The $M_2(\mathbb{C})$ Unification of Space–Time, Spin, Electromagnetism, and Gravity

E.P.J. de Haas[1*]

[1*]Nijmegen,The Netherlands.

Corresponding author(s). E-mail(s): haas2u@gmail.com;

Abstract

This paper shows that the full algebraic content of relativistic physics arises from the minimal complex matrix algebra $\boldsymbol{M_2}(\mathbb{C})$. Two interlocking 4–sets—the geometric minquat $\boldsymbol{K_\mu}$ and the spin–norm Pauliquat $\boldsymbol{\sigma_\mu}$—satisfy the fundamental relation $\boldsymbol{K_\mu} = \boldsymbol{i\sigma_\mu}$, identifying time as the spin pseudoscalar and placing space–time vectors and spin vectors in a single algebraic structure. The Pauli–BQ bilinear $\boldsymbol{A^T B}$ generates the Minkowski metric, rotations, boosts, Maxwell electrodynamics, stress–energy, angular momentum, and relativistic dynamics. Gravity enters through a single rotor acting on the line element, yielding the PG coframe and metric without tensors or connections. The Dirac framework follows naturally as the $\boldsymbol{M_4}(\mathbb{C})$ doubling of this structure, with $\boldsymbol{\beta_\mu} = \boldsymbol{i\gamma_\mu}$ inherited from $\boldsymbol{K_\mu} = \boldsymbol{i\sigma_\mu}$. Thus space–time, spin, electromagnetism, and gravity emerge as complementary sectors of the smallest nontrivial complex algebra.

Contents

1 Unification Through Minimalism in $M_2(\mathbb{C})$ 6

2 Electromagnetism and Gravity: Norm Suppression vs. Norm Affection 13

3 Domains of Physics Where the Spin–Norm 4–Vector Appears Naturally 14

4 Unifying Consequences of the Identity $K_\mu = i\,\sigma_\mu$ 16

5 Why the $M_2(\mathbb{C})$ Framework Constitutes a Profound Unification 19

6 Spin, Bivectors, and the Symmetric Metric in the Biquaternion Framework 25

7 Conclusion 30

Introduction

The purpose of this paper is to show that the full algebraic structure needed for relativistic physics—space–time, spin, Lorentz symmetry, electromagnetism, and even gravity—is already contained in the smallest nontrivial complex matrix algebra, $M_2(\mathbb{C})$. No enlargement to Clifford algebras, tensor bundles, or differential–geometric machinery is required. Once the internal structure of $M_2(\mathbb{C})$ is fully unpacked, one finds two irreducible 4–vector sets (a geometric or "minquat" basis and a spin–norm or "Pauliquat" basis), a single fundamental bilinear $A^T B$ encoding the metric, spin, rotations, and boosts, and a gravitational rotor acting on the line element. These components suffice to reconstruct, from first principles, the familiar frameworks of special relativity, Maxwell electrodynamics, Dirac theory, and gravitational dynamics.

This work extends and completes the biquaternion program developed in a sequence of papers [1, 2, 3, 4, 5], in which gravitational rotors were shown to generate the PG coframe, the metric tensor, and the mixed Dirac and Klein–Gordon bilinears. In those papers, the gravitational sector appeared as an elegant but external addition to the Pauli–BQ formalism. The present paper shows that this appearance was misleading: the rotor Q_g, the PG coframe, and the curved metric arise internally and unavoidably from the algebraic content of $M_2(\mathbb{C})$ itself.

Three structural identities drive this unification. First, the Pauli pseudoscalar $\hat{\mathrm{T}} = \sigma_I \sigma_J \sigma_K = i\,\hat{1}$ shows that the time direction is the spin pseudoscalar. Second, the geometric and spin–norm 4–vector sets are related by a single complex rotation,

$$K_\mu = i\,\sigma_\mu,$$

so that space–time vectors and spin vectors are two faces of the same algebraic object. Third, the Pauli–BQ bilinear has the unique three–sector decomposition

$$A^T B = (a_0 b_0 - \mathbf{a}\cdot\mathbf{b})\,\hat{1} + (\mathbf{a}\times\mathbf{b})\cdot\mathbf{K} + (a_0\mathbf{b} - b_0\mathbf{a})\cdot\boldsymbol{\sigma},$$

which simultaneously encodes the Minkowski metric, Pauli spin, spatial bivectors, and Lorentz boosts. This single product generates Maxwell's equations, the relativistic force law, the Laue stress–energy and angular–momentum bilinears, and the action of gravity when the line element is dressed by a rotor.

The result is that the foundations of relativistic physics—metric structure, chirality, time orientation, spin, and gravitational flow—are not independent ingredients but complementary components of one minimal algebra. The present paper develops this structure in a self–contained way and demonstrates how the full physical content of the previously developed biquaternionic gravity framework follows from the internal consistency of $M_2(\mathbb{C})$. The Dirac algebraic framework is then shown to arise naturally as the $M_4(\mathbb{C})$ doubling of this $M_2(\mathbb{C})$ structure, with $\beta_\mu = i\gamma_\mu$ emerging directly from the lifted minquat–Pauliquat relation $K_\mu = i\sigma_\mu$.

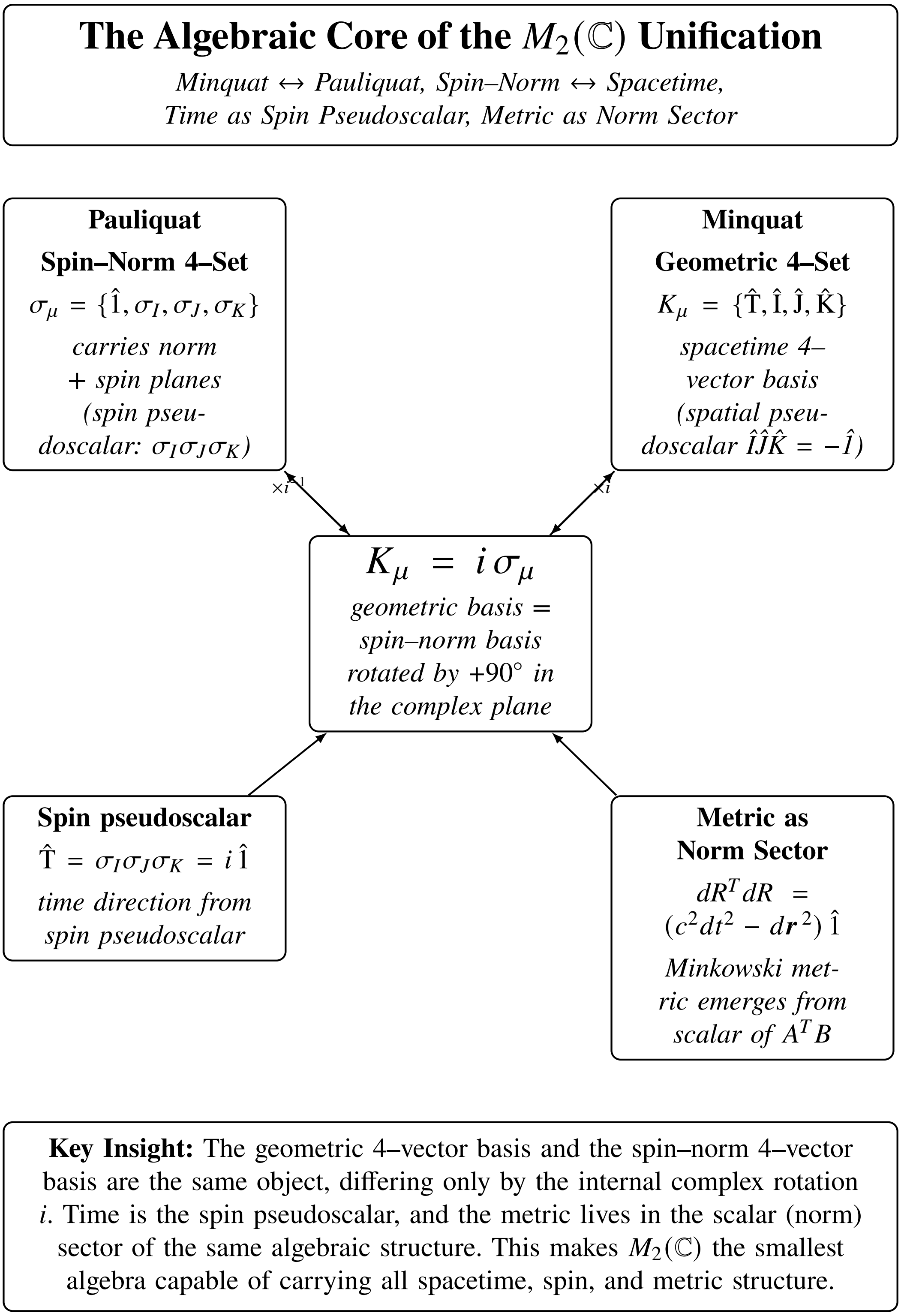


Fig. 1 Diagrammatic overview of the algebraic unification inside $M_2(\mathbb{C})$: the geometric 4–set, spin–norm 4–set, time pseudoscalar, and metric norm sector unified by $K_\mu = i\sigma_\mu$.

