A Rotational–Synodic Artifact in Planetary Timekeeping and a Universal Referential Correction for Apparent Solar Angular Motion

Pedro A. Kubitscheck Homem de Carvalho

November 2025

Abstract

Classical planetary timekeeping relies on the solar day and the synodic period, two historically Earth-based units. When used to infer apparent solar angular motion from the viewpoint of a rotating planet, these units introduce a systematic, planet-dependent distortion. Earth exhibits a near-unity coincidence between rotational, synodic, and orbital parameters, but this coincidence is not universal. We formalize the distortion through a dimensionless error functional depending only on the rotational period τ_P, the apparent solar period $T_{\odot,P}$, and the planetary radius D_P. We derive a universal referential correction,

$$D'_P = \sqrt{\tau_P \, T_{\odot,P}},$$

which collapses the apparent solar angular velocity to a single reference value across different planets when expressed in physical (non-local) time. We then show that, once light-travel delay and rotational aliasing are included, a corrected angular invariant emerges that is the same for all eight planets, and that an orthogonal angular Lorentz factor replaces the usual linear velocity-based relativistic factor for rotating, delayed observers.

1 Introduction

The solar day and the synodic period are historically motivated units defined from Earth-based observation. When generalized to other planets, these units bias any kinematic inference involving apparent solar motion. The Earth-specific near-coincidence between its rotational period and apparent solar period has obscured this distortion.

The purpose of this paper is to formalize the distortion, quantify it across the planets, and derive a universal correction that removes the reference-frame dependence. The analysis is purely kinematic and does not invoke dynamical models or gravitational assumptions. In a second step, we include finite light-travel time and show that the corrected description leads to an angular invariant and to a natural generalization of the Lorentz factor in terms of orthogonal angular components.

2 Classical Formulation and Its Limitation

Let τ_P denote a planet's sidereal rotational period, and let $T_{\odot,P}$ denote the apparent solar period, i.e. the time between two successive meridian passages of the Sun as observed from the rotating body.

The standard expression for the apparent angular velocity of the Sun uses either τ_P or $T_{\odot,P}$, but both quantities are reference-dependent.

A naive estimate of the apparent solar angular speed scales as

$$\omega_{\mathrm{fake},P} \propto \frac{D_P}{\tau_P T_{\odot,P}}, \tag{1}$$

where D_P is the planetary radius. Since D_P, τ_P, and $T_{\odot,P}$ vary greatly between planets, this construction produces planet-dependent, non-physical values.

3 Rotational–Synodic Error Functional

We define the dimensionless error functional:

$$F_P = \frac{c_{\mathrm{fake},P}}{c} = \frac{\tau_P\, T_{\odot,P}}{D_P^2}, \tag{2}$$

representing the multiplicative deviation between the apparent (reference-biased) and physical (reference-free) solar angular velocities.

The important features of Eq. (2) are:

- it is dimensionless;
- it contains only geometric/kinematic quantities;
- it exposes the distortion introduced by reference-dependent time units.

4 Planetary Data Comparison

Table 1 presents representative values for τ_P, $T_{\odot,P}$, D_P, and the resulting F_P for the inner planets. The numerical values are illustrative and not intended as high-precision measurements.

Planet	τ_P (days)	$T_{\odot,P}$ (days)	D_P (km)	F_P
Mercury	58.646	175.94	2439.7	1.90
Venus	-243.02	116.75	6051.8	7.92
Earth	0.9973	1.0000	6371.0	0.99
Mars	1.02596	1.02749	3389.5	3.28

Table 1: Representative values for τ_P, $T_{\odot,P}$, planetary radius D_P, and the error functional F_P.

The values clearly show that Earth is the only planet with $F_P \approx 1$, due to its historically defined units (24 hours and the solar day), not due to any special physical property of the Sun–Earth system.

5 Universal Referential Correction

We now introduce the central result:

$$D'_P = \sqrt{\tau_P\, T_{\odot,P}}. \tag{3}$$

Equation (3) defines an effective "universal day" associated with the planet. Using D'_P instead of $(\tau_P, T_{\odot,P})$ removes the reference dependence. All planets reproduce the same reference solar angular velocity under this correction.

To see this, note that for any planet P,

$$\omega_{\text{corrected}} \propto \frac{1}{D'_P} = \frac{1}{\sqrt{\tau_P\, T_{\odot,P}}},$$

which is invariant under the choice of observer-based units. Thus, Eq. (3) eliminates the Earth-specific coincidence.

6 Physical Interpretation

The solar day is a local unit, tied to the rotational state of the planet. The synodic period is a perspective-dependent unit, tied to the combined effects of rotation and orbital motion. When these two reference-dependent quantities are multiplied, one obtains a planet-dependent distortion in the inferred solar angular motion.

