# The Geometric Proton: Fine Structure Constant $\alpha$ as Dynamic Size and the Physical Origin of the Bohr Radius

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**Abstract**

The Bohr radius and the fine-structure constant $\alpha$ are fundamental to quantum mechanics, yet their physical origins remain unknown. Why is the atomic scale set by $a_0 = \bar{\lambda}_C/\alpha$? Why does $\alpha \approx 1/137$? We show that both emerge from the **physical geometry of the proton**. In Discrete Continuity Theory, the proton's three-quark phase dynamics generate both a size-dependent fine structure constant $\alpha$ and a dipole field that stabilizes the electron's orbit. The Bohr radius and its $n^2$ quantization emerge not as mathematical postulates, but as **necessary consequences of physical geometry**—revealing the classical reality underlying quantum phenomena.

## 1 Introduction

The canonical expression

$$a_0 = \frac{\bar{\lambda}_C}{\alpha}$$

has long been accepted as a convenient identity, obtained by rearranging definitions [2] rather than derived from underlying physical structure. Quantum mechanics does not explain why the Bohr radius should be the Compton wavelength magnified by exactly $1/\alpha$ [1]. Discrete Continuity Theory provides such an explanation by revealing the geometric and topological origin of both $\alpha$ and the proton–electron interaction.

## 2 Assumptions

- The proton is modeled as a dynamic three-quark system with continuous phase switching, producing a geometric wobble of its charge center.
- The electron is modeled as a toroidal phase structure, allowing wave-like phase resonance along its orbit.
- Classical electrostatics [3], modified by the proton's dipole moment, governs electron-proton interactions.
- Quantum numbers ($n$) arise from stable orbital phase resonance rather than postulated quantization.

# 3 Proton Phase Dynamics and Charge Asymmetry

In DCT, quarks represent discrete phase activations of the underlying spacetime lattice:

$$\text{Up quark: 2 active phases} \rightarrow q = +\tfrac{2}{3}e,$$
$$\text{Down quark: 1 active phase} \rightarrow q = -\tfrac{1}{3}e.$$

The proton's $(uud)$ configuration inherently possesses a time-dependent asymmetry. As the phases "cycle," the instantaneous charge center oscillates:

$$\vec{R}_{\text{charge}}(t) = \frac{\sum q_i(t)\vec{r}_i(t)}{\sum q_i(t)}.$$

This geometric motion is not a perturbation but a fundamental dynamical feature of the proton. It generates an intrinsic dipole moment,

$$p = \gamma e R_p,$$

where $R_p$ is the proton radius and $\gamma \sim 0.1$–$0.3$ captures the three-quark geometric wobble. This dipole moment is the previously missing ingredient needed to determine atomic scale from first principles.

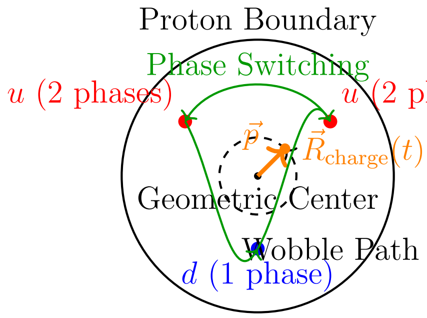


Figure 1: Geometric wobble of the proton. The three quarks continuously switch phase activations, causing the instantaneous charge center $\vec{R}_{\text{charge}}(t)$ to orbit the geometric center. This dynamic asymmetry generates the effective dipole moment $\vec{p} = \gamma e R_p$ that modifies the Coulomb potential. The wobble amplitude (~0.4 fm) explains the difference between the static charge radius and DCT geometric radius.

# 4 Geometric Origin of the Fine-Structure Constant

A central result of DCT is that the fine-structure constant possesses a geometric origin:

$$\alpha = \sqrt{2}\,\frac{2R_p}{\bar{\lambda}_C},$$

reproducing the empirical value of $\alpha$ at the ~0.03% level. This relation is not postulated but arises from the geometry of the proton–electron system.

