

# The Heisenberg Cut as a Resource Boundary: An Operational Outlook from Coherence Maintenance Costs

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December 2025

## Abstract

This article is a synthesis/outlook paper. Its quantitative content is organized around two inputs proved in companion preprints (not yet peer reviewed): (i) operational lower bounds on minimal coherence-maintenance power under battery-assisted thermal operations at bath temperature  $T$  (Imported Results from [2]), and (ii) model-specific geometric suppression and recoverability behavior in massive Gaussian split configurations (Imported Results from [1]). We do not propose a modification of quantum dynamics, do not derive the Born rule, and do not claim a solution to the measurement problem in the strong foundational sense. Instead, we propose an operational reinterpretation of the quantum–classical transition: classical-like behavior can coincide with regimes in which sustaining coherence is resource-infeasible under available control budgets. We isolate the unique nontrivial interface leap needed to connect static geometric suppression to maintenance power—the inheritance of collar suppression by relaxation rates—state it as a falsifiable interface hypothesis, and provide two worked prototypes: (a) an exact qubit dephasing computation showing how  $\mathcal{C}_{\text{loss}} \leq \kappa \mathcal{C}$  arises in a standard Markovian model, and (b) an  $\epsilon$ -tunable buffer-chain surrogate with an executed first-pass numerical plot of an  $\epsilon$ -dependent coherence-loss rate proxy consistent with an envelope-class form.

### Status of claims (read first).

- **Imported Results.** Statements labeled **Imported Result** are proved in companion preprints [2, 1] (not yet peer reviewed). They are treated as inputs.
- **Assumptions and interfaces.** **Interface Assumptions** and **Interface Hypotheses** are explicit bridges connecting geometry, information, and dynamics.
- **Conditional Propositions.** Statements labeled **Conditional Proposition** are proved here assuming the imported results plus explicitly stated assumptions.
- **Conjectures.** Scaling claims beyond the stated interfaces are conjectural and come with falsification routes.

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# 1 Scope, non-claims, and validation plan

**Scope.** This paper is an operational perspective. It reorganizes two technical inputs into a falsifiable program about *minimal control resources* required to maintain coherence, and proposes laboratory-facing tests.

**Non-claims.** We do not propose a collapse mechanism, do not derive the Born rule, and do not claim a strong-resolution of the measurement problem. The claims concern control feasibility under an explicit thermodynamic bookkeeping model.

## 1.1 Validation and reproducibility plan (checkable deliverables)

Because key quantitative inputs are currently companion preprints, we include a plan with checkable deliverables:

- (i) **Finite-dimensional maintenance inequalities.** The maintenance results in [2] are finite-dimensional. A revision will include an appendix-style proof skeleton: a list of lemmas used and where they enter (variational steps over strategy pairs, entropy identities, and monotonicity/data-processing inputs).
- (ii) **Gapped envelope surrogate (executed first pass).** We include an executed numerical buffer-chain surrogate (Prototype 8.2) and a plot (Figure 1). The intended target is an *envelope class* test (exponential suppression in  $m\epsilon$  up to polynomial prefactors), not an exact special-function fit.
- (iii) **Artifacts.** The simulation script used to generate Figure 1 outputs the plot files and a raw data file (e.g. `results_kappa_int_best.npz`) containing arrays and parameters for independent checking.

*Remark 1.1* (Entropy convention). We use natural logarithms. All entropies are measured in nats.

## 2 Operational framework: control model and meaning of power

**Definition 2.1** (Thermal state). Let  $X$  be finite-dimensional with Hamiltonian  $H_X$  and inverse temperature  $\beta := 1/(k_B T)$ . Define

$$\gamma_X := \frac{e^{-\beta H_X}}{\text{Tr}(e^{-\beta H_X})}.$$

**Definition 2.2** (Battery-assisted thermal operations (control primitive)). A battery-assisted thermal operation on  $S$  is any reduced map induced by an energy-conserving global unitary  $U$  on  $SBW$  satisfying  $[U, H_S + H_B + H_W] = 0$ , acting as

$$\rho_S \mapsto \text{Tr}_{BW}(U(\rho_S \otimes \gamma_B \otimes \sigma_W)U^\dagger).$$