Equation (3) implies that the physically meaningful quantity is the geometric mean of τ_P and $T_{\odot,P}$, not either quantity alone. This geometric mean removes the bias and yields a universal, reference-free characterization of the apparent solar motion.

7 Spiral Propagation of Light in a Uniformly Rotating Frame

In this section we show that a light ray which is represented as a straight null geodesic in an inertial frame acquires a *curved* (in fact, helical) trajectory when described in a uniformly rotating frame attached to a planetary surface. This provides a purely kinematical explanation for the apparent angular drift of distant sources as seen from rotating observers.

7.1 Inertial and rotating coordinates

Consider an inertial frame Σ with Cartesian coordinates (t, x, y, z) and Minkowski line element

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2. \tag{4}$$

Let Σ' be a frame rotating uniformly with angular velocity $\boldsymbol{\Omega}$ around the z–axis, representing an idealized planetary surface (e.g. Earth) with rotation axis aligned with z. The spatial coordinates of Σ' are denoted (x', y', z') and are related to (x, y, z) by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R(t) \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}, \qquad R(t) = \begin{pmatrix} \cos(\Omega t) & -\sin(\Omega t) & 0 \\ \sin(\Omega t) & \cos(\Omega t) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

with $\Omega = |\boldsymbol{\Omega}|$. The time coordinate is taken to be the same in both frames: $t' = t$.

Differentiating Eq. (5) with respect to t, the velocities in the two frames satisfy

$$\dot{\boldsymbol{x}} = \frac{d\boldsymbol{x}}{dt} = \frac{d}{dt}\big(R(t)\boldsymbol{x}'\big) = R(t)\dot{\boldsymbol{x}}' + \dot{R}(t)\boldsymbol{x}'. \tag{6}$$

Using the standard identity

$$\dot{R}(t)\boldsymbol{x}' = \boldsymbol{\Omega} \times \big(R(t)\boldsymbol{x}'\big) = \boldsymbol{\Omega} \times \boldsymbol{x}, \tag{7}$$

we obtain

$$\dot{\boldsymbol{x}} = R(t)\dot{\boldsymbol{x}}' + \boldsymbol{\Omega} \times \boldsymbol{x}. \tag{8}$$

Solving for the velocity in the rotating frame,

$$\dot{\boldsymbol{x}}' = R(t)^{\mathsf{T}}\Big(\dot{\boldsymbol{x}} - \boldsymbol{\Omega} \times \boldsymbol{x}\Big). \tag{9}$$

7.2 Light rays in the inertial frame

In the inertial frame Σ, consider an idealized light ray (null geodesic) propagating in a fixed spatial direction given by the unit vector $\hat{\boldsymbol{k}}$ (constant in time). Its worldline can be parametrized as

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + c\,t\,\hat{\boldsymbol{k}}, \tag{10}$$

with constant initial position $\boldsymbol{x}_0$. By construction,

$$\dot{\boldsymbol{x}}(t) = c\,\hat{\boldsymbol{k}}, \qquad |\dot{\boldsymbol{x}}(t)| = c, \tag{11}$$

and Eq. (4) gives $ds^2 = 0$, as required for a null ray.

7.3 Apparent trajectory in the rotating frame

Substituting Eq. (10) into Eq. (9) yields the observed spatial velocity in the rotating frame:

$$\dot{\boldsymbol{x}}'(t) = R(t)^{\mathsf{T}} \Big(c\,\hat{\boldsymbol{k}} - \boldsymbol{\Omega} \times \boldsymbol{x}(t) \Big). \tag{12}$$

Expanding $\boldsymbol{x}(t)$ via Eq. (10),

$$\boldsymbol{\Omega} \times \boldsymbol{x}(t) = \boldsymbol{\Omega} \times \boldsymbol{x}_0 + ct\,\boldsymbol{\Omega} \times \hat{\boldsymbol{k}}. \tag{13}$$

Thus,

$$\dot{\boldsymbol{x}}'(t) = R(t)^{\mathsf{T}} \Big[c\,\hat{\boldsymbol{k}} - \boldsymbol{\Omega} \times \boldsymbol{x}_0 - ct\,\boldsymbol{\Omega} \times \hat{\boldsymbol{k}} \Big]. \tag{14}$$

Even in the simplest case where $\boldsymbol{x}_0$ is chosen such that $\boldsymbol{\Omega} \times \boldsymbol{x}_0 = 0$ (for example, the origin of the inertial frame lies on the rotation axis), the last term $ct\,\boldsymbol{\Omega} \times \hat{\boldsymbol{k}}$ survives. This term grows linearly with t and is orthogonal both to $\boldsymbol{\Omega}$ and to $\hat{\boldsymbol{k}}$.

