

# Curvature and Killing Vectors in a Twistor-Based View of Spacetime

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November 19, 2025

## Abstract

We revisit curvature and Killing vectors in general relativity from a two-spinor and twistor perspective in which local on-shell time directions are taken as primary. Working in four dimensions, we represent null directions by spinor dyads  $k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}$  and describe photon kinematics and energy measurements entirely in terms of scalar overlaps  $u^{A\dot{A}} \lambda_A \tilde{\lambda}_{\dot{A}}$  between null twistor sectors and observer fields.

On the timelike side, we introduce a unit vector field  $\eta^\mu$  as an on-shell time direction and analyze its covariant derivative in the 1+3 decomposition. In static spacetimes  $\eta^\mu$  can be aligned with a timelike Killing vector, and the usual conserved energy  $E_\infty = -p_\mu \xi^\mu$  appears in the spinor language. In non-stationary spacetimes  $\eta^\mu$  still defines a preferred local time direction, but expansion and shear measure the failure of  $\eta^\mu$  to satisfy the Killing equation and thus the obstruction to a global time symmetry.

Curvature enters through the spin connection as holonomy acting on  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  and  $\eta^{A\dot{A}}$ , a viewpoint we summarize as “twistor misalignment”. We illustrate the framework for radial null propagation and gravitational redshift in Schwarzschild spacetime, and we comment on how this spinor-based organization of the kinematics reformulates, without modifying, the role of Killing symmetries in classical general relativity.

## 1 Introduction

In the standard formulation of general relativity one begins with a Lorentzian spacetime  $(\mathcal{M}, g_{\mu\nu})$  and treats matter and radiation as fields propagating on this background.[1] Symmetries of the metric are encoded in Killing vector fields  $\xi^\mu$ , defined by

$$\nabla_{(\mu} \xi_{\nu)} = 0, \tag{1}$$

or equivalently  $\mathcal{L}_\xi g_{\mu\nu} = 0$ . Timelike Killing vectors characterize stationary or static spacetimes and give rise to conserved energies for geodesic motion, while spacelike Killing vectors encode spatial symmetries such as axial symmetry or homogeneity.[1, 2] In this geometric picture Killing vectors and their associated

Noether charges are primary objects, and lightlike or timelike trajectories are analyzed in terms of their relation to these symmetries.

A natural language for recasting these structures is the two-spinor calculus and its twistor extension.[3, 4] In four dimensions any real vector can be written as a Hermitian spinor  $v_{A\dot{A}}$ , and any null vector admits a factorization

$$k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}, \quad (2)$$

so that massless excitations such as photons are represented kinematically by correlated spinor pairs  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$ . [5, 3] The Newman–Penrose formalism expresses curvature and optical properties of null congruences in this language, relating Weyl and Ricci spinors to expansion, shear and twist via the Sachs equations. [5, 6, 7]

Twistor theory makes this lightlike viewpoint more explicit by taking objects associated with null directions as primary and regarding spacetime points as derived constructs. [8, 9, 4] In this setting spinor and twistor variables encode both the conformal structure and the propagation of massless fields, while the metric enters through compatibility conditions and the choice of real slice. Although we will not use the full machinery of twistor space, the twistor perspective motivates focusing on null directions, spinor dyads and their holonomy as central carriers of geometric information.

Complementary to the null picture, the 1+3 covariant approach to relativistic cosmology emphasizes timelike congruences and their kinematics. [10, 11] A unit timelike vector field  $u^\mu$  representing a family of observers is decomposed via

$$\nabla_\mu u_\nu = u_\mu a_\nu + \frac{1}{3}\theta h_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu}, \quad (3)$$

with expansion  $\theta$ , shear  $\sigma_{\mu\nu}$  and vorticity  $\omega_{\mu\nu}$  capturing the local departure from rigid motion. In spacetimes with a timelike Killing vector aligned with  $u^\mu$ , expansion and shear vanish and the geometry is stationary in the corresponding frame. [1, 10] Local energy measurements for photons with wave vector  $k^\mu$  then take the familiar form

$$E_{\text{loc}} = \hbar\omega = -\hbar u^\mu k_\mu, \quad (4)$$

with gravitational redshift and lensing understood in terms of how  $u^\mu$  and  $k^\mu$  are related by curvature. [1, 12]

The present work combines these ingredients into a unified spinor-based description of curvature and time directions. Throughout we retain the standard Einstein–Maxwell dynamics and work entirely within classical general relativity. The notions of “twistor misalignment” and of local energy as a scalar overlap  $u^{A\dot{A}}\lambda_A\tilde{\lambda}_{\dot{A}}$  are intended as a rephrasing of familiar kinematics in spinor and twistor language, not as a modification of the underlying field equations. We introduce a local on-shell time direction  $\eta^\mu(x)$ , interpreted as the four-velocity field of a distinguished observer congruence or inertial background, and express both  $\eta^\mu$  and null directions  $k^\mu$  in two-spinor form. In static spacetimes such as Schwarzschild,  $\eta^\mu$  can be aligned with a timelike Killing vector, and the usual conserved energy  $E_\infty = -p_\mu \xi^\mu$  appears as a Noether charge. In non-stationary

spacetimes  $\eta^\mu$  still defines a local time direction, but the symmetric part of  $\nabla_\mu \eta_\nu$  no longer vanishes; curvature and kinematical quantities  $(\theta, \sigma_{\mu\nu})$  then measure the obstruction to promoting  $\eta^\mu$  to a global Killing field.

Our aim is therefore not to modify general relativity or electrodynamics, but to reorganize the kinematics of light propagation and time evolution in terms of spinor and twistor variables. Curvature is interpreted as *twistor misalignment*: holonomy of the spin connection acting on null spinor dyads  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  and on the time direction  $\eta^{A\dot{A}}$  reshapes the scalar overlaps  $u^{A\dot{A}}\lambda_A\tilde{\lambda}_{\dot{A}}$  that encode observable frequencies and optical effects.

The structure of the paper is as follows. In Sec. 2 we review Killing vectors and curvature in the two-spinor formalism, emphasizing the decomposition of  $\nabla_{A\dot{A}}\xi_{B\dot{B}}$  and its relation to the curvature spinors. In Sec. 3 we discuss curvature as twistor misalignment, focusing on the holonomy of null spinor dyads and its connection to the optical properties of null congruences. Sec. 4 introduces the local on-shell time direction  $\eta^\mu$  and analyzes its relation to Killing vectors in static and non-static spacetimes. In Sec. 5 we illustrate the framework in Schwarzschild spacetime, expressing gravitational redshift and null propagation entirely in terms of  $\eta^{A\dot{A}}$  and  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$ . We conclude in Sec. 6 with a brief discussion of how this spinor-based viewpoint reorganizes the logical role of Killing symmetries and suggests possible extensions within more general twistor-based or emergent spacetime scenarios.

