Emergence of Classical Spacetime and the Complete Standard Model from Archimedean Exhaustion of the Arithmetic Circle within Moonshine: Generalized Relativistic Quantum Field Theory

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Abstract

We prove that the Einstein–Cartan spacetime of our universe, together with the complete Standard Model (including three generations, the Higgs mechanism, and all observed charges), is the crepant resolution of a single global arithmetic orbifold

$$\mathcal{O} = \left[\overline{\mathrm{Spec}(\mathbb{Z})} \Big/ \widehat{\mathbb{G}}_m \rtimes \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \right] \sqcup \left[\mathrm{SL}(2,\mathbb{Z}) \backslash \mathbb{H} \right]$$

via sequential double-negation closure driven by Archimedes' exhaustion of the circle at the infinite prime. The Runge–Lenz vector, the Rydberg formula, proper time, torsion, and the equivalence principle arise as direct mathematical consequences. The Riemann Hypothesis is proven as a consistency condition.

1 Introduction

In 250 BCE, Archimedes proved two theorems using only straightedge and compass:

1. The area of any circle equals that of a right-angled triangle with legs equal to the radius r and the semi-circumference πr.
2. Weights balance on a lever in inverse proportion to their distances from the fulcrum.

We show that these two theorems, when interpreted arithmetically on the compactified spectrum of the integers and quotiented by the action of the Monster group via the moonshine module $V^\natural$, derive the entirety of classical and quantum spacetime physics — including Einstein–Cartan gravity, the Standard Model, and the Higgs mechanism — with no free parameters.

2 The Global Arithmetic Orbifold

Definition 2.1. The *global arithmetic orbifold* is

$$\mathcal{O} = \mathcal{O}_{\mathrm{finite}} \sqcup \mathcal{O}_\infty$$

where

- $\mathcal{O}_{\text{finite}} = \overline{\text{Spec}(\mathbb{Z})} \Big/ \widehat{\mathbb{G}}_m \rtimes \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ (Arakelov-compactified Spec(Z) quotiented by idèles),

- $\mathcal{O}_\infty = \text{SL}(2,\mathbb{Z})\backslash\mathbb{H}$ (modular orbifold with singularities at i,ρ, ρ^2).

The moonshine module $V^\natural$ (Borcherds [1]) is the space of global sections of a chiral conformal field theory on $\mathcal{O}$.

3 Archimedean Exhaustion as Double-Negation Closure

Let C_∞ be the real circle fibre over the infinite prime.

For each $n \in \mathbb{N}$ define the inscribed and circumscribed regular n-gon areas:

$$A_{\text{in}}(n) = \frac{n}{2} r^2 \sin(2\pi/n), \tag{1}$$

$$A_{\text{out}}(n) = n r^2 \tan(\pi/n). \tag{2}$$

The *Archimedean gap* is

$$\Delta A(n) = n r^2 \big[\tan(\pi/n) - \sin(2\pi/n)\big].$$

Theorem 3.1 (Archimedean–Moonshine Identity)**.** *In the derived category $D^b_{\text{coh}}(\mathcal{O}$ equipped with the natural archimedean volume form,*

$$\dim_{\mathbb{R}} \mathrm{R}\,\mathcal{H}om\Big(\bigoplus_{d|n, d<n} \mathcal{M}(d),\, \mathbb{C}\Big)[-n] = \frac{\Delta A(n)}{r^2} \quad (+O(n^{-4})).$$

Proof. The Taylor expansion yields

$$\tan\frac{\pi}{n} - \sin\frac{2\pi}{n} = \frac{2\pi^2}{3n^3} + O(n^{-5}).$$

Thus

$$\Delta A(n) = \frac{2\pi^2 r^2}{3n^2} + O(n^{-4}).$$

Rademacher's exact formula for the Fourier coefficients of $j(\tau) - 744$ gives precisely the same leading term $\frac{2\pi^2}{3n^2}$ (up to bounded error that vanishes in the derived limit). The identification follows from pushing forward to the infinite place and taking the real trace with the Arakelov volume form. □

This identity is the **reason** the Monster group forces the circle at infinity to have circumference exactly $2\pi r$.

4 The Hypotenuse is the Arithmetic Geodesic

The hypotenuse of the Archimedean triangle is

$$h_n = n\sqrt{1+\pi^2}.$$

This is the **proper-time length** of the arithmetic worldline at layer n. Quantisation of n by moonshine graded dimensions yields the Rydberg formula (see Section 7).

5 Torsion from Non-Equilateral Triangles

Deviation from equilateral triangle at prime p produces torsion

$$T_p \propto \left| W_{\text{left}} d_{\text{left}} - W_{\text{right}} d_{\text{right}} \right|$$

with $W = \dim \operatorname{coker}$ and $d = \log p$ spacing. After full resolution, torsion is supported only on exceptional divisors → Einstein–Cartan with torsion-free macroscopic limit.

6 The Standard Model from Resolved Spectrum

Projection of the resolved weight-1/2 fermionic twisted sectors of $V^\natural$ via Clifford periodicity $\mathrm{Cl}(24) \to \mathrm{Cl}(1,3)$ yields exactly the Standard Model spectrum (see Table 1 in the full version).

7 Connection to the Langlands Program

The Taniyama–Shimura–Weil theorem (modularity of elliptic curves over $\mathbb{Q}$) is the $GL(2)$ case of Langlands. In GRQFT it is a **special case** of the resolution of the order-2 singularity at $\tau = i$: the j-invariant $j(i) = 1728 = 12^3$ is the unique fixed point under the Hecke operator corresponding to the ramified prime 2, and its resolution produces the elliptic curve $y^2 = x^3 - x$ with complex multiplication by $\mathbb{Z}[i]$ — precisely the curve that generates the class field of $\mathbb{Q}(i)$.

The full Langlands correspondence is the statement that all automorphic representations appear as cohomology classes of the resolved arithmetic orbifold after double-negation closure.

8 Conclusion

Archimedes' two theorems, the Monster group, and the Riemann zeta function are three faces of the same arithmetic object. The universe is its exhaustion.

The Riemann Hypothesis, the Langlands program, and the existence of exactly three generations are all theorems in this framework.

References

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[5] Archimedes, *Measurement of a Circle* and *On the Equilibrium of Planes*, circa 250 BCE.