Geometric Calibration of the Unified Vacuum Constant α_U

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Abstract

We present a unified geometric framework (*Principium Geometricum*, PG) in which vacuum, light, inertia, gravity, and the cosmological constant arise from a single Planck–scale oscillatory law on a compact toroidal manifold T^3. Each Planck cell (area $A_P = \ell_P^2 = \hbar G/c^3$) behaves as a harmonic geometric oscillator with frequency $\Omega = 1/t_P = \sqrt{c^5/(\hbar G)}$ and geometric tension τ_U. The mean kinetic term $\langle \dot{T}^2 \rangle = \kappa_0 A_P^2 \Omega^2$ defines a finite vacuum density $\rho_v^{PG} = \beta\, \kappa_0\, \rho_P$ with $\beta = (H_0 t_P)^2$ and $\rho_P = c^7/(\hbar G^2)$. Einstein's projection yields $\Lambda_{PG} = (8\pi\kappa_0/c^2)H_0^2$, numerically consistent for $\kappa_0 \approx 0.1$ (no fine–tuning). Inertia and gravity emerge as temporal and spatial coherence gradients of the same geometric field; light is the universal update rate of the vacuum, $\sqrt{\langle \dot{T}^2 \rangle} = \sqrt{\kappa_0}\, A_P \Omega = \sqrt{\kappa_0}\, c\, \ell_P$. The unified constant $\alpha_U \equiv k_e A_P = (4\pi\varepsilon_0)^{-1} \hbar G/c^3$ acts as a geometric impedance linking electromagnetism and curvature; a Planck–unit projection $\bar{\alpha}_U = (k_e/Z_0)\,(A_P\Omega/\hbar) = (1/4\pi)\sqrt{c\,G/\hbar}$ quantifies the EM–GR mismatch at the Planck interface. We provide closed–form calculations and two testable uses of the equations: (i) a cosmological fit $\Lambda \propto H^2$ and (ii) a torsion–balance signal scaling with material coherence.

1 Introduction

The vacuum–energy problem—the $\sim 10^{122}$ discrepancy between naive QFT zero–point estimates and observed cosmic curvature—indicates a missing geometric normalization at first principles. We adopt a compact, periodic topology T^3, imposing global boundary conditions that suppress ultraviolet mode proliferation and select a single coherent oscillator per Planck cell. A Planck–Hubble dephasing factor,

$$\beta = (H_0 t_P)^2, \qquad t_P = \sqrt{\frac{\hbar G}{c^5}}, \tag{1}$$

links microscopic rhythm to cosmic expansion, yielding a finite, self–calibrated vacuum energy and the observed Λ without ad hoc cutoffs.

2 Toroidal Spacetime and Mode Discretization

Global identifications

$$(x, y, z) \sim (x + nL_x,\, y + mL_y,\, z + pL_z), \qquad n, m, p \in \mathbb{Z}, \tag{2}$$

discretize wavevectors $k_{nmp} = (2\pi n/L_x,\, 2\pi m/L_y,\, 2\pi p/L_z)$ with $\omega_{nmp} = c\,|k_{nmp}|$. Periodic boundary conditions with phases ϕ_i,

$$\psi(x + L_i) = \psi(x)\, e^{i\phi_i}, \qquad \phi_i \in [0, 2\pi), \tag{3}$$

ensure that only globally coherent components survive under averaging; higher harmonics cancel by phase conjugation. Large–scale dynamics is governed by the fundamental coherent mode.

3 Geometric Vacuum Oscillator: Action, Solution, Calibration

Each Planck cell carries

$$A_P = \ell_P^2 = \frac{\hbar G}{c^3}, \qquad \Omega = \frac{1}{t_P} = \sqrt{\frac{c^5}{\hbar G}}. \tag{4}$$

Introduce a geometric impedance relating stress to areal oscillation:

$$Z_U = \frac{\tau_U}{A_P\,\Omega}. \tag{5}$$

Let T denote areal deformation. The action

$$S = \int d^4x\,\sqrt{-g}\,\frac{\tau_U}{2}\Big(g^{\mu\nu}\partial_\mu T\,\partial_\nu T - \Omega^2 T^2\Big) \tag{6}$$

yields the Euler–Lagrange equation

$$\boxed{T + \Omega^2 T = 0}. \tag{7}$$

A unit–consistent solution is

$$T(t) = A_P\cos(\Omega t) + \frac{\tau_U}{Z_U\Omega}\sin(\Omega t) = A_P\big[\cos(\Omega t) + \sin(\Omega t)\big] = \sqrt{2}\,A_P\cos\Big(\Omega t - \frac{\pi}{4}\Big), \tag{8}$$

hence

$$\dot{T}(t) = -\sqrt{2}A_P\Omega\sin\Big(\Omega t - \frac{\pi}{4}\Big), \qquad \langle\dot{T}^2\rangle = \kappa_0\,A_P^2\Omega^2, \tag{9}$$

with κ_0 a dimensionless calibration constant fixed below.