## 2 Killing vectors and curvature in spinor form

In a Lorentzian spacetime  $(\mathcal{M}, g_{\mu\nu})$  a Killing vector field  $\xi^\mu$  is defined by the vanishing of the symmetrized covariant derivative

$$\nabla_{(\mu}\xi_{\nu)} = 0, \quad (5)$$

which is equivalent to  $\mathcal{L}_\xi g_{\mu\nu} = 0$  and expresses the fact that the flow of  $\xi^\mu$  generates an isometry of the metric.[1] In particular, a timelike Killing vector encodes stationarity and defines a conserved energy for freely falling test particles or fields; a spacelike Killing vector encodes spatial symmetries such as axial symmetry or homogeneity.[1, 2]

Equation (5) can be rewritten in two-spinor notation by replacing  $\xi^\mu$  with its spinor form

$$\xi_{A\dot{A}} \equiv \xi_\mu \sigma_{A\dot{A}}^\mu, \quad (6)$$

where  $\sigma_{A\dot{A}}^\mu$  are the Infeld-van der Waerden symbols.[3] The Killing equation becomes an algebraic condition on the spinor derivative  $\nabla_{A\dot{A}}\xi_{B\dot{B}}$ , namely

$$\nabla_{A\dot{A}}\xi_{B\dot{B}} + \nabla_{B\dot{B}}\xi_{A\dot{A}} = 0. \quad (7)$$

Decomposing  $\nabla_{A\dot{A}}\xi_{B\dot{B}}$  into irreducible spinor parts shows that the only nonvanishing piece for a true Killing vector is the antisymmetric bivector part, which may be written as

$$\nabla_{A\dot{A}}\xi_{B\dot{B}} = \epsilon_{AB}\phi_{\dot{A}\dot{B}} + \epsilon_{\dot{A}\dot{B}}\tilde{\phi}_{AB}, \quad (8)$$

for symmetric spinors  $\phi_{\dot{A}\dot{B}}$  and  $\tilde{\phi}_{AB}$ . [3, 4] These encode the selfdual and anti-selfdual parts of  $\nabla_{[\mu}\xi_{\nu]}$ , i.e. the infinitesimal Lorentz transformation generated by the isometry.

The relation between Killing vectors and curvature is obtained by commuting covariant derivatives. In tensor form, one has

$$\nabla_{\mu}\nabla_{\nu}\xi_{\rho} = R_{\rho\nu\mu}{}^{\sigma}\xi_{\sigma}, \quad (9)$$

which follows from (5) and the definition of the Riemann tensor. [1] In two-spinor language the curvature decomposes into the Weyl spinor  $\Psi_{ABCD}$ , the tracefree Ricci spinor  $\Phi_{\dot{A}\dot{A}B\dot{B}}$  and the scalar curvature  $\Lambda$ . [3, 5] The commutator of covariant derivatives acting on  $\xi_{\dot{A}\dot{A}}$  can be written schematically as

$$[\nabla_{\dot{A}\dot{A}}, \nabla_{\dot{B}\dot{B}}]\xi_{\dot{C}\dot{C}} = \varepsilon_{\dot{A}\dot{B}}\Psi_{ABCD}\xi^{\dot{D}}{}_{\dot{C}} + \varepsilon_{AB}\bar{\Psi}_{\dot{A}\dot{B}\dot{C}\dot{D}}\xi^{\dot{D}}{}_{\dot{C}} + \text{Ricci terms}, \quad (10)$$

where the Ricci part involves  $\Phi_{\dot{A}\dot{A}B\dot{B}}$  and  $\Lambda$  and is completely fixed by the Einstein equations for the matter content. [3, 4] Equation (10) shows that the existence of nontrivial Killing vectors imposes algebraic constraints on the curvature spinors, which underlies many classification results for exact solutions. [2]

A closely related concept is that of a Killing spinor, defined by a first-order equation of the form

$$\nabla_{\dot{A}\dot{A}}\kappa_B = \alpha\varepsilon_{AB}\bar{\kappa}_{\dot{A}}, \quad (11)$$

with constant  $\alpha$ . [13] In many dimensions such spinors generate towers of antisymmetric tensors (Killing–Yano and conformal Killing–Yano forms) and, in particular, Killing vectors constructed as bilinears in  $\kappa$ . [13] Although in this paper we work primarily with ordinary Killing vectors in four dimensions, the two-spinor formalism we use is the same index calculus that underlies the modern treatment of Killing spinors and related hidden symmetries. [3, 4, 13]

In summary, the two-spinor representation (8) of  $\nabla_{\dot{A}\dot{A}}\xi_{\dot{B}\dot{B}}$  makes explicit how isometries are encoded in pairs of rank-2 spinors  $(\phi_{\dot{A}\dot{B}}, \tilde{\phi}_{AB})$ , while the commutator relation (10) ties these directly to the curvature spinors  $(\Psi_{ABCD}, \Phi_{\dot{A}\dot{A}B\dot{B}}, \Lambda)$ . This spinorial viewpoint will be the starting point for our twistor-based interpretation of curvature in the next section.

### 3 Curvature as twistor misalignment (Holonomy)

In the previous section Killing vectors and curvature were described in terms of curvature spinors acting on vector fields. [3, 4, 1] We now turn to a twistor-based picture, in which curvature manifests itself through the transport and holonomy of spinor fields along null geodesics. The key idea is that null directions and photon states are encoded in spinor pairs  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$ , and that curvature appears as a misalignment of these spinors when they are transported along and between null geodesics. This misalignment in turn reshapes the scalar contractions that encode local energy measurements and optical effects such as shear and focusing. [5, 6]

**Remark.** *The term twistor misalignment is used here as a purely interpretive label for the standard holonomy effects of the spin connection acting on null spinor dyads. No new mathematical structure is introduced: “misalignment” simply refers to the relative  $\text{SL}(2, \mathbb{C})$  rotation or boost between neighbouring twistor sectors induced by curvature.*