**Definition 2.3** (Battery-accounted power of a strategy). Define battery non-equilibrium free energy  $\mathcal{F}_W(\sigma) := k_B T S(\sigma || \gamma_W)$ . For a control strategy  $\mathfrak{s}$  with battery trajectory  $\sigma_W^{\mathfrak{s}}(t)$ , define asymptotic power

$$P(\mathfrak{s}) := \limsup_{t \rightarrow \infty} \frac{1}{t} \left( \mathcal{F}_W(\sigma_W^{\mathfrak{s}}(t)) - \mathcal{F}_W(\sigma_W^{\mathfrak{s}}(0)) \right).$$

*Remark 2.1* (Wall-plug vs battery-accounted power). Measured electrical power typically upper-bounds battery-accounted power up to an implementation-dependent inefficiency factor. Experimental comparisons must state which proxy is used.

### 3 Operational definitions (minimal)

**Definition 3.1** (Relative entropy). For states  $\rho, \sigma$  on the same finite-dimensional Hilbert space, define

$$S(\rho\|\sigma) := \text{Tr}[\rho(\log \rho - \log \sigma)]$$

when  $\text{supp}(\rho) \subseteq \text{supp}(\sigma)$ , and  $S(\rho\|\sigma) := +\infty$  otherwise.

**Definition 3.2** (Energy pinching and default coherence functional). Let  $H = \sum_n E_n \Pi_n$  be the system Hamiltonian spectral decomposition. Define energy pinching

$$\Delta[\rho] := \sum_n \Pi_n \rho \Pi_n.$$

Unless stated otherwise, we take the coherence functional to be

$$\mathcal{C}(\rho) := S(\rho\|\Delta[\rho]).$$

**Definition 3.3** (Coherence-loss rate). For uncontrolled evolution  $\rho_t = e^{t\mathcal{L}}(\rho)$ , define

$$\dot{\mathcal{C}}_{\text{loss}}(\rho) := - \left. \frac{d}{dt} \mathcal{C}(\rho_t) \right|_{t=0}.$$

**Definition 3.4** (Default leakage proxy). For an operational bipartition into an accessible subsystem  $S$  and an effective exterior  $\text{Env}$ , define

$$\text{Leak}(\rho_{S\text{Env}}) := \frac{1}{2} \|\rho_{S\text{Env}} - \rho_S \otimes \rho_{\text{Env}}\|_1.$$

**Definition 3.5** (Incremental power proxy). Let  $\mathfrak{s}$  be a coherence-stabilizing strategy maintaining  $\rho$ , and let  $\mathfrak{s}_{\text{diag}}$  be a baseline strategy maintaining  $\Delta[\rho]$ . Define the incremental power

$$P_{\text{inc}} := P(\mathfrak{s}) - P(\mathfrak{s}_{\text{diag}}).$$

## 4 Logical status and dependency map

### 4.1 Limitations and scope clarifications (read before interpreting envelopes)

**(i) The rate bridge is the only non-derived interface.** The manuscript separates two logically distinct steps: (A) *rate domination*, i.e. the existence of an effective  $\kappa(\epsilon)$  such that  $\dot{\mathcal{C}}_{\text{loss}}(\rho(\epsilon)) \leq \kappa(\epsilon)\mathcal{C}(\rho(\epsilon))$ , and (B) *envelope inheritance*, i.e. that  $\kappa(\epsilon)$  admits a gapped-suppression envelope class (e.g.  $\kappa(\epsilon) \lesssim \text{poly}(m\epsilon)e^{-m\epsilon}$  in an appropriate regime). Step (A) is illustrated in standard Markovian models (Prototype 8.1). Step (B) is *not derived* here and is stated as a falsifiable interface hypothesis. What would count as substantive progress on (B) includes either: (i) a microscopic derivation in a Davies/weak-coupling setting with localized couplings where the relevant bath/coupling correlators across a tunable separation  $\epsilon$  can be shown to obey a gapped envelope, implying an explicit bound on  $\kappa(\epsilon)$ ; or (ii) converged numerical evidence in a controlled surrogate showing that an empirically extracted rate proxy decreases with  $\epsilon$  consistent with a gapped envelope class, with documented convergence checks.

**(ii) Type III AQFT content is a conditional blueprint, not a theorem.** Any discussion of Type III extensions should be read as *cartography of obstacles* rather than a completed derivation. In particular, the paper does not construct full Type III maintenance-power bounds. The genuine technical obstacles include: the use of Haagerup  $L_p$  spaces for noncommutative  $L_p$ -control, domain issues for modular generators, and physically meaningful energy constraints (or cutoff schemes) required to make control tasks operational. The role of the present manuscript is to identify which parts of the finite-dimensional argument structure plausibly extend and which steps require new operator-algebraic input.