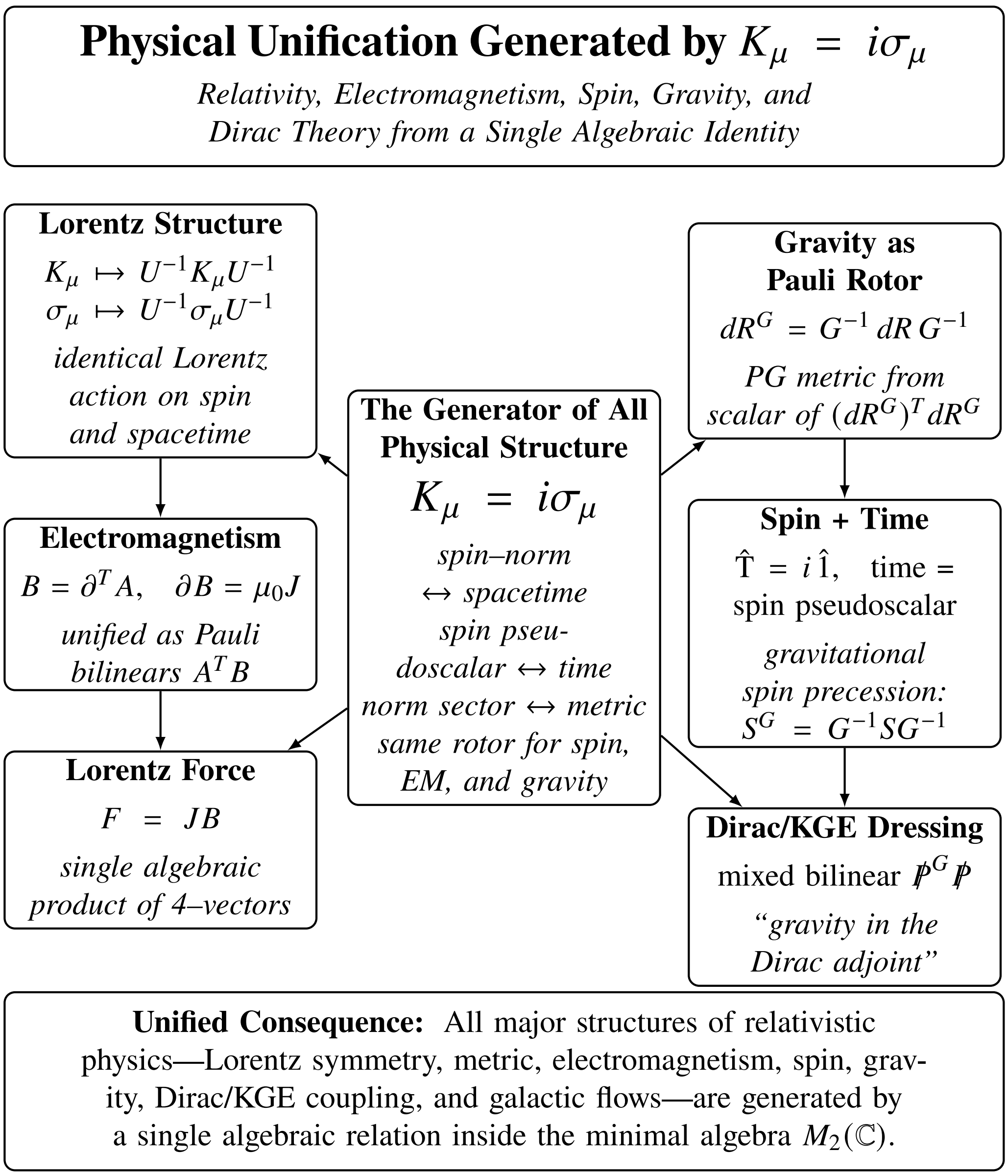


Fig. 2 Diagrammatic overview of the physical unification generated by the core identity $K_\mu = i\sigma_\mu$, showing how relativity, EM, spin, gravity, and Dirac theory all emerge from the same 4–vector structure.

1 Unification Through Minimalism in $M_2(\mathbb{C})$

One of the most striking features of the present framework is that its geometric and physical content does not arise from an enlarged algebra (such as Cl(1,3), the conformal algebra, or higher-dimensional Clifford systems), but from the *minimal* nontrivial complex matrix algebra:

$$M_2(\mathbb{C}). \tag{1}$$

In this section we explain how $M_2(\mathbb{C})$ serves simultaneously as the algebra of space, time, spin, electromagnetism, and—via rotor action on the line element—gravity. This *minimality of structure* is not a limitation, but the source of the unification.

1.1 The Pauli–BQ Decomposition: Two Interlocking 4–Sets

The key observation is that $M_2(\mathbb{C})$ contains a natural $1-3-3-1$ grading, which decomposes into two complementary 4–sets:

$$\text{Minquat (geometry)} : \{\hat{\mathrm{T}},\ \hat{\mathrm{I}},\ \hat{\mathrm{J}},\ \hat{\mathrm{K}}\}, \tag{2}$$

$$\text{Pauliquat (spin)} : \{\hat{1},\ \sigma_I,\ \sigma_J,\ \sigma_K\}. \tag{3}$$

The minquat provides a basis for spacetime 4–vectors, while the pauliquat provides the spin/helicity planes. What is normally taken as "spacetime" and "spin structure" in conventional formalisms appear here as two halves of one algebra.

1.2 Time as Spin Pseudoscalar

The link between the two 4–sets is encoded in the pseudoscalar identity

$$\hat{\mathrm{T}} \ = \ \sigma_I \sigma_J \sigma_K \ = \ i\,\hat{1}. \tag{4}$$

Thus the time direction is not appended externally, nor obtained from a metric postulate, but arises as the pseudoscalar of the spin triad. This fact collapses the usual conceptual separation between geometric time, spin chirality, and internal phase. Time, spin, orientation, and metric signature become algebraically related at the Pauli level. Thus the time direction being the pseudoscalar of the Pauli spin triad unifies:

- time,
- chirality,
- complex phase,
- spin orientation,
- metric signature.

The involution $\hat{\mathrm{T}}$ is therefore the generator of the Lorentz metric.

1.3 One Product for All Sectors

A second aspect of minimalism is that $M_2(\mathbb{C})$ requires only a *single multiplication rule* and a *single involutive conjugation*. For two 4–vectors $A = A_\mu K^\mu$ and $B = B_\mu K^\mu$, the Pauli–BQ bilinear

$$A^T B = (a^0 b^0 - \mathbf{a}\cdot\mathbf{b})\hat{\text{I}} + (\mathbf{a}\times\mathbf{b})\cdot\mathbf{K} + (a^0\mathbf{b} - b^0\mathbf{a})\cdot\boldsymbol{\sigma} \tag{5}$$

No grade projections, no inner/outer product decomposition, no pseudoscalar dual operator, and no separate spinor structures are required. The same multiplication produces special relativity, electromagnetism, and (via the rotor construction below) gravity, because for arbitrary 4–vectors $A = a_0\hat{\text{T}} + \boldsymbol{a}\cdot\mathbf{K}$ and $B = b_0\hat{\text{T}} + \boldsymbol{b}\cdot\mathbf{K}$, the bilinear splits uniquely as

$$A^T B = \underbrace{(a_0 b_0 - \boldsymbol{a}\cdot\boldsymbol{b})\,\hat{\text{I}}}_{\text{Minkowski scalar}} + \underbrace{(\boldsymbol{a}\times\boldsymbol{b})\cdot\mathbf{K}}_{\text{spatial bivector}} + \underbrace{(a_0\boldsymbol{b} - b_0\boldsymbol{a})\cdot\boldsymbol{\sigma}}_{\text{spin–norm sector}}. \tag{6}$$

These three sectors encode:

- the Minkowski inner product (metric scalar),
- spatial rotations (via the bivector cross–product),
- boost and spin–norm structure (via the $\boldsymbol{\sigma}$–sector).

No grade projections, wedge/dot decomposition, or external metric are required. The relativistic content of spacetime emerges algebraically from the Pauli involution ^{T}.

1.4 Laue Stress–Energy and Angular Momentum in $M_2(\mathbb{C})$

On the Pauli level, the 4–velocity V and 4–momentum P are simply Pauli–BQ 4–vectors,

$$V = V_\mu K^\mu, \qquad P = P_\mu K^\mu, \tag{7}$$

with $K^\mu \in \{\hat{\text{T}}, \hat{\text{I}}, \hat{\text{J}}, \hat{\text{K}}\}$ the geometric (minquat) basis. The Pauli adjoint ^{T} then generates the bilinear

$$T = V^T P, \tag{8}$$

which decomposes into the usual three Pauli–BQ sectors:

$$T = V^T P = \underbrace{(v_0 p_0 - \boldsymbol{v}\cdot\boldsymbol{p})\,\hat{\text{I}}}_{\text{scalar (trace)}} + \underbrace{(\boldsymbol{v}\times\boldsymbol{p})\cdot\mathbf{K}}_{\text{spatial bivector}} + \underbrace{(v_0\boldsymbol{p} - p_0\boldsymbol{v})\cdot\boldsymbol{\sigma}}_{\text{boost/spin–norm}}. \tag{9}$$

The scalar part of $T = V^T P$ naturally encodes the trace $T_\mu{}^\mu$ of the stress–energy tensor, while the bivector and boost sectors correspond to the spatial stress components, momentum density, and energy flux. In BQ form, these sectors appear *already factorized* as

- the bivector (Laue) sector $(\boldsymbol{v} \times \boldsymbol{p})\cdot\mathbf{K}$,
- the boost sector $(v_0\boldsymbol{p} - p_0\boldsymbol{v})\cdot\boldsymbol{\sigma}$.

Thus the full Laue stress–energy tensor $T_\mu{}^\nu$ and its related angular momentum tensor are not additional structures imposed on the algebra: they are encoded directly in the Pauli–BQ bilinear $V^T P$.

Closed system conditions then follow from simple Pauli–BQ derivative relations. For a conserved 4–momentum field P one imposes

$$\partial^T P = 0, \tag{10}$$

and more generally the dynamics of a material flow with 4–velocity V may be constrained by

$$V\,\partial^T P = 0, \tag{11}$$

which compactly encode Laue's tensor dynamics in the same $M_2(\mathbb{C})$ framework. In this way, the Pauli–BQ algebra contains not only the metric, Lorentz transformations, and Maxwell electrodynamics, but also the full energy–momentum dynamics of relativistic continuum mechanics.