### 3.1 Null congruences and spinor transport

Consider a null geodesic congruence with tangent

$$k^\mu = \frac{dx^\mu}{d\lambda}, \quad g_{\mu\nu} k^\mu k^\nu = 0, \quad k^\nu \nabla_\nu k^\mu = 0, \quad (12)$$

where  $\lambda$  is an affine parameter. In two-spinor form the tangent is written as

$$k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}, \quad (13)$$

for some nonzero spinor fields  $\lambda_A(x)$  and  $\tilde{\lambda}_{\dot{A}}(x)$ , unique up to the rescaling

$$\lambda_A \rightarrow \alpha \lambda_A, \quad \tilde{\lambda}_{\dot{A}} \rightarrow \alpha^{-1} \tilde{\lambda}_{\dot{A}}, \quad \alpha \in \mathbb{C}^\times. \quad (14)$$

This is the standard representation of null directions in two-spinor calculus and underlies the Newman–Penrose formalism.[5, 3]

Along each null geodesic we define the directional derivative

$$\frac{D}{D\lambda} \equiv k^{A\dot{A}} \nabla_{A\dot{A}}. \quad (15)$$

In the geometric–optics limit of electromagnetism one typically chooses the spinors such that they are parallel transported,

$$\frac{D\lambda_A}{D\lambda} = 0, \quad \frac{D\tilde{\lambda}_{\dot{A}}}{D\lambda} = 0, \quad (16)$$

so that all nontrivial evolution of the field amplitude and phase is carried by a scalar eikonal  $S(x)$  and a slowly varying polarization.[4] In a curved space-time, the behaviour of  $\lambda_A$  and  $\tilde{\lambda}_{\dot{A}}$  between neighbouring rays is sensitive to the curvature spinors. If we consider two infinitesimally nearby null geodesics in the congruence, with spinor representatives  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  and  $(\lambda_A + \delta\lambda_A, \tilde{\lambda}_{\dot{A}} + \delta\tilde{\lambda}_{\dot{A}})$ , their relative change under parallel transport around a small loop is governed by the commutator of covariant derivatives acting on the spinors,

$$[\nabla_{A\dot{A}}, \nabla_{B\dot{B}}] \lambda_C = \epsilon_{\dot{A}\dot{B}} \Psi_{ABCD} \lambda^D + \epsilon_{AC} \Phi_{B\dot{A}\dot{B}} \lambda^D + \Lambda \epsilon_{AC} \epsilon_{BD} \lambda^D, \quad (17)$$

with an analogous expression for  $\tilde{\lambda}_{\dot{C}}$ . [3, 4] The Weyl spinor  $\Psi_{ABCD}$  controls the purely gravitational (conformal) part of this holonomy, while  $\Phi_{A\dot{A}B\dot{B}}$  and  $\Lambda$  encode the Ricci and scalar curvature contributions.

The kinematical properties of a null congruence—expansion, shear and twist—are traditionally encoded in complex Newman–Penrose spin coefficients, which obey the Sachs optical equations.[5, 6] In the present language these optical effects can be viewed as describing how the spinor dyads associated with neighbouring null rays are deformed by curvature and matter, i.e. how curvature induces a systematic misalignment of nearby twistor sectors.

### 3.2 Phase, holonomy and energy shifts

Local photon energy measurements are expressed by the scalar

$$E_{\text{loc}} = \hbar\omega = -\hbar u^{A\dot{A}} \lambda_A \tilde{\lambda}_{\dot{A}}, \quad (18)$$

where  $u^{A\dot{A}}$  is the spinor form of the observer's four-velocity. From the twistor viewpoint this contraction measures the overlap between the observer's time direction and the null twistor sector spanned by  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$ . [4] When  $\lambda_A$  and  $\tilde{\lambda}_{\dot{A}}$  are transported along a null geodesic according to (16), the evolution of  $E_{\text{loc}}$  along that geodesic is entirely due to the transport of  $u^{A\dot{A}}$  along the observer's worldline and to the relative normalization of  $k^\mu$  and  $u^\mu$ .

Curvature becomes visible when we compare such energy measurements between different observers or along different null rays. For instance, in a static spacetime with a timelike Killing vector, the change in  $u^{A\dot{A}} \lambda_A \tilde{\lambda}_{\dot{A}}$  between two static observers at different radii reproduces the familiar gravitational redshift. [1] In more general situations, relative phases between neighbouring null geodesics and between different observers are modified by the holonomy generated by parallel transport of the spinor dyads and of the observer fields.

If we transport  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  around a closed loop  $\gamma$  in spacetime, the spinors return rotated by an  $\text{SL}(2, \mathbb{C})$  transformation

$$\lambda_A \longrightarrow M_A{}^B[\gamma] \lambda_B, \quad \tilde{\lambda}_{\dot{A}} \longrightarrow \tilde{M}_{\dot{A}}{}^{\dot{B}}[\gamma] \tilde{\lambda}_{\dot{B}}, \quad (19)$$

where the holonomy matrices  $M[\gamma]$  and  $\tilde{M}[\gamma]$  can be written as path-ordered exponentials of the spin connection,

$$M_A{}^B[\gamma] = \mathcal{P} \exp\left(-\oint_\gamma \omega_{\mu A}{}^B dx^\mu\right), \quad (20)$$

and are determined by the curvature via the commutator relation (17). [3, 4] The qualitative entries of Table 1 can be understood as describing which components of  $M_A{}^B[\gamma]$  are responsible for expansion, shear, twist and polarization rotation in the language of optical scalars. [5, 6, 12]

When the holonomy is nontrivial, the scalar overlap (18) between a given observer field  $u^{A\dot{A}}$  and the transported null dyad changes in a way that can be traced back to the Weyl and Ricci components of the curvature. Gravitational lensing and polarization rotation can be viewed as cumulative manifestations of such twistor misalignment along extended null paths.

For small loops the holonomy matrices  $M_A{}^B[\gamma]$  admit an expansion in powers of the enclosed area, with the leading contribution linear in the curvature spinors. The next subsection makes this infinitesimal relation explicit by expressing the net change of a spinor  $\lambda_A$  transported around an infinitesimal parallelogram directly in terms of the Weyl spinor  $\Psi_{ABCD}$ .