Claim / object	Statement (short)	Status	Depends on / falsification
Maintenance inequality (incremental)	$P_{\text{extra}} \geq k_{\text{B}}T \dot{C}_{\text{loss}}$	Imported	Control-model mismatch or systematic violation after calibration
Collar amplitude envelope	$\text{Leak}(\epsilon)$ $(m\epsilon)^\alpha K_\nu(m\epsilon)$	$\lesssim$ Imported (model-specific)	No $m\epsilon$ collapse in a validated gapped Gaussian regime
One-shot work envelope	$W_{\text{min}}(\epsilon)$ $k_{\text{B}}T (m\epsilon)^{2\alpha} K_\nu(m\epsilon)^2$	$\gtrsim$ Conditional	Requires static-to-work interface + weak-correlation identification
Rate inheritance (envelope class)	$\kappa(\epsilon) \lesssim \text{poly}(m\epsilon)e^{-m\epsilon}$	Interface Hypothesis + Prototype	Measure $\kappa(\epsilon)$ proxies and refute envelope class
Power-vs- $\epsilon$ envelope	$P_{\text{extra}}(\epsilon)$ $k_{\text{B}}T \kappa(\epsilon) \mathcal{C}(\rho(\epsilon))$	$\gtrsim$ Conditional	Requires maintenance inequality + rate-domination interface

Table 1: Dependency map: what is imported, what is assumed, what is derived, and how to falsify.

**(iii) Regularization is a modeling choice; stability claims are limited.** The imported geometric envelopes are established in regularized massive Gaussian frameworks. Regularization is used here as a controlled setting in which a tunable separation parameter  $\epsilon$  is well-defined and scaling regimes can be probed. The manuscript does not claim that all constants or functional forms survive continuum limits. The intended robustness claim is *qualitative*: in gapped regimes, correlation proxies and (conjecturally) induced rates should exhibit exponential-in- $m\epsilon$  suppression up to polynomial prefactors. A full continuum statement would require specifying a limit procedure and proving that the relevant envelope class is stable under that procedure within the stated hypotheses.

## 5 Imported Results (from companion preprints)

### 5.1 Maintenance power bounds

**Imported Result 5.1** (Extra-power coherence maintenance bound [2, Thm. 4.12]). *Assume a finite-dimensional system at bath temperature  $T$ , uncontrolled Markovian evolution with a thermal fixed point, and a battery-assisted thermal-operations control model. Assume strategies exist that maintain  $\rho$  and that maintain  $\Delta[\rho]$ . Then*

$$P_{\text{extra}}(\rho; \mathcal{L}, T) \geq k_{\text{B}}T \dot{C}_{\text{loss}}(\rho).$$

### 5.2 Geometric suppression in Gaussian split regimes

**Imported Result 5.2** (Collar suppression/recoverability envelopes in massive Gaussian split regimes [1]). *In a regularized massive quasi-free (Gaussian) split configuration with mass gap parameter  $m > 0$  and collar width  $\epsilon > 0$ , cross-region correlation/recoverability quantities admit envelopes controlled by modified Bessel functions  $K_\nu(m\epsilon)$ , up to polynomial prefactors in  $m\epsilon$ , in the regime of validity (typically  $m\epsilon \gg 1$ ).*

*Remark 5.1* (Physical intuition for Bessel-type envelopes). In massive continuum models,  $K_\nu(m\epsilon)$  reflects the decay of the massive propagator with separation, so Bessel-type collar suppression can be viewed as a direct imprint of mass-gap-mediated attenuation across the interface.

## 6 Interface assumptions (static)

**Assumption 6.1** (Interface A: amplitude envelope for a leakage proxy). In a platform/regime where a split-like operational interface of width  $\epsilon$  is meaningful, assume there exist constants  $A > 0$  and  $\alpha \geq 0$  such that

$$\text{Leak}(\epsilon) \leq A (m\epsilon)^\alpha K_\nu(m\epsilon)$$

in the regime of interest (e.g.  $m\epsilon \gg 1$ ).