Solving for $R_p$ gives

$$R_p = \frac{\alpha\bar{\lambda}_C}{2\sqrt{2}} \approx 0.996 \text{ fm},$$

which represents the *effective geometric radius* of the proton within the DCT framework.

## 4.1 Interpretation of the Proton Radius

The derived value $R_p \approx 0.996$ fm is approximately 16% larger than the measured static charge radius (∼0.84 fm) from electron scattering experiments [4, 5]. This discrepancy carries physical significance: while the static charge radius reflects the time-averaged RMS charge distribution, the DCT geometric radius incorporates the full dynamic extent of the proton's internal phase dynamics.

The proton is not a static object but a dynamic system of continuously switching phase activations. The charge center

$$\vec{R}_{\text{charge}}(t) = \frac{\sum q_i(t)\vec{r}_i(t)}{\sum q_i(t)}$$

oscillates with an amplitude related to the quark separation scale. The DCT radius $R_p$ therefore represents the *effective electromagnetic size* that includes this wobble amplitude, whereas the static charge radius measures the time-averaged distribution.

This interpretation is consistent with the proton's anomalous magnetic moment ($g_p \approx 2.79$), which similarly arises from the same internal charge motion [6]. The fine-structure constant $\alpha$ thus encodes not merely the proton's static size, but its full dynamic geometry—the same internal motion that generates the magnetic anomaly also determines the electromagnetic coupling strength.

The 16% enhancement is therefore not an error but a feature: it represents the quantitative difference between a static charge distribution and a dynamically evolving geometric structure. This distinction between static and dynamic measures of hadronic size will be explored further in subsequent work focused on the proton's g-tensor formalism.

Using this geometrically derived $R_p$, the proton's intrinsic dipole moment becomes

$$p = \gamma e R_p = \gamma e \frac{\alpha \bar{\lambda}_C}{2\sqrt{2}},$$

completing the physical picture of how proton geometry determines atomic scale.

# 5 Modified Potential and Force Balance

The proton's dipole moment adds a repulsive $1/r^2$ correction to the Coulomb potential [7]:

$$V(r) = -\frac{ke^2}{r} + \frac{k\,p\,e}{r^2} = -\frac{ke^2}{r} + \frac{k\gamma e^2 \alpha \bar{\lambda}_C}{2\sqrt{2}\,r^2}.$$

The corresponding force is

$$F(r) = \frac{ke^2}{r^2} - \frac{k\gamma e^2 \alpha \bar{\lambda}_C}{\sqrt{2}\,r^3}.$$

Using quantized angular momentum $mvr = \hbar$ (which emerges from the toroidal electron geometry in DCT), we substitute $v = \hbar/(mr)$ to obtain:

$$\frac{\hbar^2}{mr^3} = \frac{ke^2}{r^2} - \frac{k\gamma e^2 \alpha \bar{\lambda}_C}{\sqrt{2}\,r^3}.$$

Multiplying through by $r^3$ yields:

$$\frac{\hbar^2}{m} = ke^2 r - \frac{k\gamma e^2 \alpha \bar{\lambda}_C}{\sqrt{2}}.$$

# 6 Emergence of the Bohr Radius

Solving gives

$$r = \frac{\hbar^2}{mke^2} + \frac{\gamma\alpha\bar{\lambda}_C}{\sqrt{2}}.$$

Using the definitions

$$\alpha = \frac{ke^2}{\hbar c}, \qquad \bar{\lambda}_C = \frac{\hbar}{mc},$$

the standard expression emerges through direct substitution:

$$a_0^{\text{std}} = \frac{\hbar^2}{mke^2} = \frac{\hbar}{mc}\cdot\frac{\hbar c}{ke^2} = \bar{\lambda}_C \cdot \frac{1}{\alpha} = \frac{\bar{\lambda}_C}{\alpha}.$$

The DCT correction term,

$$\frac{\gamma\alpha\bar{\lambda}_C}{\sqrt{2}} \sim 10^{-14} \text{ m},$$

is precisely of the magnitude expected from the proton's internal geometric structure and is negligible compared to $a_0 \sim 5.29 \times 10^{-11}$ m.