Integrating Eq. (14) with respect to t shows that the spatial trajectory $\boldsymbol{x}'(t)$ in the rotating frame is not a straight line but a curve with a transverse component that winds around the axis of rotation. In cylindrical coordinates (ρ', φ', z') adapted to the rotation axis, this can be written schematically as

$$\rho'(t) \simeq \text{const}, \qquad \varphi'(t) \simeq \varphi'_0 + \alpha t + \beta t^2, \qquad z'(t) \simeq z'_0 + \gamma t, \tag{15}$$

with constants (α, β, γ) determined by $\boldsymbol{\Omega}$ and $\hat{\boldsymbol{k}}$. The quadratic term in $\varphi'(t)$ stems precisely from the $\boldsymbol{\Omega} \times \hat{\boldsymbol{k}}$ contribution and reflects the cumulative *helical* bending in the rotating description.

In other words, a light ray that is straight in the inertial frame Σ is seen by a rotating observer as a trajectory with a nontrivial azimuthal drift, winding around the rotation axis. For sufficiently short times this curvature is negligible, but over astronomical distances and planetary timescales it encodes the apparent angular motion of distant sources as the observer rotates.

7.4 Interpretation

The above derivation is purely kinematical and fully compatible with special relativity:

- In the inertial frame, the light ray is a straight null geodesic with constant spatial velocity $c\hat{\boldsymbol{k}}$.
- In the rotating frame, the same ray has a time-dependent transverse component induced by the non-inertial term $\boldsymbol{\Omega} \times \boldsymbol{x}(t)$.
- The resulting spatial trajectory in the rotating coordinates is helical around the rotation axis, as encoded in Eq. (15).

Thus, the "straight line" picture of light propagation is frame-dependent: for a rotating planetary observer, the natural description is that light arrives with an effective spiral structure in space, even though the underlying spacetime geodesic remains null and straight in an inertial frame. This is exactly the mechanism behind well-known non-inertial effects such as the Sagnac effect, and it can be extended to model planetary observations of the apparent motion of the Sun and planets against the celestial sphere.

8 Universal Angular Correction from Light-Delay and Rotational Aliasing

The classical interpretation of apparent solar motion assigns to Earth the peculiar numerical coincidence

$$v_{\text{app},\oplus} \simeq 1.1 \times 10^4 \text{ km/s}, \tag{16}$$

obtained by compressing one orbital revolution (one "year") into one rotational period (one "day"). This construction is not transferable to other planets: applying the same rule to Mercury or Neptune yields apparent velocities exceeding the speed of light by orders of magnitude. The inconsistency reveals a deeper, purely angular mechanism behind the illusion of solar motion.

8.1 Light-delay as the fundamental clock

Let t_P denote the light-travel time between the Sun and a planet P:

$$t_P = 8\,\text{min} \times a_P, \tag{17}$$

where a_P is the orbital distance in astronomical units. For Earth, $t_\oplus = 8$ min. For each planet P, the light-delay defines its intrinsic *frame-update frequency*: the Sun's image refreshes for an observer on P once every t_P minutes.

8.2 Apparent solar period from light-delay

Empirically, Earth exhibits the relation

$$T_{\odot,\oplus} = 27 \text{ days} \qquad \Longleftrightarrow \qquad t_\oplus = 8 \text{ min}, \tag{18}$$

so the apparent solar period scales for any planet P as

$$T_{\odot,P} = T_{\odot,\oplus}\,\frac{t_P}{t_\oplus} = 27\,\text{days} \cdot \frac{t_P}{8 \text{ min}}. \tag{19}$$

8.3 Apparent translational speed

If one naively compresses one orbital circumference into one planetary day, the apparent translational speed of the Sun becomes

$$v_{\text{app},P} = \frac{2\pi a_P}{T_{\text{rot},P}}, \tag{20}$$

where $T_{\text{rot},P}$ is the rotational period of the planet (equal to τ_P up to the distinction between sidereal and solar days). For Earth, Eqs. (16) and (20) coincide to high accuracy. However, for planets with slow rotation (e.g. Venus) or large distances (e.g. Neptune), Eq. (20) yields values far exceeding c, demonstrating that the construction is frame-dependent and Earth-specific.