**Assumption 6.2** (Interface B: information identification in a weak-correlation regime). Assume that in the weak-correlation regime relevant for the chosen proxies,

$$I(S : \text{Env}) \asymp \text{Leak}(\epsilon)^2,$$

up to platform-dependent constants.

**Assumption 6.3** (Interface C: static-to-work link (correlation erasure cost)). At temperature  $T$ , any admissible protocol that reduces correlations from a baseline  $I_{\text{base}}(\epsilon)$  to a target  $I_{\text{tgt}}$  obeys

$$W_{\min}(\epsilon) \geq k_{\text{B}}T [I_{\text{base}}(\epsilon) - I_{\text{tgt}}]_+.$$

## 7 Conditional consequences: work vs power

**Conditional Proposition 7.1** (One-shot work envelope from static collar suppression). *Assume Assumptions 6.1, 6.2, and 6.3. Then*

$$W_{\min}(\epsilon) \gtrsim k_{\text{B}}T (m\epsilon)^{2\alpha} K_\nu(m\epsilon)^2,$$

up to constants and regime-dependent prefactors.

## 8 The unique missing bridge: static-to-dynamic rate inheritance

**Interface Hypothesis 8.1** (Rate interface hypothesis (two-part)). *Fix a platform/regime where a tunable interface width  $\epsilon$  is meaningful, and fix a target family  $\rho(\epsilon)$ .*

(A) **Rate domination.** *There exists an effective rate  $\kappa(\epsilon) \geq 0$  such that*

$$\dot{\mathcal{C}}_{\text{loss}}(\rho(\epsilon)) \leq \kappa(\epsilon) \mathcal{C}(\rho(\epsilon)).$$

(B) **Envelope inheritance (class).** *In a massive gapped regime with parameter  $m > 0$ , the effective rate admits an exponentially suppressed envelope in  $m\epsilon$  (up to polynomial prefactors), i.e. there exist  $B > 0$  and  $\beta \geq 0$  such that*

$$\kappa(\epsilon) \leq B (m\epsilon)^\beta e^{-m\epsilon}$$

in the regime of interest. In models where the envelope is known to be controlled by modified Bessel functions, one expects compatibility with  $K_\nu(m\epsilon)$  up to polynomial prefactors.

*Remark 8.1* (Model-dependent plausibility route). In Davies-type weak-coupling limits, transition and decoherence rates are controlled by bath spectral densities, i.e. Fourier transforms of bath correlation functions evaluated at Bohr frequencies [3, 4, 5]. If the operational collar width  $\epsilon$  suppresses the relevant system-bath coupling correlators across the interface by a massive envelope controlled by  $e^{-m\epsilon}$  (and, in certain continuum regimes, by  $K_\nu(m\epsilon)$  up to polynomial factors), it is plausible that induced rates inherit a comparable envelope (possibly squared depending on microscopic structure).

## 8.1 Prototype 1: single-qubit dephasing (exact computation)

**Prototype 8.1** (Qubit pure dephasing (exact  $\dot{C}_{\text{loss}}$ ; weak-coherence rate domination)). Consider a qubit in the energy basis with uncontrolled pure dephasing

$$\frac{d}{dt}\rho_{01}(t) = -\Gamma_{\phi}\rho_{01}(t), \quad \rho_{00}(t) \text{ constant.}$$

Let

$$\rho = \begin{pmatrix} p & c \\ c^* & 1-p \end{pmatrix}, \quad \Delta[\rho] = \begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix}.$$

Define  $z := 2p - 1$  and  $r := \sqrt{z^2 + 4|c|^2}$ . Then the coherence-loss rate for  $\mathcal{C}(\rho) = S(\rho||\Delta[\rho])$  satisfies (cf. [2, App. B])

$$\dot{C}_{\text{loss}}(\rho) = \frac{2\Gamma_{\phi}|c|^2}{r} \log\left(\frac{1+r}{1-r}\right).$$

Moreover, in the small-coherence regime  $|c| \ll 1$  at fixed  $p$ ,

$$\dot{C}_{\text{loss}}(\rho) = 2\Gamma_{\phi}\mathcal{C}(\rho) + O(|c|^4).$$

## 8.2 Prototype 2: buffer-chain surrogate (executed numerical first pass)

**Prototype 8.2** (Buffer-chain surrogate for  $\epsilon$ -dependent rate suppression (numerical first pass)). We simulate a gapped lattice surrogate in which  $\epsilon$  is a tunable operational separation. Consider a 1D transverse-field Ising chain of length  $N_{\text{tot}}$  with Hamiltonian

$$H = -J \sum_{i=0}^{N_{\text{tot}}-2} \sigma_x^{(i)} \sigma_x^{(i+1)} - h \sum_{i=0}^{N_{\text{tot}}-1} \sigma_z^{(i)}.$$

