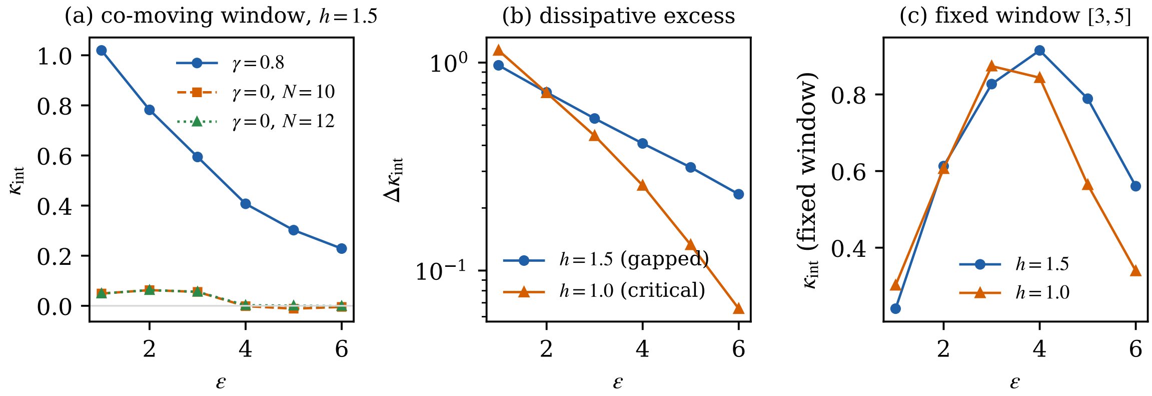

Figure 2: Exact re-examination of the v1 Fig. 1 protocol ($N = 10$, $\gamma = 0.8$, $\tau = 2$). (a) The co-moving-window proxy (9) at $h = 1.5$ reproduces the v1-style decay (blue); the unitary baselines $\gamma = 0$ at $N = 10$ and $N = 12$ (orange, green) are small, so the decay sits in the dissipative excess. (b) That excess, $\Delta\kappa_{\rm int} = \kappa^{\gamma}_{\rm int} - \kappa^{0}_{\rm int}$, decays with slope 0.28/site in the gapped phase — and 0.57/site at the *critical* point, where there is no gap at all. (c) With a fixed absolute window $[3, 5]$ the "exponential" disappears entirely: the same data are non-monotonic, peaking near the arrival time of the dissipator's influence. The proxy tracks causal kinematics, not spectral suppression.

ε without any spectral gap; the critical control shows they in fact dominate. There is also a design-level reason the protocol could never have isolated the effect: the site S is *resonant with the magnon band* of its own chain, so its coherence dephases through in-band channels that no gap protects (cf. the grey curve of Fig. 1a). Gap protection requires the protected frequency to lie below the gap — which is precisely what the probe construction of Section 3 arranges and the v1 protocol did not.

4.3 Corrected protocols

For future numerics in this program (including the stress-test companion paper [38]; its v2 applies this discipline to its own Dirichlet-form diagnostics, distinguishing ceiling from floor envelopes and quantifying the secular nonlocality of its zero-frequency channel), the corrected practice is: extract rates from the Liouvillian spectrum or from long-time exponential fits at fixed absolute times, never from co-moving windows; always subtract the $\gamma = 0$ baseline; always run the gap-removed control at criticality; and ensure the protected degree of freedom is spectrally sub-gap, e.g. by using a detuned probe as in Eq. (5). Under those rules the effect is real, clean, and exactly as large as Eq. (7) says.

5 Relation to known physics: what is imported, what is new

Intellectual honesty requires stating plainly that the *mechanism* established in Section 3 is not new physics. Suppression of decay for emitters whose frequency lies in a spectral gap is the physics of photonic band gaps [14, 15, 16]; the evanescent bound state that a sub-gap emitter forms with its structured environment, with localization length diverging at the band edge, is the atom–photon bound state of band-gap waveguide QED [17, 18], observed experimentally near photonic-crystal band edges [19]. Mathematically, Eq. (6) is a lattice Combes–Thomas estimate [20]. On the functional-analytic side, the rate-domination step (A) of v1 lives in the modified-log-Sobolev literature [12, 13]. And the general locality inputs — Lieb–Robinson bounds [21, 22] and exponential clustering in gapped phases [23] — are standard. *Status of the mechanism: imported.*

What this paper adds is (a) the *transplant*: reading that mechanism as a quantitative theory of Heisenberg-cut placement, with the maintenance inequality (1) converting rates into agent-facing power budgets and Eq. (8) converting geometry into coherence lifetime; (b) the exact, frequency-resolved form of rate inheritance in the quasi-free class, including the ω_b refinement and the squared exponent, stated at Liouvillian-spectrum rigor rather than fit rigor; (c) the negative methodological result of Section 4, which we have not found stated elsewhere in this form and which applies to any co-moving-window decoherence diagnostic; and (d) the framing of the open problem, next.