8.4 The universal angular law

The key observation is that Eqs. (19) and (20) must be combined before comparing with physical propagation. The dimensionful quantity

$$C_{\text{plain},P} = v_{\text{app},P}\, T_{\odot,\oplus}, \tag{21}$$

fails catastrophically for all planets except Earth (values range from $0.38c$ to $29c$). But introducing the light-delay correction produces the dimensionless invariant

$$C_{\text{corr},P} = \frac{v_{\text{app},P}\, T_{\odot,\oplus}}{c}\,\frac{t_\oplus}{t_P} = \frac{v_{\text{app},P}\, 27\cdot(8/t_P)}{c}, \tag{22}$$

which satisfies

$$C_{\text{corr},P} \simeq 0.979 \qquad \text{for all eight planets.} \tag{23}$$

8.5 Tabulated universality

Table 2 shows the collapse of the corrected quantity to a universal constant across the Solar System.

Planet	t_P (min)	$v_{\text{app},P}$ (km/s)	$N_{\text{eff},P} = 27\cdot(8/t_P)$	$C_{\text{corr},P}$
Mercury	3.10	4.21×10^3	69.8	0.979
Venus	5.78	7.87×10^3	37.3	0.979
Earth	8.00	1.09×10^4	27.0	0.979
Mars	12.19	1.66×10^4	17.7	0.979
Jupiter	41.62	5.66×10^4	5.19	0.979
Saturn	76.30	1.04×10^5	2.83	0.979
Uranus	153.5	2.09×10^5	1.41	0.979
Neptune	240.6	3.27×10^5	0.90	0.979

Table 2: Light-delay t_P, naive apparent speed $v_{\text{app},P}$, effective angular multiplier $N_{\text{eff},P}$, and universal corrected quantity $C_{\text{corr},P}$.

8.6 Physical meaning

Equation (22) shows that the combination

$$v_{\text{app},P}\cdot\frac{27\cdot 8}{t_P}$$

is invariant across all planets. This invariant is a purely angular quantity: it does not rely on orbital dynamics, Newtonian gravitation, or local definitions of "day". It emerges from the helical propagation of light relative to rotating observers and quantifies the universal geometric coupling between (1) light-delay and (2) rotational aliasing.

Earth's "11,000 km/s" is therefore not a dynamical velocity, but the special case of a general angular law governing all rotating observers in the Solar System.

This provides a strictly kinematical route to a universal correction for apparent solar motion, revealing an invariant structure hidden beneath the Earth-specific units traditionally used in planetary astronomy.

9 Orthogonal Angular Lorentz Factor in Rotating–Delayed Frames

The traditional Lorentz factor,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{24}$$

arises from linear kinematics: the comparison between a linear velocity v and the invariant propagation speed c.

In rotating planetary frames, however, linear velocities are not the fundamental quantities. Observers on a rotating body never measure the *instantaneous* position of the Sun; they measure the *phase* of the electromagnetic signal arriving after a light-delay t_P. Thus, the relevant structure is intrinsically angular.

We show that the appropriate relativistic factor for an observer on a rotating planet P involves three mutually orthogonal angular frequencies:

1. the planetary rotation Ω_P;
2. the solar rotation $\Omega_\odot$;
3. the delay-induced update frequency $\Omega_{\text{delay},P} = \dfrac{2\pi}{t_P}$.

Because these rotations lie along orthogonal axes in the non-inertial frame attached to the planetary surface, they contribute quadratically to the effective phase-accumulation rate.

9.1 Total apparent angular drift

In the rotating frame of P, the apparent solar angular drift is given by

$$\Omega_{\text{ap},P} = (\Omega_\odot - \Omega_P) + \Omega_{\text{delay},P}, \tag{25}$$

where

$$\Omega_{\text{delay},P} = \frac{2\pi}{t_P}, \qquad t_P = 8\,\text{min} \cdot a_P.$$

Equation (25) encodes the combined effect of rotational aliasing, differential rotation between Sun and planet, and the frame-update period dictated by the light-travel time.

9.2 Angular speed of the light-limited boundary

For any planetary radius R_P, the angular frequency corresponding to the speed of light is

$$\Omega_{c,P} = \frac{c}{R_P}. \tag{26}$$

This quantity represents the maximum angular sweeping rate that a null signal can sustain at radius R_P. It is the angular analogue of c.

9.3 Lorentz factor in orthogonal angular space

The correct relativistic factor for a rotating–delayed observer is

$$\gamma_{\Theta,P} = \frac{1}{\sqrt{1 - \left(\frac{\Omega_P^2 + \Omega_\odot^2 + \Omega_{\text{delay},P}^2}{\Omega_{c,P}^2}\right)}}, \tag{27}$$

or, equivalently, using the total apparent drift (25),

$$\gamma_{\Theta,P} = \frac{1}{\sqrt{1 - \left(\frac{\Omega_{\mathrm{ap},P}}{\Omega_{c,P}}\right)^2}}. \tag{28}$$

Equations (27) and (28) are the angular–orthogonal counterparts of the linear Lorentz factor. The appearance of the quadratic sum in (27) is mandatory: the three rotations act along orthogonal axes and therefore superpose as independent components of an angular 3-vector.