Define the system  $S$  to be site 0. For a buffer length  $L$ , define  $\epsilon := La$  and place a Markovian amplitude-damping channel on site  $j_{\text{diss}} = L + 1$  (first exterior site), with collapse operator  $\sqrt{\gamma} \sigma_-^{(j_{\text{diss}})}$ . To remain feasible on commodity hardware, we use quantum trajectories (Monte Carlo wave-function method) [5].

**Coherence and rate proxy.** We measure coherence on the reduced state  $\rho_S(t)$  of site  $S$  in the  $\sigma_x$  basis using relative-entropy coherence

$$\mathcal{C}_x(\rho_S) := S(\Delta_x[\rho_S]) - S(\rho_S),$$

and define the integrated post-lightcone rate proxy

$$\kappa_{\text{int}}(\epsilon) := \frac{1}{t_1(\epsilon) - t_0(\epsilon)} \log\left(\frac{\mathcal{C}_x(\rho_S(t_0(\epsilon)))}{\mathcal{C}_x(\rho_S(t_1(\epsilon)))}\right),$$

with  $t_0(\epsilon) = 0.8\epsilon/v$  and  $v = 2J$ . We use an adaptive window  $t_1 = t_0 + \tau$  where  $\tau = 1.5$  for  $\epsilon \leq 3$  and  $\tau = 2.5$  otherwise.

**First-pass result.** Figure 1 shows  $\kappa_{\text{int}}(\epsilon)$  decreasing with  $\epsilon$  in this gapped surrogate, consistent with an envelope-class version of rate suppression. This result is not claimed to be an exact  $K_{\nu}(m\epsilon)$  fit; the goal is to demonstrate a tunable-separation numerical testbed and an  $\epsilon$ -dependent suppression compatible with  $\text{poly}(m\epsilon)e^{-m\epsilon}$  in spirit.

**Numerical convergence note (first pass).** The displayed trend is qualitatively stable under increased sampling (more seeds and trajectories), though quantitative precision is limited by finite-size effects ( $N_{\text{tot}} = 12$ ) and residual trajectory noise. The raw arrays and run parameters are recorded by the simulation script in a data artifact suitable for reproduction.