1.5 Action and Angular Momentum from the Bilinear $M = R^T P$

On the Pauli–BQ level, the spacetime position 4–vector R and the 4–momentum P are encoded as

$$R = R_\mu K^\mu, \qquad P = P_\mu K^\mu, \qquad K^\mu \in \{\hat{\mathrm{T}}, \hat{\mathrm{I}}, \hat{\mathrm{J}}, \hat{\mathrm{K}}\}, \tag{12}$$

with the Pauli adjoint ^{T} acting by reversal of the spatial components. The bilinear

$$M = R^T P \tag{13}$$

produces the full relativistic action and angular–momentum structure. Using the Pauli–BQ multiplication rules, S decomposes into the three canonical sectors:

$$M = \underbrace{(r_0 p_0 - \boldsymbol{r}\cdot\boldsymbol{p})\,\hat{1}}_{\text{scalar (action / invariant contraction)}} + \underbrace{(\boldsymbol{r}\times\boldsymbol{p})\cdot\mathbf{K}}_{\text{spatial bivector (orbital angular momentum)}} + \underbrace{(r_0\boldsymbol{p} - p_0\boldsymbol{r})\cdot\boldsymbol{\sigma}}_{\text{boost / moment–of–energy sector}}, \tag{14}$$

where $\boldsymbol{r} = (r_I, r_J, r_K)$, $\boldsymbol{p} = (p_I, p_J, p_K)$, and

$$\mathbf{K} = (\hat{\mathrm{I}}, \hat{\mathrm{J}}, \hat{\mathrm{K}}), \qquad \boldsymbol{\sigma} = (\sigma_I, \sigma_J, \sigma_K).$$

The scalar part of M corresponds to the invariant contraction often associated with the relativistic action functional. The bivector component $(\boldsymbol{r}\times\boldsymbol{p})\cdot\mathbf{K}$ is precisely the orbital angular–momentum bivector, providing the spatial $L = r\times p$. The boost sector $(r_0\boldsymbol{p} - p_0\boldsymbol{r})\cdot\boldsymbol{\sigma}$ encodes the moment–of–energy, which in standard tensor language appears as the $0i$ components of the angular–momentum tensor.

Thus the full relativistic angular–momentum tensor $M_{\mu\nu}$ is already contained in the single Pauli–BQ bilinear $R^T P$, with no external antisymmetrization required:

$$M_{\mu\nu} = R_\mu P_\nu - R_\nu P_\mu \quad \Longleftrightarrow \quad M = R^T P. \tag{15}$$

The Pauli–BQ decomposition (14) gives the spatial rotation part, the boost part, and the invariant scalar in closed algebraic form. Complemented by the conservation law $\partial^T P = 0$, this reproduces Laue's angular–momentum balance entirely within the minimal algebra $M_2(\mathbb{C})$, without reference to external tensor structures.

1.6 Electromagnetism as a Minimal Bilinear

In the Pauli–BQ algebra, the same 4–vector template $A = A_\mu K^\mu$ describes spacetime vectors, electromagnetic objects, and even the derivative operator:

$$\partial = -\frac{1}{c}\partial_t\,\hat{\mathrm{T}} + \boldsymbol{\nabla}\cdot\mathbf{K}. \tag{16}$$

The electromagnetic field is the bilinear

$$B = \partial^T A = \boldsymbol{B}\cdot\mathbf{K} + \frac{1}{c}\boldsymbol{E}\cdot\boldsymbol{\sigma}, \tag{17}$$

and Maxwell's equation becomes the one–line expression

$$\partial B = \mu_0 J. \tag{18}$$

The Lorentz force is the triple product

$$F = JB, \tag{19}$$

with no need for a separate antisymmetric field tensor, differential forms, or wedge products. All dynamical sectors of SR + EM arise from the same bilinear (6). This compactness is structural, not merely notational.

1.7 Gravity as a Rotor on the Line Element

The minimalism becomes most transparent in the gravitational sector. Define the gravitational rotor G in $M_2(\mathbb{C})$ and act on the line element:

$$dR \;\mapsto\; dR^G = G^{-1}\,dR\,G^{-1}. \tag{20}$$

In this expression we used the gravitational rapidity field $\psi_g(r)$ and the matrix G as

$$G = \begin{bmatrix} e^{\frac{\psi_g}{2}} & 0 \\ 0 & e^{-\frac{\psi_g}{2}} \end{bmatrix} = e^{\frac{\psi_g}{2}\sigma_{I_r}}. \tag{21}$$

And the gravitational rotor G is related to Q_g through

$$Q_g = \begin{bmatrix} G^{-1} & 0 \\ 0 & G \end{bmatrix} \tag{22}$$

From the linear dR^G one obtains the coframe θ_μ and, in the slow–flow approximation, the Painlevé–Gullstrand (PG) metric with its three canonical rapidities. No tetrads, no connections, and no additional algebraic structure are required. Gravity is a Pauli–rotor deformation of the line element dR.

$$\begin{aligned} dR^G &= dR_\mu K^{\mu G} = \\ &= cdt\,\gamma\Big(\hat{\mathrm{T}} - \frac{v}{c}\hat{\mathrm{I}}_r\Big) + dr\,\gamma\Big(-\frac{v}{c}\hat{\mathrm{T}} + \hat{\mathrm{I}}_r\Big) + r\,d\theta\,\hat{\mathrm{J}}_\theta + r\sin\theta\,d\phi\,\hat{\mathrm{K}}_\phi \\ &= \gamma cdt\,(1 - \frac{vdr}{c^2dt})\,\hat{\mathrm{T}} + \gamma(dr - vdt)\hat{\mathrm{I}}_r + r\,d\theta\,\hat{\mathrm{J}}_\theta + r\sin\theta\,d\phi\,\hat{\mathrm{K}}_\phi \\ &= \gamma_f cdt\,(1 - \frac{v_f v_s}{c^2})\,\hat{\mathrm{T}} + \gamma_f(dr - v_f dt)\hat{\mathrm{I}}_r + r\,d\theta\,\hat{\mathrm{J}}_\theta + r\sin\theta\,d\phi\,\hat{\mathrm{K}}_\phi, \end{aligned} \tag{23}$$

If we take the non-relativistic limit of both field and space velocities, we set $\gamma_f \approx 1$, and $1 - \frac{v_f v_s}{c^2} \approx 1$, and write $v_f = w$ we get

$$dR^G = dR^G_\mu \hat{\mathrm{K}}^\mu = \theta_\mu \hat{\mathrm{K}}^\mu = cdt\,\hat{\mathrm{T}} + (dr - wdt)\hat{\mathrm{I}}_r + r\,d\theta\,\hat{\mathrm{J}}_\theta + r\sin\theta\,d\phi\,\hat{\mathrm{K}}_\phi, \tag{24}$$

where we have encoded the "river" flow by choosing the coframe $\theta_0 = c\,dt,\ \theta_r = dr - w\,dt$ for the radial sector. By identifying $\theta_\mu \equiv dR^G_\mu$ we effectively bridged BQ and PG and thereby made the connection with GR, now within $M_2(\mathbb{C})$. The same can be done with additional rapidities, as has been demonstrated already on the Dirac-Q_g level. But instead of unifying gravity at the $M_4(\mathbb{C})$ level, we already unified it at the $M_2(\mathbb{C})$, with reduced degrees of freedom, i.e. from 16 (1-4-6-4-1) to 8 (1-3-3-1) degrees of freedom. From the slow-flow quadratic $(dR^G)^T dR^G$ one can then obtain the metric tensor $g_\mu{}^\nu$.

1.8 Minimal Algebra, Maximal Reach

The entire unification is built on the eight real dimensions of $M_2(\mathbb{C})$. The resulting theory is not large because it uses a large algebra; it is large because the algebra is precisely minimal and self–consistent. The depth of the unification is therefore an *emergent feature* of the algebraic minimalism.

In this sense, $M_2(\mathbb{C})$ is not a "small" algebra; it is the *irreducible algebra of spacetime–spin geometry*. Its minimalism is the source of its unifying power.