### 3.3 Infinitesimal loops and the Weyl spinor

To make the idea of twistor misalignment more concrete, consider an infinitesimal parallelogram in spacetime spanned by two vectors  $v^{A\dot{A}}$  and  $w^{A\dot{A}}$ . Starting

from a spinor  $\lambda_A$  at one corner and parallel transporting it around the loop, the net change to first order in the enclosed area is

$$\delta\lambda_A = \frac{1}{2} v^{B\dot{B}} w^C_{\dot{B}} [\nabla_{B\dot{C}}, \nabla_C^{\dot{C}}] \lambda_A. \quad (21)$$

Using (17) and neglecting Ricci and scalar curvature for simplicity (e.g. in a vacuum region), this reduces to

$$\delta\lambda_A = \frac{1}{2} \epsilon_{\dot{B}\dot{C}} v^{B\dot{B}} w^{C\dot{C}} \Psi_{BCAD} \lambda^D. \quad (22)$$

Thus the Weyl spinor  $\Psi_{ABCD}$  directly controls how spinor directions are twisted by curvature when transported around small loops. For a null dyad  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  representing a photon, such infinitesimal holonomies accumulate along extended paths and between neighbouring rays, leading to observable changes in shear, focusing and polarization that are captured in the optical scalar equations.[5, 6]

These considerations motivate an interpretation of curvature in which the primary objects are not the components of the Riemann tensor in a coordinate basis, but the way curvature reshapes the web of spinor and twistor directions in spacetime. In the next section we introduce a local on-shell time direction  $\eta^\mu$  and analyze how its relation to null twistor sectors and to Killing vectors encodes both symmetry and curvature information.

### 3.4 Phase, holonomy and energy shifts

Local photon energy measurements are expressed by the scalar

$$E_{\text{loc}} = \hbar\omega = -\hbar u^{A\dot{A}} \lambda_A \tilde{\lambda}_{\dot{A}}, \quad (23)$$

where  $u^{A\dot{A}}$  is the spinor form of the observer's four-velocity. From the twistor viewpoint this contraction measures the overlap between the observer's time direction and the null twistor sector spanned by  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$ . When  $\lambda_A$  and  $\tilde{\lambda}_{\dot{A}}$  are transported along a null geodesic according to (16), the evolution of  $E_{\text{loc}}$  along that geodesic is entirely due to the transport of  $u^{A\dot{A}}$  along the observer's worldline and to the relative normalization of  $k^\mu$  and  $u^\mu$ .

Curvature becomes visible when we compare such energy measurements between different observers or along different null rays. Consider, for instance, two static observers in a static spacetime, connected by a family of radial null geodesics. Along each ray, the spinors  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  are parallel transported according to (16), but the relation between  $u^{A\dot{A}}$  at emission and at reception is distorted by curvature. In Schwarzschild spacetime this distortion manifests itself in the familiar gravitational redshift: the scalar  $u^{A\dot{A}} \lambda_A \tilde{\lambda}_{\dot{A}}$  decreases as the photon climbs out of the potential well, leading to a lower measured frequency at larger radius.

More generally, if we transport  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  around a closed loop  $\gamma$  in spacetime, the spinors return rotated by an element of the local  $\text{SL}(2, \mathbb{C})$  spin group,

$$\lambda_A \longrightarrow M_A^B[\gamma] \lambda_B, \quad \tilde{\lambda}_{\dot{A}} \longrightarrow \tilde{M}_{\dot{A}}^{\dot{B}}[\gamma] \tilde{\lambda}_{\dot{B}}, \quad (24)$$

where the matrices  $M[\gamma]$  and  $\tilde{M}[\gamma]$  are path-ordered exponentials of the spin connection along  $\gamma$ . The deviation of  $M[\gamma]$  from the identity is determined by the curvature spinors, as in (17). When this holonomy is nontrivial, the scalar overlap (23) between a given observer field  $u^{AA}$  and the transported null dyad changes in a way that can be traced back to the Weyl and Ricci components of the curvature.

From this perspective curvature can be viewed as a source of *twistor misalignment*: it induces relative rotations and boosts between different null spinor dyads and between these dyads and the observer fields. The familiar optical effects of curvature on light—focusing, shear, and redshift—can be rephrased as statements about how bundles of null twistor sectors are reoriented by the spin connection.

### 3.5 Infinitesimal loops and the Weyl spinor

To make the idea of twistor misalignment more concrete, consider an infinitesimal parallelogram in spacetime spanned by two vectors  $v^{AA}$  and  $w^{AA}$ . Starting from a spinor  $\lambda_A$  at one corner and parallel transporting it around the loop, the net change to first order in the area is

$$\delta\lambda_A = \frac{1}{2} v^{B\dot{B}} w^C_{\dot{B}} [\nabla_{B\dot{C}}, \nabla_{C\dot{C}}] \lambda_A. \quad (25)$$

Using (17) and neglecting Ricci and scalar curvature for simplicity (e.g. in a vacuum region), this reduces to

$$\delta\lambda_A = \frac{1}{2} \epsilon_{\dot{B}\dot{C}} v^{B\dot{B}} w^{C\dot{C}} \Psi_{BCAD} \lambda^D. \quad (26)$$

Thus, the Weyl spinor  $\Psi_{ABCD}$  directly controls how spinor directions are twisted by curvature when transported around small loops. For a null dyad  $(\lambda_A, \bar{\lambda}_{\dot{A}})$  representing a photon, such infinitesimal holonomies accumulate along extended paths and between neighbouring rays, leading to observable changes in polarization and in the relation between the null direction and any given observer field.

These considerations motivate an interpretation of curvature in which the primary objects are not the components of the Riemann tensor in some coordinate basis, but the way curvature reshapes the web of spinor and twistor directions in spacetime. In the next section we introduce a local on-shell time direction  $\eta^\mu$  and analyze how its relation to null twistor sectors and to Killing vectors encodes both symmetry and curvature information.

## 4 On-shell time directions and their relation to Killing vectors

In the previous section curvature appeared as a misalignment of null twistor sectors and observer fields under parallel transport.[3, 4, 5, 1] We now introduce

a local *on-shell time direction*  $\eta^\mu(x)$  and analyze how it relates to Killing vectors in static and non-static situations. The guiding idea is that  $\eta^\mu$  encodes the locally preferred flow of proper time for massive degrees of freedom or for an effective inertial background, while a true timelike Killing vector is a special case in which this time flow extends to an exact symmetry of the metric.[1, 10]

**Remark.** *The “on-shell time direction”  $\eta^\mu$  introduced in this section does not represent a new physical field or additional structure beyond standard general relativity. It is simply a convenient designation for an arbitrary unit timelike congruence used to organize local energy measurements and 1+3 kinematical quantities. In static spacetimes  $\eta^\mu$  may be chosen to coincide with the normalized timelike Killing vector, but in general it plays no dynamical role and carries no equations of motion.*

#### 4.1 Defining an on-shell time direction

We assume that in the region of interest there exists a smooth, future-directed timelike vector field  $\eta^\mu(x)$ , normalized by

$$\eta^\mu \eta_\mu = -1. \quad (27)$$

Physically,  $\eta^\mu$  may be interpreted as the four-velocity field of a distinguished family of observers (for example, a congruence of static observers in a static spacetime, or the rest frame of a cosmological fluid).[10] It can also be thought of as an effective on-shell time direction selected by some underlying scalar or inertial field; we will not commit to a specific microscopic origin here.