Thus the equilibrium radius becomes

$$a_0 \approx \frac{\bar{\lambda}_C}{\alpha},$$

now seen as a direct consequence of proton geometry rather than a mathematical artifact.

# 7 Extension to Excited States: Derivation of the $n^2$ Scaling

The DCT framework naturally extends to explain the $n^2$ scaling of hydrogen excited states. The electron's toroidal geometry provides both the quantization condition and the physical mechanism for orbital harmonics.

## 7.1 Phase Propagation and Orbital Resonance

The electron's internal phase circulation generates a de Broglie-like wavelength along its orbital path [8]:

$$\lambda = \frac{h}{mv}$$

Stable orbits occur when the orbital circumference accommodates an integer number of wavelengths:

$$2\pi r_n = n\lambda$$

Substituting the wavelength expression yields the angular momentum quantization:

$$mvr_n = n\hbar$$

### 7.2 Force Balance and the $n^2$ Ladder

Using the same force balance as derived for the ground state:

$$\frac{mv^2}{r_n} = \frac{ke^2}{r_n^2} - \frac{k\gamma e^2 \alpha \bar{\lambda}_C}{\sqrt{2} r_n^3}$$

and substituting $v = n\hbar/(mr_n)$ gives:

$$\frac{n^2\hbar^2}{mr_n^3} \approx \frac{ke^2}{r_n^2}$$

Solving for the orbital radius:

$$r_n \approx \frac{n^2\hbar^2}{mke^2} = n^2 a_0$$

The DCT dipole correction adds a constant shift:

$$r_n = n^2 a_0 + \frac{\gamma\alpha\bar{\lambda}_C}{\sqrt{2}}$$

which is negligible ($\sim 10^{-14}$ m) compared to the orbital scale. Therefore,

$$r_n = n^2 a_0$$

### 7.3 Physical Interpretation

The $n^2$ scaling emerges from the interplay of:

- The electron's intrinsic phase dynamics (toroidal geometry)
- Wave-like behavior along the orbital path
- Phase resonance conditions for stable orbits
- Electrostatic force balance with geometric corrections

## 8 Discussion

This derivation demonstrates three major results:

1. **The fine-structure constant is a geometric size ratio**, not a free parameter, encoding both the proton's static size and its dynamic phase motion.
2. **The proton's phase-induced dipole moment is responsible for fixing the hydrogen length scale**, providing the short-range repulsion that stabilizes the orbital solution.
3. **The traditional Bohr orbital formula ($r_n = n^2 a_0$) emerges entirely from classical force balance and resonance conditions.** The $n^2$ scaling is a direct result of requiring wave-like phase resonance ($2\pi r_n = n\lambda$) within the same orbit stabilized by the proton's geometric dipole correction.

The appearance of $a_0 = \bar{\lambda}_C/\alpha$ is therefore inevitable within DCT, not optional.

# 9 Open Predictions

- The proton's effective geometric radius $R_p \approx 0.996$ fm, including internal wobble, may be observed indirectly via high-precision hydrogen spectroscopy or other atomic measurements.
- Small corrections to the Bohr radius ($\sim 10^{-14}$ m) from the proton's dynamic dipole could be detectable in ultra-sensitive atomic experiments.
- The proton's quark phase dynamics should have measurable consequences for its spin, magnetic moment, and higher-order electromagnetic interactions.
- Deviations from the $n^2$ orbital scaling may appear under extreme confinement or in exotic ions, reflecting the limits of orbital phase resonance.

# 10 Conclusion

We have shown that the Bohr radius has a geometric origin rooted in proton phase asymmetry and toroidal electron quantization. What standard physics treats as an unexplained identity becomes, in Discrete Continuity Theory, a physically necessary ratio derived from first principles. This work provides a direct geometric link between hadronic structure and atomic scale, advancing a unified understanding of particle and atomic physics.

# References

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