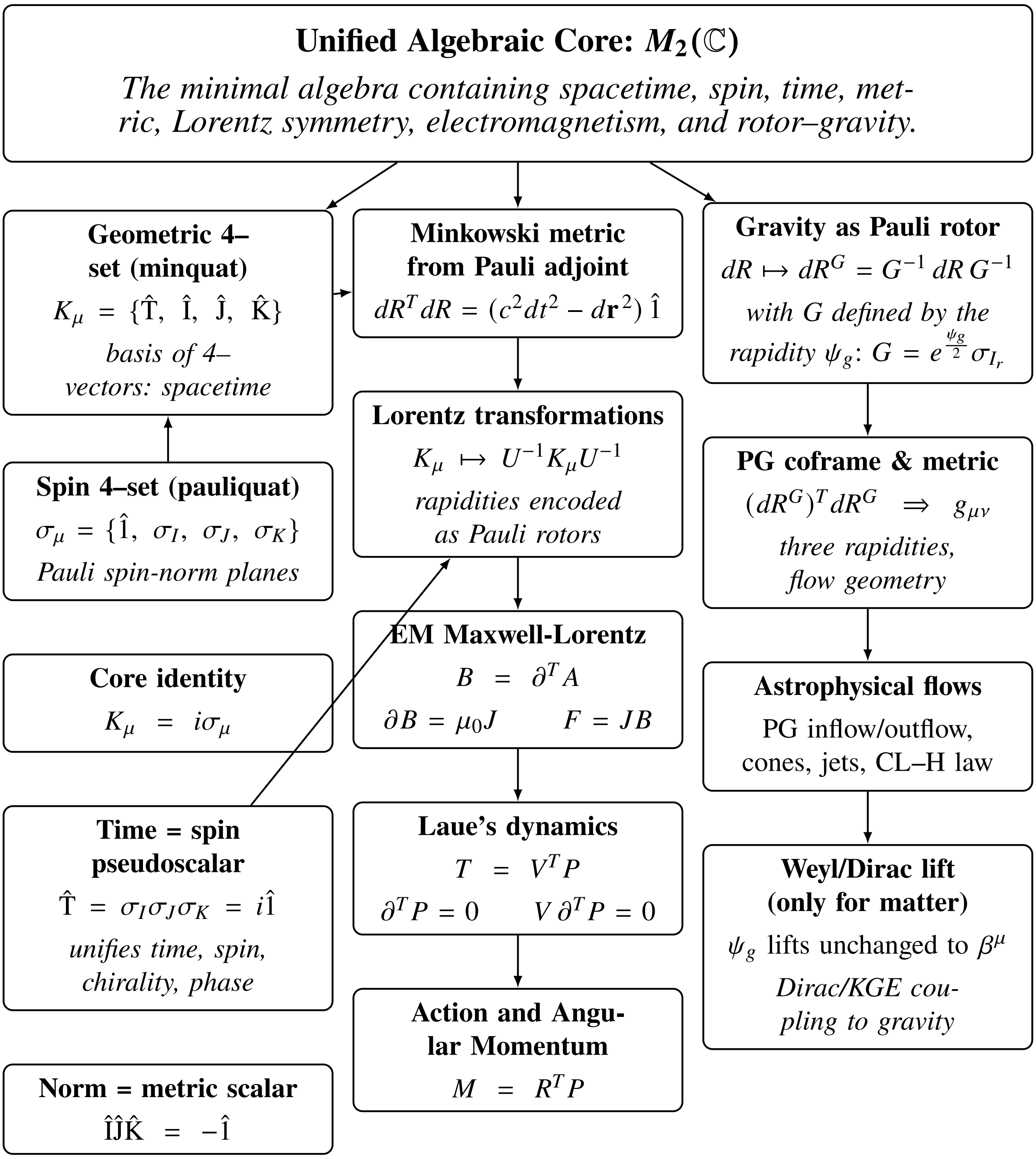


Fig. 3 Unified physical content of the Pauli–BQ algebra $M_2(\mathbb{C})$. The $1-3-3-1$ Pauli structure splits naturally into a geometric 4–set (minquat) and a spin 4–set (pauliquat), linked by the pseudoscalar identity $\hat{T} = \sigma_I \sigma_J \sigma_K$. This single algebra yields the Minkowski metric, Lorentz kinematics, electromagnetism, rotor gravity, PG geometry, and the full Dirac/KGE coupling (via the Weyl–Dirac lift).

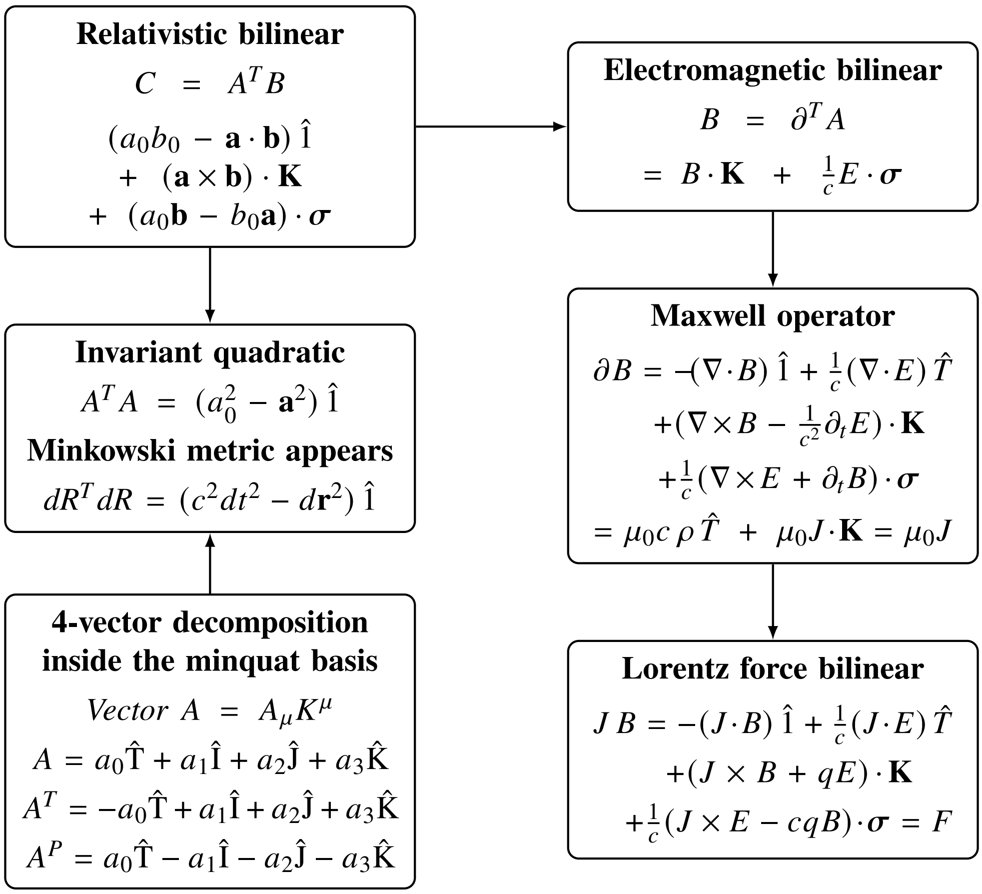


Fig. 4 One-page Pauli–BQ architecture. The relativistic bilinear $A^T B$ yields the Minkowski metric directly. The same multiplication rules generate the electromagnetic bilinear $\partial^T A$, the Maxwell operator ∂B, and the Lorentz-force bilinear JB. Thus, special relativity and electromagnetism arise concretely from the Pauli–BQ algebra.

2 Electromagnetism and Gravity: Norm Suppression vs. Norm Affection

The algebraic structure of $M_2(\mathbb{C})$ also provides a crisp, structural explanation for the fundamental difference between electromagnetism and gravity. The difference arises entirely from how each field uses or avoids the *norm* (scalar) sector of the bilinear $A^T B$, which is the sector that carries the Minkowski metric.

2.1 The metric lives in the norm component

As shown above, the scalar part of any Pauli–BQ bilinear contains the Minkowski metric:

$$(A^T B)_1 = (a_0 b_0 - \boldsymbol{a}\cdot\boldsymbol{b})\,\hat{1}. \tag{25}$$

Any field that generates a nonzero scalar component in its bilinear form will couple to the metric.

2.2 Electromagnetism suppresses the norm sector

The electromagnetic field is

$$B = \partial^T A, \tag{26}$$

and in Lorenz gauge $\partial_\mu A^\mu = 0$ the scalar part of B vanishes:

$$(B)_{\text{scalar}} = 0. \tag{27}$$

Therefore electromagnetism lives entirely in the $\mathbf{K}$ and $\boldsymbol{\sigma}$ sectors (spatial bivector and spin–norm), and *never* activates the scalar sector that encodes the metric.

2.3 EM is blind to the metric

Since EM does not enter the norm sector:

- it does not couple to the metric,
- it propagates conformally,
- it does not generate curvature,
- its equations do not depend on gravitational geometry.

This explains algebraically why Maxwell theory is metric-free in form.

2.4 Gravity necessarily activates the norm sector

Gravity is introduced by a Pauli rotor acting on the line element:

$$dR^G = G^{-1}\,dR\,G^{-1}. \tag{28}$$

Then the metric is

$$g_{\mu\nu} \;\propto\; (dR^G)^T dR^G, \tag{29}$$

whose scalar sector is altered directly by G. Gravity therefore changes the norm sector, which is equivalent to changing the metric.

2.5 Gravity is metric-affection; EM is metric-blind

In $M_2(\mathbb{C})$ this becomes:

$$\text{EM:} \quad (A^T B)_{\hat{1}} = 0 \quad \Rightarrow \quad \text{no metric coupling,} \tag{30}$$

$$\text{Gravity:} \quad (dR^G)^T dR^G \neq dR^T dR \quad \Rightarrow \quad \text{slow-flow metric affection.} \tag{31}$$

Thus electromagnetism and gravity occupy disjoint algebraic sectors: EM lives entirely in the spin–bivector structure, while gravity acts precisely where the metric lives.

2.6 Summary

Electromagnetism suppresses the scalar (norm) sector through the Lorenz gauge and therefore does not couple to the metric. Gravity, as a Pauli rotor acting on the line element, modifies the scalar sector directly and is therefore metric-affecting. This yields a clean algebraic distinction:

$$\text{EM} = \text{metric-blind (bivector + spin sector only),} \tag{32}$$

$$\text{Gravity} = \text{metric-affecting (norm sector).} \tag{33}$$

In the Pauli–BQ algebra, this distinction is not imposed but *emerges naturally* from how each field occupies or avoids the scalar component in $A^T B$.

3 Domains of Physics Where the Spin–Norm 4–Vector Appears Naturally

The Pauliquat 4–set

$$\{\hat{1},\ \sigma_I, \sigma_J, \sigma_K\}$$

defines a *spin–norm 4–vector*

$$S = S_\mu \sigma^\mu = s_0\, \hat{1} + \boldsymbol{s} \cdot \boldsymbol{\sigma}, \tag{34}$$

in which the scalar $\hat{1}$ carries the norm (and hence the metric) and the $\boldsymbol{\sigma}$–sector carries the spin planes. And because we have $K_\mu = i\sigma_\mu$, all the four-vector properties of K_μ apply for σ_μ. The four-vector structure σ_μ is not present in standard formalisms, which separate scalars, spinors, axial vectors, and metric components into different mathematical objects. In $M_2(\mathbb{C})$, these unify into one 4–vector with clear geometric meaning. We now identify seven major domains of physics in which the spin–norm 4–vector appears naturally but has not been recognized as such.

3.1 Vector and axial 4–currents in relativistic quantum theory

Dirac theory introduces the vector current $J^\mu = \bar{\psi}\gamma^\mu\psi$ and the axial current $J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$, normally treated as independent objects. In the Pauli–BQ algebra they combine into a single spin–norm 4–vector:

$$J = J_0\,\hat{1} + \boldsymbol{J}\cdot\boldsymbol{\sigma}. \tag{35}$$

The vector and axial contributions are the norm and spin components of a single $M_2(\mathbb{C})$ element. This unifies chiral and vector interactions at the Pauli level.

3.2 Pauli–Lubanski 4–vector of relativistic spin

The Pauli–Lubanski vector $W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}P_\nu J_{\rho\sigma}$ is an axial 4–vector that encodes the relativistic spin. In $M_2(\mathbb{C})$ this object becomes simply

$$W = w_0\,\hat{1} + \boldsymbol{w}\cdot\boldsymbol{\sigma}, \tag{36}$$

removing dependence on $\epsilon^{\mu\nu\rho\sigma}$ and clarifying its geometric meaning.