Relative to the decoherence program [2, 3, 4] and quantum Darwinism [5], the shift of emphasis is from the quantum side of the cut (what the environment does) to the classical side (what maintaining the alternative would cost an agent), in the spirit of macroscopicity measures [28] but tied to a concrete geometric handle, ε.

6 Conjecture: interacting gapped buffers

The quasi-free result rides on exact solvability. The version of rate inheritance that would be a genuine theorem — and that nothing in the band-gap literature settles — is the interacting one.

Conjecture 1 (rate inheritance, interacting case)**.** *Let the buffer be a finite-range interacting spin chain of width ε with a unique gapped ground state (gap Δ uniform in ε), prepared near its ground state at temperature $k_{\mathrm{B}}T \ll \Delta$. Couple a system S, all of whose Bohr frequencies lie in $(-\Delta + \delta, \Delta - \delta)$ for some $\delta > 0$, to one boundary with strength g, and a Markovian environment to the other. Then the induced generator on S in the weak-coupling (Davies [24]) regime satisfies*

$$\|\mathcal{L}_S\| \;\le\; \mathrm{poly}(\varepsilon)\, e^{-\varepsilon/\xi_*} \;+\; O\big(e^{-\Delta/k_{\mathrm{B}}T}\big), \tag{10}$$

with ξ_* controlled by the Hastings–Koma correlation length [23] of the buffer at the relevant sub-gap frequencies.

Proof strategy. Davies rates are boundary-to-boundary spectral densities of the buffer-plus-sink medium evaluated at the Bohr frequencies of S; exponential clustering of gapped ground states bounds the static part, and Lieb–Robinson bounds [21, 22] propagate the bound to the finite-frequency, finite-time correlations that enter the Davies integral. *Known obstacles,* stated so that partial progress is checkable: uniformity of the Davies limit in ε; the buffer is driven out of its ground state by the sink itself, so one needs local stability or rapid-mixing input (the MLSI results for commuting Hamiltonians [13] are the natural tool); non-Markovian corrections at the band edge; and the thermal floor, which is physical and belongs in the statement. A proof of Conjecture 1, even for commuting interactions, would be the first version of rate inheritance not already implicit in quasi-free band-gap physics, and is in our view the correct next target for this program.

7 What this does and does not say about the measurement problem

It is useful to separate three questions that the word "cut" entangles: where the cut *can* be placed; why its placement does not affect predictions; and why there is a unique outcome at all.

On the first, this paper's contribution is genuine and now partly exact: the cut can be placed at any surface where the maintenance power $k_{\mathrm{B}}T\,\kappa\,C$ exceeds the agent's budget, and κ has known exponents, Eq. (7), with the logarithmic cost (8) quantifying how expensive it is to push the boundary outward. Moreover the boundary is a *spectral* one: what becomes maintainable is the sub-gap subalgebra, which is a sharper statement than any spatial placement. A corollary worth stating is a "Wigner's friend with a budget" bound: for an external agent to keep the friend-plus-lab superposition maintainable in the sense of Eq. (1) requires power exponential in the lab's decoherence bandwidth over the isolating gap — a quantitative version of why the Frauchiger–Renner [35] agents' mutual modelling is FAPP-impossible long before it is paradoxical.

On the second question — consistency of moving the cut — nothing here goes beyond standard decoherence theory [3, 4], and we do not pretend otherwise. On the third, the position of v1 stands verbatim: this framework *presupposes* the trace rule in every object it manipulates ($\mathrm{Tr}(\rho A)$, $S(\rho\|\sigma)$, the Lindblad structure), so no internal derivation of the Born rule could be non-circular. Reconstructions exist — Gleason [29] and its POVM form [30], envariance [31], decision-theoretic routes [32, 33], operational axiomatics [34] — but each trades the Born postulate for another axiom, and this paper adds nothing to that exchange. What it offers instead is a precise account of *effective* classicality: below budget, no admissible strategy can maintain, exploit, or reverse the coherence over the relevant operational timescale — collapse as a resource horizon, with $k_{\mathrm{B}}T$ in front of every inequality.

8 Conclusions

Version 1 proposed that the Heisenberg cut is a resource boundary and conjectured that gapped buffers make isolation exponentially cheap. Version 2 delivers the conjecture exactly in the quasi-free class — with a frequency-resolved exponent $2q(\omega_b)$ that closes at the band edge, is verified to four decimals over ten decades, and settles the squared-exponent question of v1's Remark 8.1 — while withdrawing the numerical evidence v1 offered, because its own missing control (the critical point) shows that evidence measured window kinematics. The mechanism is band-gap QED transplanted to the cut-placement question; the honest novelty is the transplant, the exact frequency-resolved law, the negative methodological result, and the statement of the interacting-buffer conjecture where a real theorem is still to be had. The measurement problem's hard core — outcomes and probabilities — is exactly as untouched as v1 said it was. But the soft shell, the placement and price of the cut, now has exponents, and that is the part Bell asked to see written down.