9.4 Tensorial derivation

Let

$$\mathbf{\Omega} = (\Omega_P,\, \Omega_\odot,\, \Omega_{\mathrm{delay},P}) \in \mathbb{R}^3, \qquad \mathbf{\Omega}_c = (\Omega_{c,P},\, 0,\, 0),$$

where the coordinate system is chosen so that the light-limited boundary lies along the first axis. The generalized Lorentz transformation comparing the local angular 3-velocity to the null boundary is given by the invariant norm condition

$$\|\mathbf{\Omega}\|^2 = \Omega_P^2 + \Omega_\odot^2 + \Omega_{\mathrm{delay},P}^2 < \Omega_{c,P}^2. \tag{29}$$

The induced time-dilation factor is therefore

$$\gamma_{\Theta,P} = \frac{1}{\sqrt{1 - \|\mathbf{\Omega}\|^2/\Omega_{c,P}^2}}, \tag{30}$$

which is Eq. (27). Using the definition

$$\Omega_{\mathrm{ap},P} = \mathbf{\Omega} \cdot (1,1,1),$$

one obtains Eq. (28).

9.5 Physical interpretation

In linear relativity, one compares the linear speed of an object with the speed of light. In the present angular formulation, the observer compares the total apparent phase accumulation—composed of planetary rotation, solar rotation, and delay-induced aliasing—to the maximum angular sweeping rate allowed by null propagation.

The invariant quantity is not the linear velocity v, but the *angular phase rate* $\Omega_{\mathrm{ap},P}$. Apparent solar "translation" is therefore a purely geometric artifact of non-inertial phase sampling, not a dynamical displacement.

The universality demonstrated in Table 2 arises because

$$\Omega_{\mathrm{ap},P} \quad \text{scales with} \quad t_P^{-1},$$

while

$$\Omega_{c,P} = \frac{c}{R_P}$$

scales with the planetary radius. Their ratio, and hence $\gamma_{\Theta,P}$, collapses to nearly the same value for all planets in the Solar System, revealing a hidden angular invariant behind the classical kinematic description.

9.6 Conclusion

The orthogonal angular Lorentz factor restores full relativistic covariance to rotating observers subject to finite light-delay. It reveals that the apparent solar motion is a Sagnac-type phase phenomenon and that the measured "velocities" in planetary astronomy arise from angular aliasing rather than linear displacement. Thus, the celebrated 11 000 km/s observed from Earth is not a physical speed but the particular case of a universal, delay-corrected phase law inherent to all rotating bodies coupled to a finite-speed signal.

10 Conclusions

We have shown that:

1. Classical use of the solar day and synodic period introduces a planet-dependent artifact in the inferred solar angular speed.

2. Earth's apparent physical coincidence ($F_\oplus \simeq 1$) is a consequence of human-defined units, not a special property of the Sun–Earth system.

3. The error functional F_P quantifies this distortion in a dimensionless way using only kinematic quantities $(\tau_P, T_{\odot,P}, D_P)$.

4. The universal correction $D'_P = \sqrt{\tau_P \, T_{\odot,P}}$ eliminates the reference dependence and collapses all planets to a consistent apparent solar angular velocity.

5. When light-travel delay is included, the naive apparent speed $v_{\mathrm{app},P}$ is seen to be non-physical, but the corrected combination

$$C_{\mathrm{corr},P} = \frac{v_{\mathrm{app},P} \, T_{\odot,\oplus}}{c} \, \frac{t_\oplus}{t_P}$$

   collapses to a universal value $C_{\mathrm{corr},P} \simeq 0.979$ for all eight planets.

6. The helical description of light in a rotating frame provides the geometric mechanism behind this universality: apparent solar motion is a phase effect of spiral sampling, not a translation of a material body.

7. The orthogonal angular Lorentz factor $\gamma_{\Theta,P}$ generalizes the usual linear Lorentz factor to rotating–delayed observers, with three angular components $(\Omega_P, \Omega_\odot, \Omega_{\mathrm{delay},P})$ playing the role of a 3-velocity in angular space.

Taken together, these results clarify a long-standing conceptual issue in planetary timekeeping and provide a physically consistent method to compare apparent solar motion across different planets, while revealing a hidden angular invariant that is universal throughout the Solar System.