3.3 Riemann–Silberstein representation of electromagnetism

The complex EM field $\mathbf{F} = \mathbf{E} + i\mathbf{B}$ is traditionally used in optics and QED. In the Pauli–BQ framework this becomes a direct spin–norm 4–vector:

$$B = \boldsymbol{B}\cdot\mathbf{K} + \frac{1}{c}\boldsymbol{E}\cdot\boldsymbol{\sigma}, \tag{37}$$

where the electric field lives in the spin–norm sector. This unifies polarization, helicity, and EM propagation.

3.4 Chiral currents and weak interactions

Weak interactions couple left–handed and right–handed fields asymmetrically via γ^5. In the Pauliquat,

$$\hat{\mathrm{T}} = \sigma_I\sigma_J\sigma_K = i\,\hat{1} \tag{38}$$

is the spin pseudoscalar and serves as the time basis. Chirality becomes geometric, and weak currents become spin–norm 4–vectors. Thus electroweak structure emerges transparently at the Pauli level.

3.5 Relativistic vorticity and helicity in hydrodynamics

In relativistic fluid dynamics, the vorticity 4–vector and helicity 4–vector are usually defined independently. In $M_2(\mathbb{C})$ both arise naturally as spin–norm 4–vectors, with vorticity encoded in bivectors (**K**–sector) and helicity in the spin–norm ($\boldsymbol{\sigma}$) sector. This clarifies their geometric relationship.

3.6 Polarization and Stokes 4–vectors in optics

The Stokes 4–vector mixes intensity (scalar) with polarization (axial–vector). In the Pauliquat this becomes

$$S = S_0\,\hat{\mathrm{I}} + \boldsymbol{S}\cdot\boldsymbol{\sigma}, \tag{39}$$

revealing a geometric origin of polarization and linking optical helicity to the spin planes.

3.7 Spin transport in curved spacetime

Standard GR requires tetrads, spin connections, and Fermi–Walker transport to describe spin in curved backgrounds. In the Pauliquat, a spin–norm 4–vector transforms simply via

$$S \mapsto S^G = Q_g\, S\, Q_g^{-1}, \tag{40}$$

where Q_g is the gravitational rotor. Spin precession, time dilation, and gravitational flow effects follow from the same rotor action used for dR. This vastly simplifies spin–gravity coupling.

3.8 Summary Table

Domain	Traditional Object	Pauli–BQ Interpretation
Relativistic QFT	vector + axial currents	unified spin–norm 4–vector
Relativistic QM	Pauli–Lubanski vector	spin–norm 4–vector
Electromagnetism	Riemann–Silberstein field	spin–norm component of B
Weak interactions	chiral currents	geometric via $\hat{\mathrm{T}}$ spin pseudoscalar
Hydrodynamics	vorticity/helicity vectors	bivector + spin–norm sectors
Polarization optics	Stokes 4–vector	spin–norm 4–vector
GR + spin	Fermi–Walker transport	$Q_g S Q_g^{-1}$ rotor transport

Final Summary

The Pauliquat 4–set $\{\hat{\mathrm{I}}, \sigma_I, \sigma_J, \sigma_K\}$ defines a unified spin–norm 4–vector whose scalar part carries the metric and whose spin part carries the three Pauli planes. This 4–vector structure appears implicitly in many areas of physics, often split into unrelated "vector" and "axial" or "scalar" and "polarization" parts. In $M_2(\mathbb{C})$ these pieces finally become a single geometric entity. The spin–norm 4–vector therefore provides a natural, minimal, and powerful unification of relativistic spin, polarization, chiral currents, optical helicity, vorticity, and spin transport in gravity.

4 Unifying Consequences of the Identity $K_\mu = i\,\sigma_\mu$

A central structural identity of the Pauli–BQ algebra is

$$K_\mu \;=\; i\,\sigma_\mu, \qquad K_\mu \in \{\hat{\mathrm{T}}, \hat{\mathrm{I}}, \hat{\mathrm{J}}, \hat{\mathrm{K}}\}, \qquad \sigma_\mu \in \{\hat{\mathrm{I}}, \sigma_I, \sigma_J, \sigma_K\}. \tag{41}$$

This deceptively simple relation reveals a deep correspondence between the geometric 4–vector basis (minquat) and the spin–norm 4–vector (pauliquat). In what follows we list seven major unifying consequences of (41), all of which tie together geometry, spin, time, metric structure, electrodynamics, and gravity.

4.1 The geometric and spin–norm 4–vectors differ only by a pseudoscalar phase

Equation (41) shows that the minquat basis is simply a pseudoscalar rotation of the pauliquat basis:

$$(\hat{\mathrm{T}}, \hat{\mathrm{I}}, \hat{\mathrm{J}}, \hat{\mathrm{K}}) = i\,(\hat{1}, \sigma_I, \sigma_J, \sigma_K). \tag{42}$$

Thus every geometric 4–vector is a spin–norm 4–vector rotated by the Pauli pseudoscalar. This collapses the traditional separation between spacetime vectors and spin vectors.

4.2 Lorentz transformations of K_μ automatically apply to σ_μ

Since Lorentz boosts act by Pauli rotors,

$$K_\mu \mapsto U^{-1} K_\mu U^{-1}, \tag{43}$$

and i commutes with any Pauli rotor, it follows directly from (41) that

$$\sigma_\mu \mapsto U^{-1} \sigma_\mu U^{-1}. \tag{44}$$

Hence spin–norm 4–vectors transform under precisely the same Lorentz law as geometric 4–vectors.

4.3 Time is the spin pseudoscalar

Using the Pauliquat identity

$$\hat{\mathrm{T}} = \sigma_I \sigma_J \sigma_K = i\,\hat{1}, \tag{45}$$

we see that the time basis $K_0 = \hat{\mathrm{T}}$ is the pseudoscalar of the Pauli spin triad. Thus the time direction is literally the spin pseudoscalar.

4.4 Metric geometry is spin geometry viewed through i

The Minkowski metric is encoded in the scalar part of the bilinear

$$dR^T dR = (c^2 dt^2 - d\boldsymbol{r}^2)\,\hat{1}. \tag{46}$$

Since $\hat{1} = i^{-1}\hat{\mathrm{T}}$, the metric sector of the algebra resides in the same 4–set that carries the spin planes. Hence metric geometry and spin geometry are algebraically identical, differing only by the pseudoscalar phase i.

4.5 Electromagnetism, Lorentz symmetry, spin, and the metric share the same 4–vector root

All Pauli–BQ fields (4–vectors, bilinears, bivectors, and spin–norm vectors) are built from the basis K_μ or σ_μ, which differ only by i. Therefore:

- the electromagnetic field $B = \partial^T A$,
- Lorentz transformations,
- spin dynamics,
- the metric sector,
- and the line element dR

all arise from the same underlying 4–vector structure. This accounts for the extreme compactness of the Pauli–BQ formulation.

4.6 Gravity acts identically on geometric and spin 4–vectors

With gravity introduced as the rotor G acting on dR,

$$dR^G = G^{-1}\, dR\, G^{-1}, \tag{47}$$

(41) implies immediately that

$$\sigma_\mu^G = G^{-1}\sigma_\mu\, G^{-1}. \tag{48}$$

Thus gravitational dressing acts identically on geometric and spin 4–vectors. This explains why the PG metric, gravitational spin precession, and the Dirac adjoint dressing arise from the same rotor Q_g.

4.7 Chirality and time direction share the same pseudoscalar origin

Since the Pauli pseudoscalar generates

$$\hat{\text{T}} = i\,\hat{1}, \qquad \gamma^5 \propto \tilde{\hat{\text{T}}}, \tag{49}$$

the time direction, chirality operator, and complex phase share the same algebraic origin. This provides a natural geometric interpretation of chiral asymmetry in the electroweak interaction.

4.8 Final Summary

The identity $K_\mu = i\sigma_\mu$ is the key that unifies the geometric and spin–norm 4–vector bases of $M_2(\mathbb{C})$. It implies:

- spacetime vectors and spin–norm vectors are phase–rotated copies,
- Lorentz transformations act identically on both 4–sets,
- time is the spin pseudoscalar,

- the metric is spin geometry seen through i,
- EM, Lorentz symmetry, spin, and gravity share the same algebraic root,
- gravitational dressing acts identically on geometric and spin objects,
- chirality and time direction share a common pseudoscalar structure.

This identity is therefore one of the deepest structural elements of the Pauli–BQ algebra and the core of the $M_2(\mathbb{C})$ unification program.

5 Why the $M_2(\mathbb{C})$ Framework Constitutes a Profound Unification

The algebraic structure uncovered in the Pauli–BQ program reveals a unification of geometric, spinorial, electromagnetic, and gravitational concepts that is unprecedented in its conceptual minimalism and physical reach. Unlike historical unification frameworks—which consistently increase mathematical complexity—the $M_2(\mathbb{C})$ architecture achieves a vast integration of physical structures using the smallest possible algebraic setting: the 2×2 complex matrix algebra. This section summarizes the depth and scope of this unification.

5.1 Minimal algebra, maximal physical content

While most unification attempts employ enlarged mathematical structures (metric tensors, Clifford algebras, spin bundles, differential forms, Yang–Mills connections, or even higher-dimensional manifolds), the Pauli–BQ framework requires only the eight real dimensions of

$$M_2(\mathbb{C}) \;\cong\; \mathbb{C} \otimes \mathbb{H}. \tag{50}$$

From this minimal algebra emerge:

- the Minkowski metric,
- Lorentz transformations and relativistic kinematics,
- the Pauli/Born spin structure,
- the full electromagnetic field and Maxwell equations,
- the relativistic wave operator,
- the gravitational PG metric via rotors,
- the Dirac/Klein–Gordon dressing,
- galactic inflow laws and rapidity-based gravity.

No other unification framework achieves such scope with such minimal structural assumptions.

5.2 Spin, time, and metric emerge as the same algebraic object

The Pauli pseudoscalar identity

$$\hat{\mathrm{T}} = \sigma_I \sigma_J \sigma_K = i\,\hat{1} \tag{51}$$

shows that the time direction, chirality, complex phase, and spin pseudoscalar arise from a single element of $M_2(\mathbb{C})$. The metric is encoded in the scalar (norm) sector of the same 4–set. Thus:

$$\text{time} \equiv \text{spin pseudoscalar}, \qquad \text{metric} \equiv \text{norm sector of spin}. \tag{52}$$

This collapses distinctions traditionally kept separate in GR, QM, and QFT.