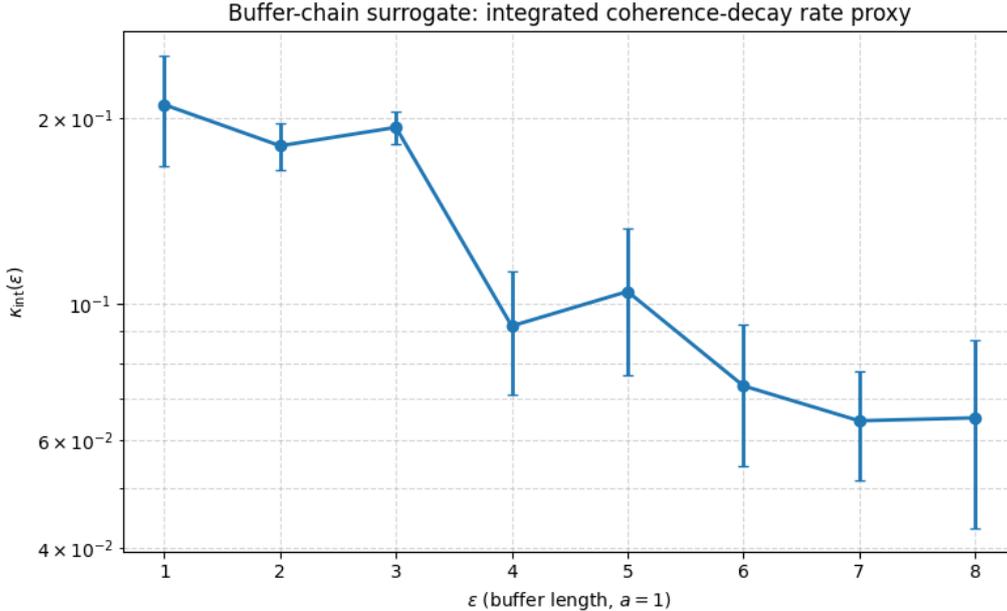


Figure 1: Buffer-chain surrogate (Prototype 8.2): integrated coherence-loss rate proxy  $\kappa_{\text{int}}(\epsilon)$  versus separation  $\epsilon = La$  for  $a = 1$ . Coherence is measured on the reduced state of site  $S$  in the  $\sigma_x$  basis as  $\mathcal{C}_x(\rho_S) = S(\Delta_x[\rho_S]) - S(\rho_S)$ . We use  $t_0(\epsilon) = 0.8\epsilon/v$  with  $v = 2J$  and an adaptive window  $t_1 = t_0 + \tau$  where  $\tau = 1.5$  for  $\epsilon \leq 3$  and  $\tau = 2.5$  otherwise. Error bars denote the standard deviation across 6 independent random seeds, each averaged over  $n_{\text{traj}} = 40$  quantum trajectories. Parameters:  $N_{\text{tot}} = 12$ ,  $J = 1$ ,  $h = 1.5$ ,  $\gamma = 0.2$ ,  $a = 1$ .

### 8.3 Maintenance power envelope given the rate interface

**Conditional Proposition 8.1** (Maintenance power envelope given rate domination). *Assume the imported maintenance inequality*

$$P_{\text{extra}}(\rho; \mathcal{L}, T) \geq k_{\text{B}}T \dot{\mathcal{C}}_{\text{loss}}(\rho),$$

and assume Hypothesis 8.1(A) holds for the target family  $\rho(\epsilon)$ . Then

$$P_{\text{extra}}(\epsilon) \geq k_{\text{B}}T \dot{\mathcal{C}}_{\text{loss}}(\rho(\epsilon)) \geq k_{\text{B}}T \kappa(\epsilon) \mathcal{C}(\rho(\epsilon)).$$

If additionally Hypothesis 8.1(B) holds and  $\mathcal{C}(\rho(\epsilon))$  is  $\epsilon$ -independent (or controlled independently), then

$$P_{\text{extra}}(\epsilon) \gtrsim k_{\text{B}}T (m\epsilon)^\beta e^{-m\epsilon},$$

up to constants and regime-dependent prefactors.

## 9 Master viewpoint: the cut as a resource boundary

**Definition 9.1** (Resource cut criterion). Fix a battery-accounted power budget  $P_{\text{avail}}$ . A resource cut occurs when required maintenance power exceeds available power, e.g.

$$P_{\text{extra}}(\rho; \mathcal{L}, T) > P_{\text{avail}}^{\text{extra}}.$$

*Remark 9.1* (Why this is not “vacuous”). The definition is a decision boundary. The scientific content lies in lower bounds and scaling relations that make the required power estimable and therefore render the criterion operational.

## 10 Macroscopic scaling: conjecture plus prototype

**Conjecture 10.1** (Macroscopic scaling under local noise and realistic control constraints). *Let  $\rho^{(N)}$  be a GHZ-like target family on  $N$  qubits and let the uncontrolled noise be approximately local dephasing with per-site rate  $\Gamma$  that is  $N$ -independent. Under physically relevant control constraints (e.g. bounded-strength, finite-range control, and explicit accounting of error-correction overhead where used), the incremental stabilization power satisfies*

$$P_{\text{extra}}(\rho^{(N)}; \mathcal{L}^{(N)}, T) = \Omega(N).$$

**Lemma 10.1** (Small-coherence scaling for relative-entropy coherence in a two-level block). *Fix  $p \in (0, 1)$ . For the  $2 \times 2$  family*

$$\rho = \begin{pmatrix} p & c \\ c^* & 1-p \end{pmatrix}, \quad |c| \text{ small},$$

*one has  $\mathcal{C}(\rho) = S(\rho \|\Delta[\rho]) \asymp |c|^2$  as  $|c| \rightarrow 0$ , with constants depending on  $p$ .*

*Proof sketch.* This follows from a Taylor expansion of  $S(\rho \|\Delta[\rho])$  in the off-diagonal parameter  $c$  at fixed diagonal  $p$ , using smoothness of eigenvalues in  $c$  for  $p \in (0, 1)$ .  $\square$

