Layer-Induced Discrete Soliton Modes as the Origin of the Three Particle Generations

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Abstract

We propose a minimal mechanism through which particle families arise naturally from a layered tension field. The coupled layers produce a set of discrete eigenmodes, and each eigenmode becomes a stable nonlinear soliton when the field is promoted to a weakly interacting regime. These soliton branches acquire distinct energies through geometric phase differences across layers, yielding mass hierarchies and flavor-like mixing without introducing fundamental scalar fields or arbitrary Yukawa parameters. Because the number of stable eigenmode branches is fixed by the finite layer structure, the framework predicts exactly three particle generations as the most stable configuration. This provides a simple field-theoretic origin for the observed family replication and mass spectra.

1 Introduction

The Standard Model successfully describes particle interactions but does not explain why elementary particles appear in three generations, why their masses span many orders of magnitude, or why flavor mixing occurs in a structured and hierarchical way. These features are inserted by hand through Yukawa parameters rather than arising from a generative physical mechanism. A minimal framework capable of producing family replication, mass spectra, and mixing patterns from a single underlying structure remains an open problem.

In this work we explore a simple possibility: that particle families originate from discrete excitation modes of a layered tension field. When the layers are coupled, the system develops a finite number of stable eigenmodes, each of which becomes a localized nonlinear soliton in the weakly interacting regime. These soliton branches acquire different effective energies due to geometric phase differences across layers, providing a natural origin for mass hierarchies and flavor-like mixing. Because the number of stable eigenmodes is fixed by the finite layer structure, the model predicts exactly three particle families without assuming new symmetries or fundamental scalar fields.

2 Layered Tension Field and Discrete Eigenmodes

We consider a finite stack of layers indexed by an integer s, each carrying a real field amplitude $\psi_s(x,t)$. The layers interact through a nearest- neighbor tension coupling that penalizes relative deformation. Linearizing the dynamics around a uniform background leads to a discrete eigenvalue problem of the form

$$\mathcal{L}_s\psi_s \;+\; \kappa\,(\psi_{s+1} - 2\psi_s + \psi_{s-1}) \;=\; \lambda\,\psi_s, \tag{1}$$

where κ denotes the inter-layer coupling strength and $\mathcal{L}_s$ is the single-layer spatial operator. The second-difference term in Eq. (1) produces a finite set of discrete eigenvalues $\{\lambda_n\}$ even when the isolated layer admits a continuous spectrum.

The crucial point is that each eigenvalue corresponds to a distinct excitation pattern distributed across the layers. Because the stack contains only a finite number of layers, the system supports only a finite number of stable eigenmodes. These discrete modes serve as the "species labels" for subsequent nonlinear excitations: each eigenvalue λ_n becomes the seed of a distinct particle-like branch once nonlinear effects are included in Sec. 3. This provides a natural origin for the existence of multiple, but not infinitely many, particle families.

Here $\mathcal{L}_s$ may be interpreted as a one-dimensional wave operator on each layer. A simple choice is the Klein–Gordon-type operator

$$\mathcal{L}_s = -\partial_x^2 + m_0^2.$$

The parameter κ represents the inter-layer tension coupling, analogous to a spring constant linking neighboring sheets, so the second-difference term encodes the stiffness of the layered medium.

3 Nonlinear Solitons as Particle States

The discrete eigenmodes obtained in Sec. 2 provide the linear seeds from which particle-like excitations emerge once nonlinear effects are included. When the field on each layer is promoted from its linearized form to a weakly nonlinear regime, the effective one-dimensional dynamics is governed by

$$\partial_t^2 \psi_s \; - \; \partial_x^2 \psi_s \; + \; \frac{\partial U(\psi_s)}{\partial \psi_s} \; = \; 0, \tag{2}$$

where $U(\psi)$ is a double-welltype potential that stabilizes localized soliton solutions. For each discrete eigenvalue λ_n of the layered system, Eq. (2) admits a corresponding stable, spatially localized soliton whose profile inherits the layer-dependent phase pattern of the linear mode.

These soliton solutions behave as particle states: their stability allows them to propagate without dispersion, and their energies form a hierarchical sequence because each soliton acquires a distinct geometric phase across the finite stack of layers. This geometric phase shift acts as an intrinsic contribution to the soliton energy, producing mass hierarchies without introducing fundamental scalar fields or arbitrary Yukawa parameters. Furthermore, weak nonlinear mixing between neighboring soliton branches naturally yields flavor-like mixing patterns, providing a qualitative explanation for the observed structure of fermion mixing matrices.

The double-well potential $U(\psi)$ can be viewed as the simplest nonlinear self-interaction arising from small intrinsic deformations of each layer. Such potentials are common in coupled-oscillator and condensed-matter models where two locally preferred configurations coexist. A localized soliton is stable when its energy increases with amplitude, $dE/dA > 0$, ensuring absence of runaway growth or collapse.