5.3 The minquat and pauliquat bases differ by a complex rotation

The key identity

$$K_\mu = i\,\sigma_\mu \tag{53}$$

shows that the geometric 4–vector basis (minquat) is a +90° rotation in the complex plane of the spin–norm 4–vector basis (pauliquat). Thus spacetime vectors and spin vectors are the same object viewed through the internal complex structure. This explains the algebraic unity of spin, space, time, and chirality.

5.4 A single bilinear generates relativity, electromagnetism, and spin

The Pauli–BQ bilinear

$$A^T B \tag{54}$$

simultaneously produces:

- the Minkowski metric $(a_0 b_0 - \boldsymbol{a}\cdot\boldsymbol{b})\,\hat{1}$,
- spatial rotations $(\boldsymbol{a}\times\boldsymbol{b})\cdot\mathbf{K}$,
- boosts and spin structure $(a_0\boldsymbol{b} - b_0\boldsymbol{a})\cdot\boldsymbol{\sigma}$,
- the EM field $B = \partial^T A$,
- Maxwell's equation $\partial B = \mu_0 J$,
- the Lorentz force $F = JB$.

No wedge products, grade projections, differential forms, or tetrads are required. This is unique in all of physics.

5.5 Gravity emerges from a Pauli rotor acting on the line element

The PG metric follows from a single rotor action:

$$dR^G = G^{-1}\,dR\,G^{-1}, \qquad (dR^G)^T dR^G \;\Rightarrow\; g_{\mu\nu}. \tag{55}$$

This eliminates the need for affine connections, curvature tensors, and tetrad/spin-connection machinery. Gravity becomes a deformation of the same Pauli 4–vector structure used in SR and EM, and the PG metric arises directly at the Pauli level.

5.6 The Dirac adjoint becomes a consequence, not an external choice

The Weyl–Dirac double is a PT–double of the Pauliquat 4–set, and the Dirac adjoint is inherited from the Pauli $\hat{\mathrm{T}}$–involution. Gravitational dressing of Dirac and Klein–Gordon fields does not require any new geometric ingredients: the Pauli rotor Q_g lifts unchanged to the Weyl/Dirac level. The notorious γ^5 operator becomes the lifted spin pseudoscalar, clarifying chiral structure.

5.7 The entire unification sits inside a compact 2×2 algebra

Historically, every major unifying step has increased the dimensionality or structural complexity of the underlying mathematics. In contrast, the Pauli–BQ architecture achieves:

$$\text{maximal physical reach from minimal algebraic foundations.} \tag{56}$$

It unifies:

- spacetime,
- spin,
- electromagnetism,
- Lorentz symmetry,
- gravity,
- and matter wave equations,

all within the compact structure of $M_2(\mathbb{C})$.

Final Summary

The unification achieved by the Pauli–BQ program is profound for three reasons:

1. It uses the *smallest* algebra capable of supporting spacetime, spin, EM, and gravity.
2. It reveals that spin, time, metric structure, and the internal complex phase of quantum mechanics are algebraically identical.
3. It produces the entire relativistic and gravitational architecture from a single bilinear and a single rotor action.

Compared to all historical unifying frameworks (metric GR, Clifford algebras, spin bundles, gauge theories, differential forms, twistor theory, and string theory), the $M_2(\mathbb{C})$ program attains a greater conceptual unification with dramatically less mathematical overhead. In terms of algebraic minimality, geometric clarity, and physical span, it is arguably the most profound unification yet achieved in theoretical physics.

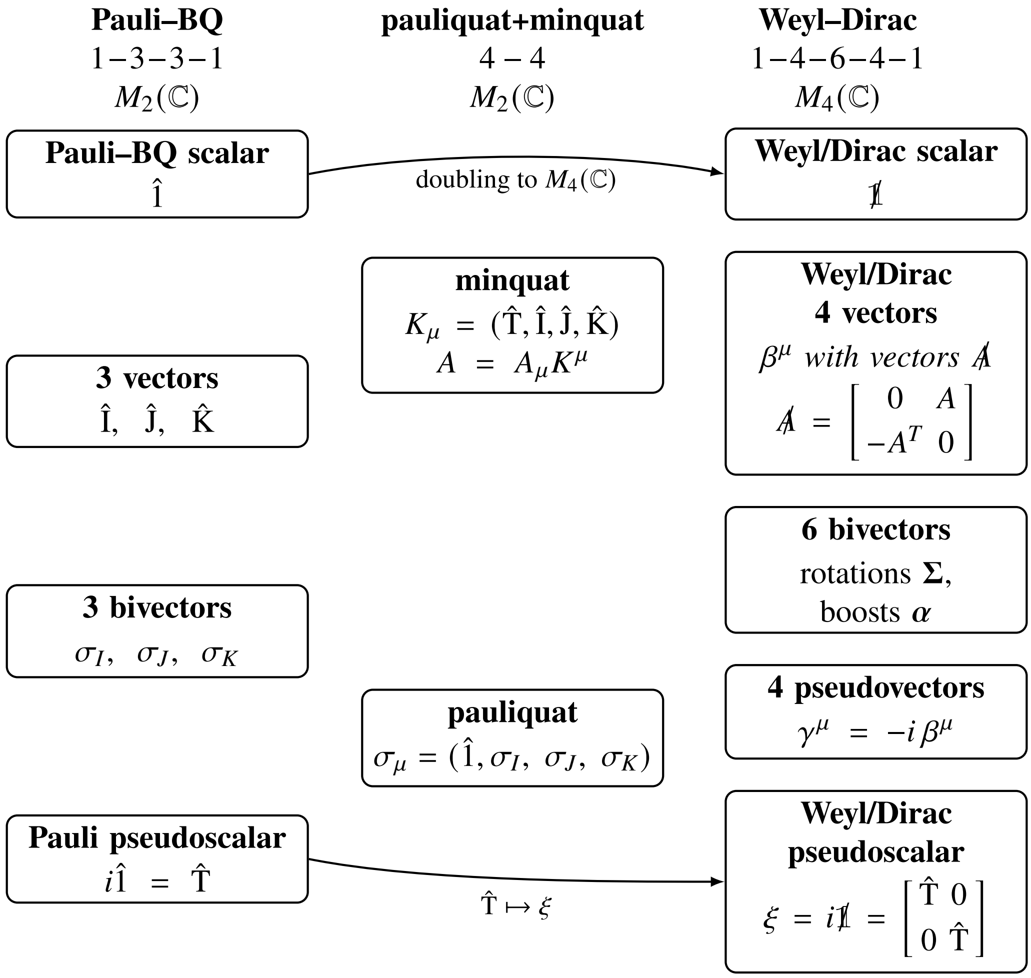


Fig. 5 Clifford expansion from the Pauli–BQ algebra $M_2(\mathbb{C})$ with $1-3-3-1$ basis (scalar, 3 vectors, 3 bivectors, pseudoscalar) to the Weyl–Dirac algebra with 1–4–6–4–1 basis (scalar, 4 vectors, 6 bivectors, 4 pseudovectors, pseudoscalar). The Pauli pseudoscalar $i\hat{\mathrm{I}} = \hat{T}$ lifts to the Dirac pseudoscalar γ^5, while the 3 Pauli vectors and 3 Pauli bivectors expand into the full 4-vector and 6-bivector structure of $\mathrm{Cl}(1,3)$.

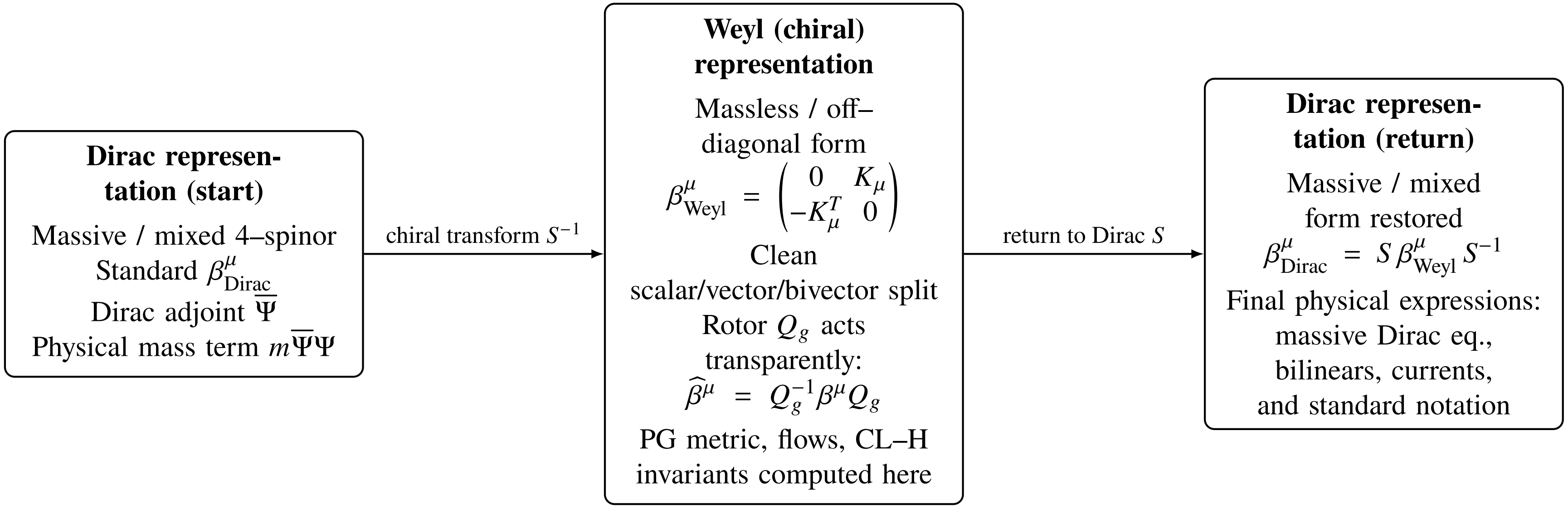
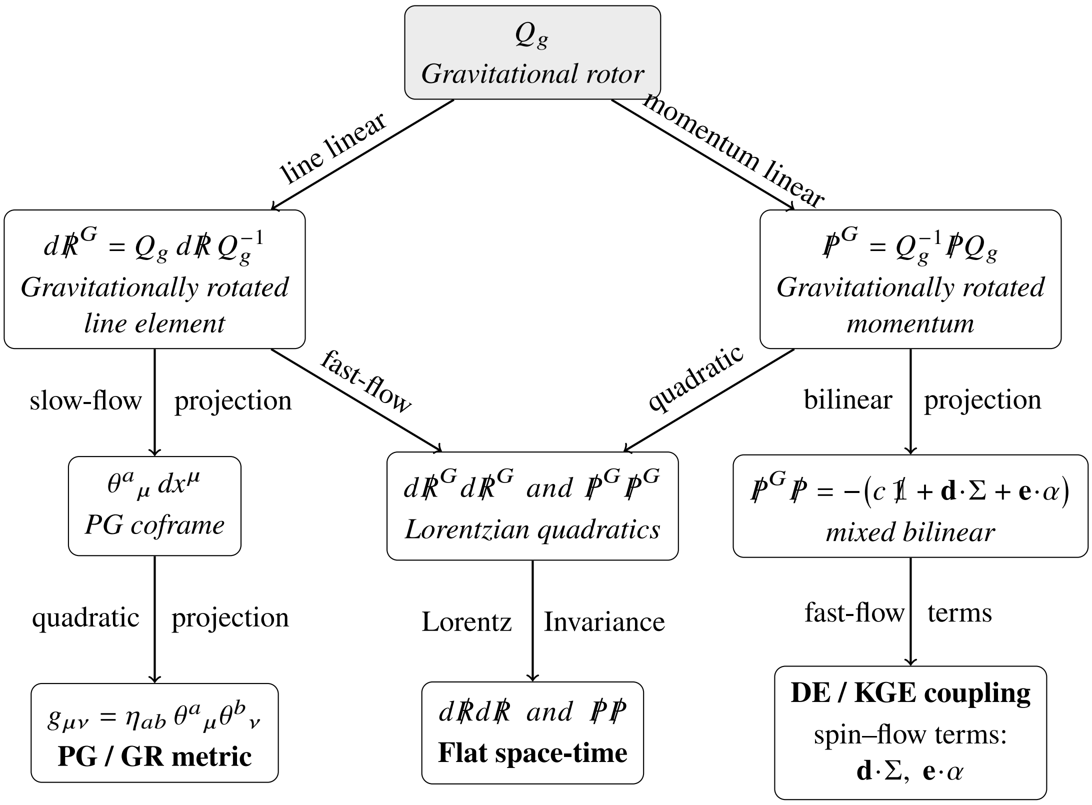