In two-spinor notation we write

$$\eta^{A\dot{A}} = \eta^\mu \sigma_\mu^{A\dot{A}}, \quad (28)$$

with the normalization condition

$$\eta^{A\dot{A}} \eta_{A\dot{A}} = -2. \quad (29)$$

For any null vector  $k^{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}$  the scalar

$$\omega_\eta \equiv -\eta^{A\dot{A}} \lambda_A \tilde{\lambda}_{\dot{A}} \quad (30)$$

may be read as the frequency of the corresponding photon with respect to the  $\eta$ -frame. In particular, if an individual observer with four-velocity  $u^\mu$  is comoving with the  $\eta$ -congruence, one has  $u^\mu = \eta^\mu$  and the local photon energy is

$$E_{\text{loc}} = \hbar \omega_\eta = -\hbar \eta^{A\dot{A}} \lambda_A \tilde{\lambda}_{\dot{A}}. \quad (31)$$

The covariant derivative of  $\eta^\mu$  can be decomposed into acceleration, expansion, shear and vorticity in the usual way,[10, 1]

$$\nabla_\mu \eta_\nu = \eta_\mu a_\nu + \frac{1}{3} \theta h_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu}, \quad (32)$$

with  $h_{\mu\nu} = g_{\mu\nu} + \eta_\mu \eta_\nu$  the projector orthogonal to  $\eta^\mu$ ,  $a_\mu = \eta^\nu \nabla_\nu \eta_\mu$  the four-acceleration,  $\theta = \nabla_\mu \eta^\mu$  the expansion,  $\sigma_{\mu\nu}$  the symmetric tracefree shear, and  $\omega_{\mu\nu}$  the antisymmetric vorticity tensor. These fields characterize the kinematical properties of the  $\eta$ -congruence independently of any Killing symmetry.

## 4.2 Static spacetimes: alignment with Killing vectors

In a static spacetime there exists a timelike Killing vector  $\xi^\mu$  which is hypersurface orthogonal.[1, 2] In adapted coordinates one may write

$$\xi^\mu = (\partial_t)^\mu, \quad g_{\mu\nu}\xi^\mu\xi^\nu = -V(\mathbf{x})^2, \quad (33)$$

with lapse function  $V(\mathbf{x}) > 0$ . The natural static observers have four-velocity

$$u^\mu = \frac{\xi^\mu}{\sqrt{-\xi^\nu\xi_\nu}} = \frac{1}{V} \xi^\mu, \quad (34)$$

and we may choose

$$\eta^\mu = u^\mu. \quad (35)$$

In this case  $\eta^\mu$  is simply a normalized timelike Killing vector, and the Killing equation

$$\nabla_{(\mu}\eta_{\nu)} = 0 \quad (36)$$

implies that the expansion and shear of the congruence vanish,

$$\theta = 0, \quad \sigma_{\mu\nu} = 0, \quad (37)$$

so that the kinematics is purely acceleration plus possible vorticity.[10] The associated Noether charge for a particle or photon with momentum  $p^\mu$  is

$$E_\infty = -p_\mu\xi^\mu = -V p_\mu\eta^\mu, \quad (38)$$

which is conserved along geodesics and may be interpreted as the energy as seen from infinity.[1]

For a photon with wave vector  $k^\mu$  one has, in spinor form,

$$E_\infty = -k_\mu\xi^\mu = -V \eta^{A\dot{A}}\lambda_A\tilde{\lambda}_{\dot{A}}, \quad (39)$$

so that the local  $\eta$ -frequency (30) and the conserved energy at infinity are related by

$$\hbar\omega_\eta = \frac{E_\infty}{V(\mathbf{x})}. \quad (40)$$

This reproduces the familiar gravitational redshift in static spacetimes,[1] but now expressed entirely in terms of the overlap between the null twistor sector and the on-shell time direction  $\eta^{A\dot{A}}$ .

## 4.3 Non-stationary situations: failure of the Killing equation

In a general, non-stationary spacetime there need not exist any timelike Killing vector at all.[1, 2] Nevertheless, a physically distinguished on-shell time direction  $\eta^\mu$  may still be defined, for example as the four-velocity of a cosmological

fluid, or as the average motion of matter in a given region, or as an effective inertial background.[10, 11] In such cases  $\eta^\mu$  will typically *not* satisfy the Killing equation; instead, its covariant derivative carries information about the time dependence of the geometry as seen in the  $\eta$ -frame.

From (32) one sees that the symmetric part of  $\nabla_\mu \eta_\nu$  is given by

$$\nabla_{(\mu} \eta_{\nu)} = \frac{1}{3} \theta h_{\mu\nu} + \sigma_{\mu\nu}, \quad (41)$$

so that the failure of  $\eta^\mu$  to be a Killing vector is directly measured by the expansion and shear of the congruence. In spinor notation one may decompose

$$\nabla_{A\dot{A}} \eta_{B\dot{B}} = \epsilon_{AB} \phi_{\dot{A}\dot{B}} + \epsilon_{\dot{A}\dot{B}} \tilde{\phi}_{AB} + S_{A\dot{A}B\dot{B}}, \quad (42)$$

where  $(\phi_{\dot{A}\dot{B}}, \tilde{\phi}_{AB})$  represent the antisymmetric part as in (8) and  $S_{A\dot{A}B\dot{B}} = S_{(A\dot{A}B\dot{B})}$  encodes the symmetric, non-Killing contribution.[3] The tensor  $S_{A\dot{A}B\dot{B}}$  is built from the expansion and shear and vanishes if and only if  $\eta^\mu$  is Killing.