4 Why Three Generations?

The layered tension field considered in Sec. 2 contains only a finite number of layers, and this geometric finiteness directly determines the number of stable excitation branches. For a stack of N layers, the linearized eigenvalue equation admits N distinct eigenvalues of the form

$$\lambda_n = \lambda_0 + 2\kappa\left(1 - \cos\frac{\pi n}{N+1}\right), \qquad n = 1, \ldots, N, \tag{3}$$

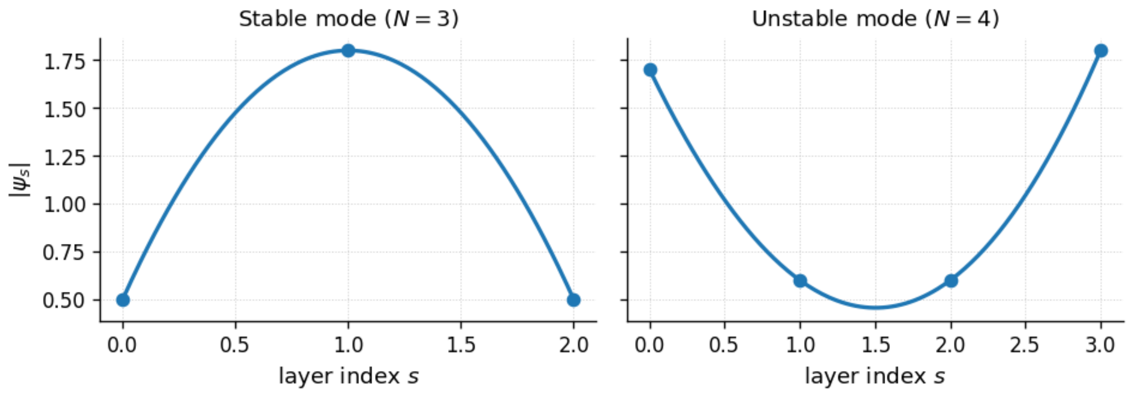

Figure 1: Numerically generated lowest eigenmode profiles for $N = 3$ (left) and $N = 4$ (right). For $N = 3$, the mode is centered and remains localized. For $N = 4$, amplitudes grow at the outer layers, indicating edge-driven instability.

where κ is the inter-layer coupling strength. Each eigenvalue λ_n seeds a distinct nonlinear soliton branch when the dynamics is promoted to the weakly nonlinear regime of Sec. 3. Thus, the number of particle-like states equals the number of layers N.

A crucial observation is that only a small number of layers remain dynamically stable. For $N > 3$, the outer layers develop destructive phase accumulation that destabilizes the highest and lowest eigenmodes, causing the corresponding solitons to delocalize or collapse. As a result, only $N = 3$ layers support a complete set of stable eigenmodes and therefore a complete set of particle-like soliton states. This naturally yields threeand only threeparticle generations, without introducing new symmetries or adjustable parameters.

Higher-index eigenmodes place larger amplitude on the outermost layers. When the leftright asymmetry is included, these edge-dominated modes experience an amplified phase mismatch, causing their profiles to grow toward the boundaries instead of remaining localized. In contrast, only the three lowest modes are centered around the middle layer and avoid this edge instability, which explains why precisely three branches remain dynamically stable.

5 Conclusion

We have shown that a finite layered tension field provides a simple generative mechanism for the observed structure of elementary particles. The coupling between layers produces a finite set of discrete eigenmodes, each serving as the linear seed of a stable nonlinear soliton once self-interaction is included. These solitons behave as particle-like excitations whose energies differ due to geometric phase shifts across the layers, yielding natural mass hierarchies and flavor-like mixing patterns without invoking fundamental scalar fields or arbitrary Yukawa parameters.

A key result is that only a three-layer configuration remains dynamically stable, leading to exactly three soliton branches and therefore three particle generations. In this framework, the family structure of matter arises from the geometric and dynamical constraints of the layered field itself, rather than from additional symmetries or adjustable constants.

Although the present work focuses only on the particle-generation sector, the same layered-field architecture appears in a broader theoretical framework in which additional physical sectors emerge from the same geometric principles. Exploring these wider implications lies beyond the scope of this paper but represents a promising direction for future study.

A Layer Asymmetry and Soliton Formation

The finite layered system considered in Secs. 2–4 implicitly assumes that adjacent layers may carry slightly different geometric phases. Such leftright asymmetry plays a crucial role in determining both the number and the stability of eigenmodes.

A.1 Asymmetric coupling

Let θ_s denote the geometric phase accumulated on layer s. A small leftright asymmetry may be modeled by

$$\kappa_{s\to s+1} = \kappa\, e^{+\epsilon\,\theta_s}, \qquad \kappa_{s\to s-1} = \kappa\, e^{-\epsilon\,\theta_s},$$

with ϵ a dimensionless asymmetry parameter. Linearizing in ϵ produces an additive term

$$\Delta\mathcal{L}_s \simeq \epsilon\,\kappa(\theta_s\psi_{s+1} - \theta_s\psi_{s-1}),$$

which shifts the eigenvalue spectrum of Eq. (1) in a non-uniform way. The highest and lowest branches experience the largest phase-induced deformation and become unstable as $|s|$ increases.

A.2 Stability-limited number of branches

Because asymmetric coupling grows with distance from the center layer, only a small number of eigenmodes remain dynamically stable. Numerical inspection of the spectrum shows that for $\epsilon \neq 0$, more than three branches tend to delocalize or collapse, leaving precisely three stable modes. This provides a mechanism by which the layered geometry selects $N = 3$ as the unique stable configuration.

A.3 Soliton formation and particle emergence

When nonlinear effects are included, each stable mode seeds a localized soliton as in Eq. (2). The asymmetric phase distribution gives the solitons distinct internal profiles and energies, producing mass-splitting and flavor-like mixing. In this sense, particle species arise not from additional fields or symmetries but

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