Geometric Reconstruction from Correlation Structure: Operational Derivation of Lorentzian Signature, Causal Cones, and Metric Dynamics from Statistical Propagation Behaviour

N. J. Kettlewell[1]

[1]Independent Researcher, UK, Email: `[redacted for anonymisation]`

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Abstract

We begin with a complex two-point correlation kernel $W(x, y)$ defined on an abstract smooth label space X with *no assumed metric, signature, causal structure, or geometric fields*. From four operational constraints—finite propagation, passivity, regularity, and local homogeneity—we show that Lorentzian cones, Hadamard singularities, and a metric emerge as *statistical summaries of propagation behaviour*. Mixed derivatives of the correlation phase reconstruct the metric, and stability of a least-change functional selects Lorentzian signature and statistically favours three spatial dimensions. Allowing coefficients of the correlation generator to vary introduces curvature, and ensemble-averaging the correlation stress yields the statistical consistency condition

$$G_{AB} + \Lambda g_{AB} = \kappa \langle E_{AB} \rangle,$$

linking curvature to averaged correlation tension. Thus spacetime geometry arises not as a background structure but as the collective behaviour of correlations satisfying operational postulates.

1 Introduction

Quantum field theory in curved spacetime treats the metric as a fixed geometric background on which correlations propagate. This raises a foundational question: to what extent is spacetime geometry *assumed*, and to what extent can it be *recovered* from the behaviour of correlations alone?

Microlocal analysis identifies deep links between correlation singularities, wavefront sets, and causal structure. In particular, the Hadamard form encodes null geodesics and local metric information through the phase function and coefficients [3, 1, 2]. Parallel work in inverse problems has shown that Lorentzian metrics can be reconstructed from wave propagation data alone [8, 9]. The classic EPS framework reconstructs metric structure from the behaviour of light rays and free fall [7]. Geroch's Einstein-algebra formulation shows that a metric is not necessary as fundamental structure [10].

Here we follow an operational–analytic path. We begin with a correlation kernel $W(x, y)$ defined on a smooth manifold X used *purely as a label space*; no geometric structure is placed on X. We impose operational postulates reflecting empirical constraints on correlations (finite propagation, passivity, regularity). From these, we derive:

- Lorentzian signature of the correlation generator;
- Hadamard singularity structure of W;
- causal cones and metric reconstruction from the correlation phase;
- statistically preferred spatial dimension $d = 3$;
- Einstein-type dynamics from ensemble-averaged correlation stress.

Geometry thus appears as a statistical summary of correlation behaviour.

2 Operational postulates (no geometric assumptions)

Let X be a smooth manifold. We treat X purely as an *abstract label space* providing differentiable structure for distributions and local operators; *no metric, signature, causal cones, or geometric fields* are assumed.

A correlation kernel $W : X \times X \to \mathbb{C}$ is *admissible* if there exists a second-order differential operator

$$P = R^{AB}(x)\,\partial_A\partial_B + \text{(lower order terms)}$$

such that $P_x W = P_y W = 0$ and:

- **(OP1) Finite propagation.** Perturbations with compact support influence only compact domains (domain-of-dependence).
- **(OP2) Passivity (stability).** A stationary configuration exists with non-negative energy; no cyclic process can extract net work.
- **(OP3) Regularity.** W is a tempered distribution with controlled short-distance scaling.
- **(OP4) Local homogeneity.** In suitable charts, the coefficients vary smoothly and approximately isotropically.

These are operational and non-geometric.

Definition 2.1 (Primitive Correlation Kernel)**.** *The two-point kernel $W : X \times X \to \mathbb{C}$ is taken to be the fundamental relational object of the theory. It is* not *defined as the correlation of any underlying field, operator, state, or probability space. The bracket notation $\langle\cdot\rangle$ denotes the statistical summary over admissible configurations determined by OP1–OP4, and should not be interpreted as a quantum expectation value. All geometric structure is reconstructed from the singular behaviour of W.*

Interpretation. $W(x, y)$ is a primitive relational kernel whose analytic structure encodes the patterns of co-variation between events. No underlying field or Hilbert-space structure is assumed; the representation $W = \langle B\,\overline{B}\rangle$ is used only as a mathematical factorisation, not as an ontology.

3 From operational postulates to hyperbolicity and Hadamard form

The operational postulates ensure that admissible correlations propagate with well-defined singular structure. The geometric objects that follow (phase function σ, null cones, metric) arise as *collective* properties of W, not imposed structures on X.

Lemma 3.1 (Finite propagation ⇒ Lorentzian inertia)**.** *Finite propagation and well-posedness of the Cauchy problem imply that the principal symbol*

$$\sigma_P(x, k) = R^{AB}(x)k_A k_B$$

has exactly one negative eigenvalue. Elliptic symbols permit instantaneous propagation; ultrahyperbolic symbols violate stability. Hence the operator is necessarily normally hyperbolic.

Proof sketch. If all eigenvalues of R^{AB} have the same sign, then the principal part is elliptic and disturbances propagate instantaneously, violating OP1. If R^{AB} has two or more negative eigenvalues, then for any fixed scale one can choose covectors k_A, k'_A in independent negative directions such that $R^{AB}k_A k_B$ becomes arbitrarily negative, making the quadratic energy form unbounded below and violating OP2. The only remaining possibility compatible with OP1–OP2 is exactly one negative eigenvalue. This yields a normally hyperbolic operator in the sense of [11, 1].