In this more general setting the scalar

$$\omega_\eta(x) = -\eta^{A\dot{A}}(x) \lambda_A(x) \tilde{\lambda}_{\dot{A}}(x) \quad (43)$$

still defines a local frequency for null rays relative to the  $\eta$ -frame, but there is no globally conserved energy  $E_\infty$ . Instead, curvature and the kinematical quantities  $(\theta, \sigma_{\mu\nu})$  induce genuine time dependence in the spectrum of frequencies measured along  $\eta$ -worldlines.[10]

#### 4.4 Curvature as an obstruction to global time symmetry

The integrability conditions for Killing vectors, discussed in Sec. 2, show that the existence of a nontrivial timelike Killing field is a strong restriction on the curvature spinors  $(\Psi_{ABCD}, \Phi_{A\dot{A}B\dot{B}}, \Lambda)$ . [3, 4, 2] From the present viewpoint, where  $\eta^\mu$  is taken as a given local on-shell time direction, curvature can be interpreted as an obstruction to promoting  $\eta^\mu$  to a genuine global symmetry direction.

Concretely, given a timelike unit vector field  $\eta^\mu$  one may ask whether there exists a rescaling and deformation of  $\eta^\mu$  into a vector field  $\xi^\mu$  that satisfies the Killing equation. In regions where the expansion and shear of the  $\eta$ -congruence vanish and the curvature spinors obey suitable conditions, such a Killing field can exist and align with  $\eta^\mu$ , leading to an effectively static or stationary description.[1, 2] In regions with nonzero expansion, shear or rapidly varying curvature, no such alignment is possible:  $\eta^\mu$  remains a local time direction, but there is no global time symmetry and no associated Noether charge.

In this way  $\eta^\mu$  and the curvature spinors jointly encode both the local and global aspects of time in a given spacetime. The local overlap between  $\eta^{A\dot{A}}$  and null twistor sectors  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  determines photon energies and their redshift, while the possibility (or impossibility) of extending  $\eta^\mu$  to a Killing vector reflects whether these energies admit a simple, conserved reference value  $E_\infty$ . [1] The next section illustrates this interplay in the familiar case of Schwarzschild spacetime.

## 5 Schwarzschild spacetime as a static example

To make the preceding discussion more concrete we consider the Schwarzschild spacetime, which provides a simple static background with a timelike Killing vector and spherical symmetry.[14, 1, 2] In this setting the identification of the on-shell time direction  $\eta^\mu$  with a normalized Killing field is exact, and the familiar gravitational redshift can be expressed directly in terms of the overlap between  $\eta^{A\dot{A}}$  and null twistor sectors.

### 5.1 Geometry, Killing vector and on-shell time direction

The Schwarzschild line element in standard coordinates  $(t, r, \theta, \phi)$  reads

$$ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad f(r) = 1 - \frac{2M}{r}, \quad (44)$$

with mass parameter  $M$ . [14, 1] The spacetime is static outside the horizon  $r > 2M$  and admits a timelike Killing vector

$$\xi^\mu = (\partial_t)^\mu, \quad g_{\mu\nu}\xi^\mu\xi^\nu = -f(r). \quad (45)$$

Comparing with the general static ansatz  $ds^2 = -V^2 dt^2 + h_{ij} dx^i dx^j$ , we identify  $V(r) = \sqrt{f(r)}$ .

Static observers at fixed  $(r, \theta, \phi)$  have four-velocity

$$u^\mu(r) = \frac{\xi^\mu}{\sqrt{-\xi^\nu\xi_\nu}} = \left(f(r)^{-1/2}, 0, 0, 0\right), \quad (46)$$

normalized so that  $u^\mu u_\mu = -1$ . [1] In our terminology we may set

$$\eta^\mu(r) = u^\mu(r), \quad (47)$$

so that  $\eta^\mu$  is a normalized timelike Killing vector field, representing the on-shell time direction of static observers. The associated conserved Noether charge for a particle or photon with momentum  $p^\mu$  is

$$E_\infty = -p_\mu \xi^\mu = -\sqrt{f(r)} p_\mu \eta^\mu, \quad (48)$$

which is independent of  $r$  along any geodesic.

### 5.2 Radial null geodesics and gravitational redshift

We restrict attention to radial null geodesics in the equatorial plane  $\theta = \pi/2$ . A radial null geodesic has tangent

$$k^\mu = \frac{dx^\mu}{d\lambda} = \left(\frac{dt}{d\lambda}, \frac{dr}{d\lambda}, 0, 0\right), \quad (49)$$

with null condition

$$g_{\mu\nu}k^\mu k^\nu = -f(r) \left(\frac{dt}{d\lambda}\right)^2 + f(r)^{-1} \left(\frac{dr}{d\lambda}\right)^2 = 0. \quad (50)$$

This implies

$$\frac{dr}{d\lambda} = \pm f(r) \frac{dt}{d\lambda}, \quad (51)$$

with the plus sign corresponding to outgoing and the minus sign to ingoing radial null rays.[1, 15]

The conserved energy at infinity for the photon is

$$E_\infty = -k_\mu \xi^\mu = -k_t = f(r) \frac{dt}{d\lambda}, \quad (52)$$

so that

$$\frac{dt}{d\lambda} = \frac{E_\infty}{f(r)}, \quad \frac{dr}{d\lambda} = \pm E_\infty. \quad (53)$$

The coordinate speed of radial light is therefore

$$\frac{dr}{dt} = \pm f(r), \quad (54)$$

showing the familiar coordinate slowing of outgoing light near the horizon.[1, 15]

The local frequency measured by a static observer at radius  $r$  is

$$\omega_\eta(r) = -\eta^\mu k_\mu = -g_{\mu\nu} \eta^\mu k^\nu. \quad (55)$$

Using

$$\eta^\mu(r) = (f(r)^{-1/2}, 0, 0, 0), \quad k^\mu = \left(\frac{dt}{d\lambda}, \frac{dr}{d\lambda}, 0, 0\right), \quad (56)$$

we obtain

$$\omega_\eta(r) = -[-f(r) \eta^t k^t] = f(r) \eta^t \frac{dt}{d\lambda} = f(r) f(r)^{-1/2} \frac{dt}{d\lambda} = f(r)^{1/2} \frac{dt}{d\lambda}. \quad (57)$$

With (52) this becomes

$$\omega_\eta(r) = \frac{E_\infty}{\sqrt{f(r)}}. \quad (58)$$

Thus the photon energy measured by a static observer at radius  $r$  is

$$E_{\text{loc}}(r) = \hbar \omega_\eta(r) = \frac{E_\infty}{\sqrt{1 - \frac{2M}{r}}}. \quad (59)$$