A Methods

Quasi-free solver. Write the $2n$ Majoranas $a_{2j} = c_j + c_j^\dagger$, $a_{2j+1} = -i(c_j - c_j^\dagger)$, $\{a_\alpha, a_\beta\} = 2\delta_{\alpha\beta}$. A quadratic Hamiltonian is $H = \frac{i}{4} a^\mathsf{T} \mathbb{H}\, a + \text{const}$ with $\mathbb{H}$ real antisymmetric, and linear jump operators $L_\mu = \ell_\mu^\mathsf{T} a$ define the Hermitian bath matrix $M = \sum_\mu \bar{\ell}_\mu \ell_\mu^\mathsf{T}$. The covariance matrix $\Gamma_{\alpha\beta} = \frac{i}{2}\langle [a_\alpha, a_\beta]\rangle$ then obeys the Lyapunov equation

$$\dot{\Gamma} \;=\; X\,\Gamma + \Gamma\, X^\mathsf{T} \;-\; 4\,\mathrm{Im}\, M\,, \qquad X = \mathbb{H} - 2\,\mathrm{Re}\, M\,, \tag{11}$$

whose eigenvalues (rapidities) generate all two-point relaxation rates as pairwise sums [26, 27]. For model (5), $L = \sqrt{\gamma}\, c_\varepsilon$ gives $\ell_{2\varepsilon} = \sqrt{\gamma}/2$, $\ell_{2\varepsilon+1} = i\sqrt{\gamma}/2$. The probe rapidity is identified by maximal eigenvector weight on components $(0, 1)$; $\kappa = -2\,\mathrm{Re}\,\lambda$, $\omega_b = |\mathrm{Im}\,\lambda|$. Validation: (i) the quadratic-to-Majorana conversion was checked against explicit Fock-space construction for random (A, B) at $n = 3$ (max deviation 1.1×10^{-16} after removing the scalar $\mathrm{Tr}A/2$); (ii) the Lyapunov dynamics of $\langle d^\dagger d\rangle(t)$ was checked against fourth-order Runge–Kutta integration of the full 8×8 Lindblad density matrix for probe plus two sites (max deviation 2.6×10^{-15}). Sub-gap slopes in Table 1 are least squares over the last six points with $\kappa > 10^{-13}$, $\varepsilon \le 60$.

Exact protocol re-examination. For Section 4: TFIM with $N = 10$ spins, $J = 1$, $h \in \{1.5, 1.0\}$; initial state $|\uparrow \cdots \uparrow\rangle$ in the σ^z basis (maximal C_x at every site, stationary under the field term alone); dissipator $\sqrt{\gamma}\,\sigma^-$ at site $\varepsilon{+}1$, $\gamma = 0.8$; full density-matrix integration (RK4, $dt = 0.032$, checked against $dt = 0.02$ at the 10^{-3} level in C_x); $C_x(t)$ from the reduced state of site 0 rotated to the σ^x eigenbasis; κ_{int} per Eq. (9) with $\tau = 2$, and the fixed-window variant on $[3, 5]$. Unitary baselines by exact statevector evolution at $N = 10$ and $N = 12$; the two agree closely over the window range used (Fig. 2a), bounding finite-size effects. The v1 runs used $N_{\text{tot}} = 12$ with trajectory sampling and a possibly different (γ, initial state); the diagnosis of Section 4 is structural — arrival kinematics, dispersive spreading, and window drift operate for any parameter choice in this protocol family, and the critical control is decisive independently of them. Code (Python/NumPy, exact solvers and all sweeps) is available from the author.

B Summary of changes from v1

(1) Section 3 is new: rate inheritance (v1 Hypothesis 8.1) is established exactly in the quasi-free class, with the frequency-resolved law (7)–(6); v1 Remark 8.1 is resolved (the rate exponent is the squared amplitude exponent, $2/\xi(\omega_b)$, hence slope $2\ln(h/J)$ at zero frequency). (2) v1 Fig. 1 and the windowed κ_{int} evidence are withdrawn, for the reasons documented in Section 4; the corrected protocols are stated there. (3) A related-work section (Section 5) now attributes the mechanism to photonic-band-gap QED and Combes–Thomas estimates, and the rate-domination step to the MLSI literature; the paper's novelty claims are re-scoped accordingly. (4) Lemma 1 makes the coherence-loss rate identity explicit. (5) The interacting-buffer conjecture (Section 6) replaces informal remarks about generality. (6) The frequency-selectivity of the protected subalgebra and the logarithmic lifetime cost (8) are new corollaries. (7) All non-claims of v1 (no Born derivation, no single-outcome selection) are unchanged. Numerical prototypes of v1 not affected by the window issue (in particular the qubit formula (4)) are retained. (8) Companion references are updated to their v2 revisions: in particular, the maintenance inequality (1) is now grounded in the corrected extra-power theorems of [37] (v2), whose free-baseline case covers exactly the pointer-basis dephasing setting of this paper (Section 2); and the geometric collar-suppression inputs cited from [36] are unaffected by that paper's v2 correction (which concerns the identification of its Gaussian reconstruction with the Petz map, not the Bessel/Combes–Thomas decay bounds used here). The verification scripts of this paper (exact rapidity model, windowed-proxy control, Combes–Thomas oracle) are distributed in the companion repository (`verification/0064`).

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