Proposition 3.2 (Passivity ⇒ time orientation)**.** *Spectral positivity with respect to the unique negative eigenvector fixes the direction of energy flow and the $+i0$ prescription, establishing a global time orientation.*

Theorem 3.3. *The general microlocal structure of the Hadamard form is due to [3, 1]. Here we observe that OP1–OP3 provide the hypotheses of those theorems—normal hyperbolicity, appropriate microlocal spectrum, and finite propagation—*without presupposing a metric. *Thus the Hadamard singularity of W is an operational consequence rather than a geometric assumption.*

4 Statistical metric reconstruction

Theorem 4.1 (Statistical metric reconstruction)**.** *For an admissible W:*

1. *The singular support of $\Delta := W - W^*$ lies on $\sigma(x, y) = 0$, defining null cones.*
2. *The metric is the local propagation tensor*

$$g_{ab}(x) \propto -\partial_{x^a}\partial_{y^b}\sigma(x, y)\big|_{y=x}.$$

   This does not *represent pre-assumed geometry; it encodes the* statistically consistent propagation pattern *of admissible kernels.*

3. *The singular scaling $W \sim \sigma^{-(d/2-1)}$ determines the dimension d.*

This is consistent with EPS-type reconstruction from null structure [7] and inverse-problem reconstructions of Lorentzian metrics [8, 9].

5 Least-change functional and emergent signature

The field $B : X \to \mathbb{C}$ is related to the correlation kernel by $W(x, y) = \langle B(x)\, \bar{B(y)}\rangle$, so variations of B represent fluctuations in the allowed correlation structure.

To understand why the emergent metric has Lorentzian signature and favours three spatial dimensions, we introduce the least-change functional

$$C[B] = \frac{1}{2}\int_X \left(R^{AB}\partial_A B\,\partial_B \bar{B} + \beta|B|^2\right)\, d\mu.$$

This functional does not impose geometry; it quantifies the *cost of locally deforming correlation behaviour* under OP1–OP4. Stability of its minimisers determines statistically preferred structure.

Lemma 5.1. *Finite propagation implies exactly one negative eigenvalue of R^{AB}.*

Proposition 5.2. *Passivity fixes the orientation of the unique negative eigenvector.*

Proposition 5.3 (Three spatial dimensions)**.** *For perturbations $B \to B + \delta B$, the dispersion relation in d spatial dimensions is*

$$\omega^2 = c^2|k|^2 + \beta.$$

The coherence cost per unit radiated flux is scale-independent only for $d = 3$. Dimensions $d < 3$ suffer infrared divergence; $d > 3$ excessively suppress interference. Thus $d = 3$ is the unique statistically stable case.

Dimensional calculation. *For the outgoing Helmholtz Green function in d spatial dimensions, $G_d(r) \sim r^{2-d}$ as $r \to 0$ for $d > 2$ (and $\sim \log r$ for $d = 2$), the near-zone contribution to the least-change cost scales as*

$$C_{near} \propto \int_0^{1/k} r^{d-1}\,|G_d(r)|^2\, dr \sim \int_0^{1/k} r^{3-d}\, dr.$$

Normalising by the radiated flux $\Phi \propto k\|A\|^2$ yields

$$\frac{C_{near}}{\Phi} \propto \begin{cases} k^{3-d}, & d \neq 3, \\ \log(kR_0), & d = 3. \end{cases}$$

For $d < 3$ the cost grows in the infrared ($k \to 0$); for $d > 3$ it grows in the ultraviolet. Only $d = 3$ yields a scale-independent (marginal) penalty, making it the unique statistically stable dimension.

6 Curvature and ensemble-averaged consistency

The admissible configurations of B form the ensemble over which statistical summaries such as $\langle E_{AB} \rangle$ are taken. Variation of the least-change functional with respect to g_{AB} yields the correlation tension tensor

$$E_{AB} := -\frac{\delta C}{\delta g^{AB}}, \qquad \nabla^A E_{AB} = 0.$$

Since the reconstructed metric summarises *collective* propagation behaviour, its curvature must satisfy a statistical consistency condition.

Ensemble. The ensemble $\langle \cdot \rangle$ refers to the collection of admissible correlation configurations generated by fluctuations of B compatible with the operational postulates OP1–OP4. No probabilistic structure beyond this is assumed: the average denotes the statistical summary of those configurations that satisfy the same local propagation and stability constraints used to define admissible kernels. In this sense the emergent metric must be consistent with the averaged correlation tension of the same ensemble that determines its propagation behaviour.

$$G_{AB} + \Lambda g_{AB} = \kappa \langle E_{AB} \rangle,$$

where $\langle E_{AB} \rangle$ is the ensemble average over correlation configurations compatible with OP1–OP4. Inverse-problem results suggest that such consistency relations are natural for reconstructed metrics [8]. In the limit of constant g_{AB} one recovers the flat Helmholtz operator.

7 Discussion

Interpretation. The reconstructed g_{ab} and its curvature should be viewed as *statistical summaries* of correlation behaviour constrained by the operational postulates. Geometry is not assumed on X; instead, signature, cones, dimensionality, and Einstein-type relations arise as collective properties of admissible kernels.

Relation to other work. Our reconstruction is distinct from but complementary to EPS geometry from null structure [7], inverse-problem reconstructions of Lorentzian metrics [8], and entanglement-based emergent spacetime scenarios [15]. Unlike causal-set approaches [14], we work directly with correlation behaviour and microlocal singularities.

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