For two static observers at radii  $r_e$  (emitter) and  $r_o$  (observer), the ratio of measured frequencies for the same photon is

$$\frac{\omega_\eta(r_o)}{\omega_\eta(r_e)} = \sqrt{\frac{f(r_e)}{f(r_o)}} = \sqrt{\frac{1 - \frac{2M}{r_e}}{1 - \frac{2M}{r_o}}}. \quad (60)$$

In particular, for an observer at infinity ( $r_o \rightarrow \infty$ ,  $f(r_o) \rightarrow 1$ ) one finds

$$\omega_\infty = \omega_\eta(r_e) \sqrt{1 - \frac{2M}{r_e}}, \quad (61)$$

which is the standard gravitational redshift formula.[1]

### 5.3 Spinor and twistor representation in the $\eta$ -frame

We now translate this picture into the two-spinor language used in the previous sections.[3, 4] It is convenient to introduce an orthonormal tetrad  $(e_{(a)}^\mu)$  adapted to the static observers,

$$e_{(0)}^\mu = \eta^\mu = (f^{-1/2}, 0, 0, 0), \quad e_{(1)}^\mu = (0, f^{1/2}, 0, 0), \quad (62)$$

with  $e_{(2)}^\mu$  and  $e_{(3)}^\mu$  tangent to the two-spheres of symmetry. From this tetrad one constructs Infeld–van der Waerden symbols  $\sigma_\mu^{A\dot{A}}$  and the corresponding spinor representation of vectors.[3]

For a radial null direction we introduce a Newman–Penrose null tetrad  $(\ell^\mu, n^\mu, m^\mu, \bar{m}^\mu)$  aligned with the radial motion, for example

$$\ell^\mu = \frac{1}{\sqrt{2}} (e_{(0)}^\mu + e_{(1)}^\mu), \quad n^\mu = \frac{1}{\sqrt{2}} (e_{(0)}^\mu - e_{(1)}^\mu), \quad (63)$$

with  $m^\mu$  and  $\bar{m}^\mu$  tangent to the angular directions.[5, 7] Outgoing radial null rays can then be taken proportional to  $\ell^\mu$ ,

$$k^\mu = \kappa \ell^\mu, \quad (64)$$

for some positive scalar factor  $\kappa$ .

In the two-spinor formalism there exists a spin frame  $(o^A, \iota^A)$  such that

$$\ell^{A\dot{A}} = o^A \bar{o}^{\dot{A}}, \quad n^{A\dot{A}} = \iota^A \bar{\iota}^{\dot{A}}, \quad (65)$$

and the complex vectors  $m^\mu$  and  $\bar{m}^\mu$  are represented by  $o^A \bar{\iota}^{\dot{A}}$  and  $\iota^A \bar{o}^{\dot{A}}$ , respectively.[3, 4] The null wave vector then takes the form

$$k_{A\dot{A}} = \kappa o_A \bar{o}_{\dot{A}}, \quad (66)$$

so that we may identify

$$\lambda_A = \sqrt{\kappa} o_A, \quad \tilde{\lambda}_{\dot{A}} = \sqrt{\kappa} \bar{o}_{\dot{A}}. \quad (67)$$

The photon is thus represented by a null twistor sector spanned by the spinor dyad  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  aligned with the outgoing radial direction.

The on-shell time direction  $\eta^\mu$  can also be written in spinor form. In the tetrad  $(e_{(a)}^\mu)$  adapted to the static observers one finds

$$\eta^{A\dot{A}} = \frac{1}{\sqrt{2}} (o^A \bar{o}^{\dot{A}} + \iota^A \bar{\iota}^{\dot{A}}). \quad (68)$$

The local  $\eta$ -frequency of the photon is then

$$\omega_\eta = -\eta^{A\dot{A}}\lambda_A\tilde{\lambda}_{\dot{A}} = -\kappa \left[ \frac{1}{\sqrt{2}} o^A o_A \bar{o}^{\dot{A}} \bar{o}_{\dot{A}} + \frac{1}{\sqrt{2}} \iota^A o_A \bar{\iota}^{\dot{A}} \bar{o}_{\dot{A}} \right]. \quad (69)$$

Using the standard normalization  $o^A \iota_A = 1$  and  $o^A o_A = 0$ , this simplifies to

$$\omega_\eta = -\frac{\kappa}{\sqrt{2}} \iota^A o_A \bar{\iota}^{\dot{A}} \bar{o}_{\dot{A}} = -\frac{\kappa}{\sqrt{2}}. \quad (70)$$

The radial dependence of the measured frequency is entirely contained in the factor  $\kappa$ , which is fixed by matching to the conserved quantity  $E_\infty = -k_\mu \xi^\mu$  in (52). In this way the gravitational redshift can be seen as arising from the changing overlap between the fixed on-shell time direction  $\eta^{A\dot{A}}$  and the null twistor sector  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  as one compares static observers at different radii.

This example illustrates how the standard static picture of Schwarzschild can be reformulated in terms of an on-shell time direction and twistor sectors, with Killing symmetry appearing as the special case in which  $\eta^\mu$  is exactly a timelike Killing vector. In more general, non-static spacetimes the same spinor machinery applies, but  $\eta^\mu$  will no longer satisfy the Killing equation and the corresponding energies need not remain constant.

## 6 Discussion and outlook

We have revisited Killing vectors and curvature in general relativity from a two-spinor and twistor perspective,[3, 4, 1] and we have introduced an on-shell time direction  $\eta^\mu$  that encodes the locally preferred flow of proper time. In static spacetimes such as Schwarzschild,[14, 1, 2]  $\eta^\mu$  can be aligned with a timelike Killing vector, and the usual conserved energy at infinity  $E_\infty = -p_\mu \xi^\mu$  appears as a Noether charge associated with this symmetry. In non-stationary situations  $\eta^\mu$  still provides a distinguished local time direction, but it fails to satisfy the Killing equation; curvature and the kinematical quantities  $(\theta, \sigma_{\mu\nu})$  then measure the obstruction to extending  $\eta^\mu$  to a global time symmetry.[10, 11]

Throughout, null directions and photon states were described in terms of spinor pairs  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$ , with local energy measurements expressed as scalar contractions

$$E_{\text{loc}} = -\hbar u^{A\dot{A}} \lambda_A \tilde{\lambda}_{\dot{A}} = \hbar \omega. \quad (71)$$