**Prototype 10.1** (GHZ under local dephasing:  $\Omega(N)$  scaling in a weak-coherence regime). *Consider the  $N$ -qubit subspace spanned by  $|0\rangle^{\otimes N}$  and  $|1\rangle^{\otimes N}$  and a state of the form*

$$\rho^{(N)} = \begin{pmatrix} p & c \\ c^* & 1-p \end{pmatrix}$$

*in that basis, with  $|c| \ll 1$ . Under local dephasing with per-site rate  $\Gamma$ , the off-diagonal decays as*

$$c(t) = c e^{-N\Gamma t}.$$

*By Lemma 10.1,  $\mathcal{C}(\rho^{(N)}) \asymp |c|^2$  at fixed  $p$ , hence*

$$\dot{\mathcal{C}}_{\text{loss}}(\rho^{(N)}) \asymp 2N\Gamma \mathcal{C}(\rho^{(N)})$$

*in this weak-coherence regime.*

## 11 Relation to standard decoherence and to resource theories

### 11.1 Relation to the decoherence program

*Remark 11.1* (Positioning relative to the decoherence program). Standard decoherence theory explains the passive suppression of interference under uncontrolled dynamics [6, 7, 8, 9]. The present framework emphasizes a complementary operational question: given a target coherent state, what is the minimum incremental stabilization power required to keep it stationary under an explicit thermodynamic control model? This shifts emphasis from decay timescales to unavoidable resource costs.

### 11.2 Relation to quantum resource theories of coherence

The present outlook uses a resource-theoretic control model (battery-assisted thermal operations), but addresses a different question than most “second-law” formulations: not which state transformations are possible, but what *minimum steady incremental power* is required to keep a target coherent state stationary under uncontrolled dynamics. The paired-strategy definition of  $P_{\text{extra}}$  is designed to isolate coherence-stabilization cost at fixed populations, complementing standard coherence/asymmetry constraints and catalytic viewpoints studied in, e.g., [10, 11, 12, 13].

## 12 Falsification protocols and candidate platforms

### 12.1 Falsification matrix (two distinct targets)

Target	Observable/proxy	Falsification criterion
Imported maintenance inequality	$P_{\text{inc}}$ and $\dot{C}_{\text{loss}}(\rho)$	Systematic observation of $P_{\text{inc}} < k_{\text{B}}T \dot{C}_{\text{loss}}(\rho)$ after controlling for population leakage and battery-accounting/proxy mismatch
Rate interface hypothesis (Hy-8.1)	$\kappa(\epsilon)$ proxies such as $\kappa_{\text{int}}(\epsilon)$	In a validated gapped regime, failure of any envelope class consistent with $\kappa(\epsilon) \lesssim \text{poly}(m\epsilon)e^{-m\epsilon}$ (and, where applicable, $K_{\nu}(m\epsilon)$ up to polynomial prefactors)

Table 2: Two separate experimental targets: one tests the thermodynamic maintenance inequality, the other tests the static-to-dynamic bridge.

### 12.2 Candidate platforms and practical proxies

Platform	Access to $\dot{C}_{\text{loss}}$	Power proxy	Main challenges
Superconducting qubits	Short-time tomography; $T_2$ -based modeling	Drive power + dissipation as upper bounds	Battery-accounted vs wall-plug gap; separating population leakage
Trapped ions	High-fidelity tomography; tunable effective couplings	Laser power monitoring (proxy)	Heating and technical noise; mapping $\epsilon$ operationally
Cold atoms / Rydberg arrays	Randomized measurements for $I_2$ proxies	Harder for power; good for correlation envelopes	Defining $\epsilon$ and isolating gapped regime; thermal environment modeling
NV centers	Well-characterized $T_2^*$ and dephasing channels	Microwave drive power proxy	Single-spin limits; environmental variability

Table 3: Laboratory-facing guidance: what is measurable and what the main obstacles are.

## 13 Conclusion

We reframed the Heisenberg cut as a resource boundary defined by control feasibility under explicit thermodynamic bookkeeping. We separated one-shot work consequences (static) from maintenance power consequences (dynamic), isolated the unique missing interface leap (rate inheritance) as a falsifiable hypothesis, and provided two worked prototypes: an exact single-qubit dephasing computation illustrating rate domination in a standard Markovian model, and an  $\epsilon$ -tunable buffer-chain surrogate with a first-pass numerical plot demonstrating  $\epsilon$ -dependent suppression of an empirical coherence-loss rate proxy in a gapped lattice setting.

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