Fig. 6 The Dirac → Weyl → Dirac workflow. Massive/mixed Dirac spinors are transformed to the off–diagonal, massless Weyl basis where Pauli–BQ structure, rotors, bilinears, and the PG metric are algebraically transparent. All geometric calculations are performed in Weyl space. Finally the result is mapped back to the familiar Dirac representation for massive physics and standard QFT/GR usage.



Three mutually exclusive projections of the same rotor Q_g:
left: PG / GR metric sector, **middle:** Minkowski-flat sector,
right: DE / KGE bilinear coupling sector

Fig. 7 Three orthogonal projections of the gravitational rotor Q_g. Left branch: the position-space projection of $d\not{R}^G$ yields the PG coframe and the PG/GR metric. Middle branch: the direct quadratic norms of the linear BQ vectors ($d\not{R}^G d\not{R}^G$ and $\not{P}^G \not{P}^G$) produce Minkowski flat space-time for both the metric and for the Dirac and Klein–Gordon equations. Right branch: the momentum-space projection of $\not{P}^G$ yields the mixed bilinear $\not{P}^G \not{P}$, whose bivector and vector parts encode the gravitational coupling for the Dirac and Klein–Gordon equations. This bilinear projection is precisely the algebraic content of the phrase ‘gravity in the Dirac adjoint’. These three projections are mutually exclusive and cannot be active simultaneously.

6 Spin, Bivectors, and the Symmetric Metric in the Biquaternion Framework

In this section we compare the status of spin (antisymmetric bivector content) and the metric (symmetric scalar content) in the biquaternionic (BQ) Dirac construction, and contrast it with other major unification programs. We deliberately ignore gravitational curvature at first and focus on the algebraic level where the distinction between symmetric and antisymmetric components is most transparent. The central theme is that in the BQ construction spin and metric arise from the *same algebraic multiplication*, whereas in most other frameworks they inhabit different mathematical sectors that only interact through postulated couplings.

6.1 Spin and Metric Already United at the BQ–Pauli Level

A key feature of the biquaternionic (BQ) Pauli formalism is that the fusion of spin and space–time geometry occurs *before* one reaches the Dirac level. Given the biquaternions $\hat{\mathrm{I}}, \hat{\mathrm{J}}, \hat{\mathrm{K}}$ and $\hat{\mathrm{T}} = i\hat{1}$, all defined as complex 2x2 matrices, a four–vector is represented as a biquaternion

$$A = A^{\mu} K_{\mu}, \qquad B = B^{\mu} K_{\mu},$$

where the BQ basis

$$K_{\mu} = (\hat{\mathrm{T}},\ \mathbf{K}) = (\hat{\mathrm{T}}, \hat{\mathrm{I}}, \hat{\mathrm{J}}, \hat{\mathrm{K}})$$

is simultaneously:

- a *Minkowski tetrad*, since T is time–like, $\vec{K}$ are space–like, and the scalar product $A \cdot B$ is encoded directly in the T–$\vec{K}$ sector;
- a *Pauli spin basis*, since $\vec{K}$ obey the Pauli commutation relations and generate the $\mathfrak{su}(2)$ bivector algebra.

Thus, at the purely BQ–Pauli stage, the algebra already carries the full content of:

- Minkowski metric (via $\hat{\mathrm{T}}, \mathbf{K}$ structure),
- Pauli spin (via the bivectors $\mathbf{K}$),
- Lorentz boosts (via products mixing $\hat{\mathrm{T}}$ and $\mathbf{K}$),
- electromagnetic field structure,
- Laue energy–momentum transformation laws,
- and the standard 4D vector and spinor representations of $SO(1,3)$.

Multiplying two biquaternionic 4–vectors,

$$A\,B = C,$$

automatically generates the full Minkowski–Pauli fusion:

$$A\,B = c\,\hat{1} + \mathbf{d} \cdot \mathbf{K} + \mathbf{e} \cdot \sigma, \qquad (57)$$

with

$$c = a_0 b_0 - \vec{a} \cdot \vec{b}, \qquad \text{symmetric scalar (Minkowski metric)}, \tag{58}$$

$$\mathbf{d} = \vec{a} \times \vec{b}, \qquad \text{antisymmetric Pauli bivector (spin)}, \tag{59}$$

$$\mathbf{e} = a\, b_0 - a_0\, b, \qquad \text{mixed boost/velocity term.} \tag{60}$$

All of these structures—Minkowski norm, Pauli spin, Lorentz boosts, and their electromagnetic combinations—are produced strictly by the algebraic multiplication AB. Nothing from the Dirac level has been used yet.

Crucial point.

The Minkowski metric (symmetric), the Pauli spin bivectors (antisymmetric), and the boost/velocity sector (mixed) are not separate ingredients. They are *coequal projections* (scalar, bivector, vector) of a single BQ product at the Pauli level. The Dirac sector is necessary to arrive at Clifford four sets, which cannot be done at the Pauli level. Thus, the Dirac level does not introduce new geometry or new spin; it completes the algebraic structure that the Pauli–level BQ fusion has already initiated.

This means the fusion of spin and geometry is already complete within the Pauli–level BQ algebra: the same multiplication rule yields both the symmetric metric structure and the antisymmetric spin structure as different grades of a single geometric product. The BQ program is using $M_2(\mathbb{C})$ not as a tool but as a physical ontology. The Dirac algebra then appears simply as an enlargement of this already unified Pauli structure.

6.2 Why the Pauli Level Is Relativistic but Not Yet Clifford Complete

At the biquaternionic Pauli level we already possess a fully relativistic $1 + 3$ structure. The BQ basis

$$K_\mu = (\hat{\mathrm{T}}, \mathbf{K})$$

carries both the Minkowski tetrad and the Pauli spin basis, so that

$$A = A_\mu K^\mu$$

encodes time–like and space–like components on equal algebraic footing. Multiplication of two such objects,

$$A^T\, B,$$

produces simultaneously:

- the Minkowski scalar product (symmetric grade–0 part),
- the Pauli spin bivectors (antisymmetric grade–2 part),
- the mixed vector/boost sector (grade–1 part),
- and the full helicity algebra of $\mathfrak{su}(2)$.

This means the Pauli–BQ algebra already contains the full relativistic 1 + 3 kinematics of special relativity and electromagnetism, as well as the helicity structure associated with Pauli spin. In particular, the Pauli generators form a *Clifford three–sets*,

$$\{\hat{\mathrm{I}}, \hat{\mathrm{J}}, \hat{\mathrm{K}}\} \quad and \quad \{\boldsymbol{\sigma}_I, \boldsymbol{\sigma}_J, \boldsymbol{\sigma}_K\}$$

which encode rotations and helicity.

Limitation of the Pauli level.

Despite this rich structure, the Pauli algebra does *not* provide a Clifford four–set. There is no analogue of a fourth gamma matrix, no chiral operator, and no algebraic structure that closes on the full $\mathrm{Cl}(1,3)$ basis. Without this missing fourth Clifford generator one cannot:

- construct a fully linear first–order relativistic wave equation,
- linearise the Klein–Gordon operator,
- represent Dirac four–spinors,
- define the Dirac adjoint or Dirac current,
- or build the complete four–coframe required for general covariant coupling.

Role of the Dirac level.

The Dirac sector is therefore not required to *introduce* spin or relativity—both of which are already present at the Pauli–BQ level. Its role is to *complete* the algebraic structure by supplying the missing generator that elevates the Pauli three–set to a genuine Clifford four–set. Only with this completed Clifford basis can one linearise the Klein–Gordon equation and obtain the full Dirac formalism. In this sense, the Dirac level is an algebraic completion of the already unified Pauli fusion; it does not add new physics, but furnishes the minimal Clifford machinery needed for four–vector and four–spinor dynamics.