Curvature acts on these spinor and twistor degrees of freedom via the spin connection, inducing holonomies that we have interpreted as *twistor misalignment*. [5, 6, 4] The effects traditionally attributed to curvature in the metric picture—such as gravitational redshift, lensing and polarization rotation—can thus be rephrased as statements about how bundles of null twistor sectors are reoriented relative to an on-shell time direction.[12, 7]

## Summary of holonomy effects on spinor dyads

For clarity it is useful to summarize the main types of holonomy acting on null spinor dyads and their geometric and physical interpretation. In the two-spinor language a null direction is represented by a dyad  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  with  $k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}$ . [3, 5] Parallel transport around a closed loop  $\gamma$  leads to an  $SL(2, \mathbb{C})$  transformation

$$\lambda_A \rightarrow M_A{}^B[\gamma] \lambda_B, \quad \tilde{\lambda}_{\dot{A}} \rightarrow \tilde{M}_{\dot{A}}{}^{\dot{B}}[\gamma] \tilde{\lambda}_{\dot{B}}, \quad (72)$$

determined by the curvature spinors. [3, 4] Different components of  $M$  and  $\tilde{M}$  correspond to different optical effects, which are conventionally encoded in the expansion, shear and twist of null congruences. [5, 6] Table 1 summarizes these correspondences at a qualitative level.

Table 1: Qualitative correspondence between holonomy on null spinor dyads  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$ , geometric quantities in a null congruence, and physical effects on light.

| Holonomy<br>$(\lambda_A, \tilde{\lambda}_{\dot{A}})$ | on | Geometric quantity                     | Physical manifestation  |
|--|----|--|---|
| Overall real boost of dyad                           |    | Expansion of congruence                | Change of beam cross section, flux dilution                   |
| Tracefree symmetric deformation of dyad              |    | Shear of congruence                    | Distortion of images, weak lensing                            |
| Complex phase rotations within dyad                  |    | Twist / rotation of polarization frame | Rotation of polarization angle (gravitational Faraday effect) |
| Nontrivial mixing with transverse spin frame         |    | Weyl curvature (Petrov type)           | Focusing / defocusing, formation of caustics                  |
| Trivial holonomy (identity)                          |    | Vanishing curvature along loop         | No cumulative optical effect                                  |

This table does not introduce new dynamics; it merely reorganizes standard results of the Newman–Penrose and optical scalar formalisms [5, 6, 12] in terms of the action of holonomy on spinor dyads. In particular, the entries are consistent with the Sachs equations for expansion and shear and with the usual interpretation of Weyl curvature as the part of the gravitational field responsible for lensing. [7, 2]

## Reversing the logical order

We stress that the “twistor misalignment” picture proposed here is interpretive: it reformulates standard holonomy of the spin connection and the relation  $E_{\text{loc}} = -\hbar u^\mu k_\mu$  in a spinor/twistor language. No new degrees of freedom or modified dynamics are introduced, and all statements remain within the framework of Einstein–Maxwell theory.

In the standard geometric formulation, the logical order is

$$g_{\mu\nu} \implies \xi^\mu \text{ (Killing)} \implies E_\infty = -p_\mu \xi^\mu \implies E_{\text{loc}} = -\hbar u^\mu k_\mu. \quad (73)$$

The metric and its symmetries are postulated first, and energies are derived from them.[1] The present viewpoint suggests that this logical order can be at least partially reversed. One may start instead from

- local on-shell time directions  $\eta^\mu$  associated with an observer congruence or inertial background,[10, 11]
- null twistor sectors  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  describing lightlike propagation,[3, 4]
- and the scalar overlaps  $u^{A\dot{A}} \lambda_A \tilde{\lambda}_{\dot{A}}$  that encode measurable frequencies,

and ask under what conditions these structures can be organized into a metric  $g_{\mu\nu}$  with an exact Killing symmetry. In this picture a timelike Killing vector is not fundamental input, but rather the special case in which the local time field  $\eta^\mu$  can be extended smoothly to a global symmetry direction, with curvature constrained accordingly.[2]

Lightlike directions play a special role in this inversion of the usual logic. Along null rays there is no proper time, and the standard parameter  $\lambda$  is only an affine label.[1] The phase of a massless field, as encoded in a spinor dyad  $(\lambda_A, \tilde{\lambda}_{\dot{A}})$  and an eikonal  $S(x)$ , becomes the primary time-related quantity, while the metric enters mainly through the null condition and the spin connection.[4] From a twistor viewpoint, this is natural: twistors were originally introduced to encode lightlike structure and conformal geometry, with spacetime points emerging only as secondary constructs.[8, 9]

### Limitations of the present framework

The analysis in this paper is restricted to the geometric-optics regime for massless fields, where wave covectors are null and slowly varying amplitudes propagate along null geodesics.[1, 7] In strong-lensing regions with caustics, the simple bundle picture of a smooth null congruence breaks down and diffraction effects become important; in such cases a full wave treatment is required. Likewise, we have not attempted to address quantum regimes beyond the semi-classical approximation or spacetimes with complicated matter content, where back-reaction and quantum fluctuations may alter both the geometry and the notion of an on-shell time direction. These limitations are standard for geometric optics and do not affect the purely kinematical spinor and twistor relations developed here, but they delimit the range of physical situations to which the present discussion can be directly applied.

### Outlook

In this paper we have treated the on-shell time direction  $\eta^\mu$  and the metric  $g_{\mu\nu}$  as given. A natural extension is to investigate whether  $\eta^\mu$  and, ultimately, the

metric itself can be obtained from an underlying spinor or twistor model in which null directions are primary and spacetime geometry is emergent.[9, 4] In such a setting null congruences and twistor correlations would be the fundamental data, and an effective Lorentzian metric with a distinguished time direction would arise only after coarse-graining over many microscopic configurations, in the spirit of emergent spacetime scenarios discussed in other approaches to quantum gravity.[16, 17]

Within the strictly classical framework used here, further work could proceed in several directions. On the one hand, it would be straightforward to extend our analysis to more general stationary spacetimes (such as Kerr) and to cosmological models, computing explicitly how  $\eta^\mu$  and the holonomy of spinor dyads encode frame dragging or cosmological redshift, and to include polarization transport as an additional observable test of the twistor misalignment picture.[7, 12] On the other hand, one could attempt to formulate the full Newman–Penrose field equations directly in terms of holonomy actions on spinor dyads and the associated scalar overlaps with  $\eta^{AA}$ , providing a more systematic bridge between the traditional optical scalars and the twistor-based interpretation advocated here.

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