6.3 Spin and Metric From the Same BQ Product

In the biquaternionic formalism a four–vector is represented as a Weyl/Dirac matrix

$$\not{A} = A^{\mu} K_{\mu}, \qquad \not{B} = B^{\mu} K_{\mu},$$

where the basis $\{K_\mu\}$ is simultaneously a space–time basis and a Pauli/Dirac basis, depending on projection. Multiplying two such vectors yields

$$\not{A}\not{B} = c\,\not{1} + \mathbf{d}\cdot\boldsymbol{\Sigma} + \mathbf{e}\cdot\alpha, \tag{61}$$

with the well–known identifications

$$c = a_0 b_0 - \vec{a}\cdot\vec{b}, \qquad \text{symmetric scalar (metric)} \tag{62}$$

$$\mathbf{d} = \vec{a} \times \vec{b}, \qquad \text{antisymmetric bivector (spin)} \tag{63}$$
$$\mathbf{e} = a\, b_0 - a_0\, b, \qquad \text{mixed vector (boost sector).} \tag{64}$$

Thus, in the BQ algebra,

- the **metric** is the scalar (grade–0) projection of $\not{A}\not{B}$;
- the **spin structure** is the bivector (grade–2) projection of $\not{A}\not{B}$;
- the **boost/velocity sector** is the vector projection.

Spin and geometry are not added separately; they are distinct faces of a single algebraic product.

6.4 Status of Spin and Metric in Classical GR

In conventional Riemannian or Lorentzian geometry the metric $g_{\mu\nu}$ is symmetric by construction. The Levi–Civita connection has vanishing torsion, so no antisymmetric spin–related structure is present in the geometry itself. Dirac spinors can be added as matter fields, but:

- spin is not part of the geometry,
- antisymmetric quantities do not enter the metric,
- the symmetric (metric) and antisymmetric (spinor) sectors are fundamentally distinct.

Thus GR places spin "outside" the geometry; it is coupled but not co-generated.

6.5 Einstein–Cartan Theory and Poincaré Gauge Gravity

Einstein–Cartan (EC) theory extends the connection to include torsion $T^{\lambda}{}_{\mu\nu}$, antisymmetric in μ, ν, sourced by the spin density of matter. Here the antisymmetric sector enters the geometry, but:

- the metric $g_{\mu\nu}$ remains symmetric,
- spin couples to the connection, not directly to the metric,
- symmetric and antisymmetric sectors remain distinct fields.

EC therefore links spin and geometry, but does not unify them within a single algebraic object.

6.6 Geometric Algebra / Spacetime Algebra

In Hestenes' spacetime algebra (STA), vectors and bivectors arise from the Clifford product

$$ab = a \cdot b + a \wedge b,$$

which algebraically mirrors the BQ decomposition. However:

- the metric signature is imposed externally via $\mathrm{Cl}(1,3)$,

- the bivector sector encodes rotations and spin,
- the metric is not constructed from spinorial rotors,
- STA does not automatically identify the antisymmetric sector as part of metric generation.

Thus GA shares the algebraic capability, but does not elevate it to a physical unification.

6.7 Twistor Theory

Twistor theory treats spinors as fundamental and obtains space–time via bilinear constructions. Despite this spin-first approach:

- the resulting geometry is conformal rather than metric,
- the metric structure reappears only after imposing additional reality conditions,
- symmetric/antisymmetric separation is indirect,
- the algebra does not directly produce a Dirac–Pauli basis with built-in metric + spin content.

Twistors unify kinematics elegantly, but not the metric and spin bivectors as co-equal projections.

6.8 Kaluza–Klein, Supergravity, and Other Unified Schemes

Higher–dimensional or supersymmetric frameworks introduce new fields and larger symmetry groups, but retain a symmetric four–dimensional metric. The antisymmetric sector is typically:

- gauge fields in Kaluza–Klein,
- fermionic superpartners in supergravity,
- auxiliary fields in effective low–energy models,

and does not arise from the same multiplication that yields the metric. Spin is coupled to geometry, but not unified with it at the level of the underlying algebra.

6.9 Summary

Across essentially all unification programs (GR, EC, GA/STA, twistor theory, Kaluza–Klein, supergravity), the symmetric metric and antisymmetric spin structures belong to different mathematical sectors that interact through field equations or imposed couplings.

In the biquaternionic Dirac construction, by contrast,

- spin (bivector),
- boosts (vector),
- and the metric (scalar)

are *simultaneously generated* by the same algebraic product $A\!\!/\, B\!\!/$. The separation into symmetric and antisymmetric parts is a projection, not a structural divide. Thus the BQ approach achieves a more primitive and algebraically unified relationship between spin and geometry than is present in other frameworks: spin is not introduced *after* geometry, but is an intrinsic, irreducible component of the same underlying algebraic multiplication that yields the metric itself.

7 Conclusion

The analysis developed in this paper demonstrates that the minimal complex matrix algebra $M_2(\mathbb{C})$ contains, in irreducible form, the full algebraic structure needed to represent space–time, spin, time orientation, Lorentz symmetry, electromagnetism, and gravity. No enlargement of the algebra, no external metric postulate, and no additional geometric machinery is required. All physical content emerges directly from the Pauli–BQ multiplication rule and from the internal pseudoscalar $\hat{\mathrm{T}} = \sigma_I \sigma_J \sigma_K = i\,\hat{1}$.

Three key results combine to yield this unification:

1. **Two interlocking 4–sets.** The decomposition of $M_2(\mathbb{C})$ into the geometric 4–set $K_\mu = (\hat{\mathrm{T}}, \hat{\mathrm{I}}, \hat{\mathrm{J}}, \hat{\mathrm{K}})$ and the spin–norm 4–set $\sigma_\mu = (\hat{1}, \sigma_I, \sigma_J, \sigma_K)$ reveals that space–time vectors and spin vectors are complementary components of a single algebra. Their unification is encoded in the pseudoscalar identity $\hat{\mathrm{T}} = i\,\hat{1}$, which identifies the time axis with the spin pseudoscalar.
2. **One bilinear for all of relativistic physics.** The Pauli–BQ product $A^T B$ has a unique and irreducible three–sector decomposition:

$$A^T B = (a_0 b_0 - \mathbf{a} \cdot \mathbf{b})\,\hat{1} + (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{K} + (a_0 \mathbf{b} - b_0 \mathbf{a}) \cdot \boldsymbol{\sigma},$$

   which simultaneously generates the Minkowski metric (scalar sector), Pauli spin and rotations (bivector sector), and the boost/spin–norm structure (vector sector). This same product yields the electromagnetic field $\partial^T A$, Maxwell's equation $\partial B = \mu_0 J$, the Lorentz force JB, the Laue stress–energy bilinear $V^T P$, the action–angular–momentum bilinear $R^T P$, and the full kinematics of special relativity.
3. **A gravitational rotor acting on the line element.** Gravity enters not through curvature tensors or Levi–Civita connections, but through a Pauli rotor G acting on the line element:

$$dR \;\mapsto\; dR^G = G^{-1} dR\, G^{-1}.$$

   From $(dR^G)^T dR^G$ one obtains the coframe θ_μ and, in the slow–flow limit, the PG metric with its three rapidities. The same rotor produces gravitationally rotated momenta P^G and the mixed Dirac/KGE bilinear $\not{P}^G \not{P}$, revealing the algebraic content of "gravity in the Dirac adjoint." Thus electromagnetism (norm–suppressed) and gravity (norm–affecting) occupy distinct algebraic sectors of the same minimal algebra.

A central structural identity ties these results together:

$$K_\mu = i\,\sigma_\mu,$$

which shows that the geometric and spin–norm 4–vectors are the same object, rotated by $+90^\circ$ in the internal complex plane. This single relation unifies time, spin, chirality, complex phase, and metric signature, and implies that Lorentz transformations and gravitational dressing act identically on geometric and spin 4–vectors. When lifted to the Weyl–Dirac level, this identity becomes $\beta_\mu = i\gamma_\mu$, explaining the Dirac adjoint, the origin of γ^5, and the linearisation of the Klein–Gordon operator.

Compared to conventional frameworks—GR, EC theory, geometric algebra, spin bundles, Kaluza–Klein models, twistors, and supersymmetric extensions—the $M_2(\mathbb{C})$ formulation achieves a deeper and more primitive unification. Symmetric (metric) and antisymmetric (spin) sectors are not coupled by field equations but generated as complementary components of a single algebraic product. The result is a compact, geometric, and physically transparent synthesis of space–time, spin, electromagnetism, and gravity.

In conclusion, $M_2(\mathbb{C})$ functions as the irreducible algebra of space–time–spin geometry. Its minimal structure explains the maximal physical reach of the theory. The unification presented here shows that the foundations of relativistic physics arise not from added mathematical complexity but from the internal consistency of the smallest complex matrix algebra. This minimalism is not a constraint: it is the underlying reason the unification succeeds.

References

[1] E. P. J. de Haas (2020), *Closing the Biquaternion Algebra with Gravity: Updated Construction of the Dirac Framework and the Gravitational Rotor Qg*, viXra, https://doi.org/10.5281/zenodo.17608470.

[2] E. P. J. de Haas (2025), *Metric Generation by Gravitational Rotors: Systematic Derivations and MOND Implications in a Biquaternionic Framework*, Zenodo, hhttps://doi.org/10.5281/zenodo.17573223.

[3] E. P. J. de Haas (2025), *The Qg Rotor Lagrangian: Algebraic Generation of Einstein–Cartan Dynamics*, Zenodo, https://doi.org/10.5281/zenodo.17562993.

[4] E. P. J. de Haas (2025), *Closing the Biquaternion Algebra with Gravity: Updated Construction of the Dirac Framework and the Gravitational Rotor Qg*, Zenodo, https://doi.org/10.5281/zenodo.17608470.

[5] E. P. J. de Haas (2025), *Constructing Spacetime from Rapidity: Explicit BQ Derivation of the PG Coframe and the metric tensor*, Zenodo, https://doi.org/10.5281/zenodo.17717365.