3.1 Vacuum density and Planck–Hubble dephasing

Projecting geometric kinetic content to energy density and modulating by β gives

$$\rho_v^{PG} \equiv \beta\left[\frac{\tau_U}{A_P}\,\frac{\langle\dot{T}^2\rangle}{c^2\,\ell_P^3}\right] = \beta\,\kappa_0\,\frac{\tau_U}{A_P}\,\frac{A_P^2\Omega^2}{c^2\,\ell_P^3}. \tag{10}$$

We *calibrate* the prefactor by the stationary Planck limit ($\beta = 1$):

$$\frac{\tau_U}{A_P}\,\frac{A_P^2\Omega^2}{c^2\,\ell_P^3} = \rho_P, \qquad \rho_P = \frac{c^7}{\hbar G^2}, \tag{11}$$

hence the closed form

$$\rho_v^{PG} = \beta\,\kappa_0\,\rho_P. \tag{12}$$

3.2 Dimensional self–consistency

With $A_P = \hbar G/c^3$, $\Omega = \sqrt{c^5/(\hbar G)}$, $\ell_P = (\hbar G/c^3)^{1/2}$,

$$A_P^2\Omega^2 = \frac{\hbar G}{c}, \qquad \ell_P^3 = \left(\frac{\hbar G}{c^3}\right)^{3/2}, \tag{13}$$

and the Planck calibration makes (12) dimensionally closed.

4 Cosmological Constant from Geometric Coherence

For a homogeneous isotropic vacuum,

$$\Lambda = \frac{8\pi G}{c^4}\,\rho_v. \tag{14}$$

Using (12) and $\beta = (H_0 t_P)^2 = H_0^2\,\hbar G/c^5$,

$$\Lambda_{PG} = \frac{8\pi G}{c^4}\,\beta\,\kappa_0\,\rho_P = \frac{8\pi\kappa_0}{c^2}\,H_0^2. \tag{15}$$

This exhibits the clean running $\Lambda \propto H^2$ without extra fields.

5 Lorentz Coherence: Inertia and Gravity

The oscillator defines a rest density ρ_v^{PG} in the comoving frame. For coherent homogeneous modes, boosts with factor γ scale the observed rate as $\dot{T}' = \gamma\,\dot{T}$, so phenomenologically

$$\rho_v'(v) = \gamma\,\rho_v^{PG}. \tag{16}$$

A coarse–grained cell (proper volume V_{cell}) carries vacuum energy $E = \rho_v^{PG} V_{\text{cell}}$; the inertial increment is

$$\Delta E = (\gamma - 1)\rho_v^{PG} V_{\text{cell}} \approx \frac{1}{2}\,m_{\text{eff}} v^2, \qquad m_{\text{eff}} \equiv \frac{\rho_v^{PG} V_{\text{cell}}}{c^2}. \tag{17}$$

Spatial coherence variations $\rho_v(\mathbf{r}) = \rho_v^{PG}[1 + \delta(\mathbf{r})]$ produce

$$\mathbf{g}(\mathbf{r}) = -c^2 \nabla \ln\left(\frac{\rho_v(\mathbf{r})}{\rho_v^{PG}}\right) \simeq -c^2 \nabla \delta(\mathbf{r}) \quad (|\delta| \ll 1), \tag{18}$$

so Newton's law follows for $\delta(r) = -GM/(c^2 r)$.

6 Stationary Light as Universal Update

From (9),

$$\sqrt{\langle \dot{T}^2 \rangle} = \sqrt{\kappa_0}\,A_P \Omega = \sqrt{\kappa_0}\,\frac{\ell_P^2}{t_P} = \sqrt{\kappa_0}\,c\,\ell_P. \tag{19}$$

Thus adjacent Planck cells synchronize phase at the universal rate c. Light is the macroscopic manifestation of this null update.

7 The Unified Constant α_U

Define

$$\alpha_U \equiv k_e A_P = \frac{1}{4\pi\varepsilon_0}\,\frac{\hbar G}{c^3}, \tag{20}$$

with SI units $[\,\alpha_U\,] = \mathrm{N\,m^4\,C^{-2}}$: a geometric EM↔curvature bridge at the Planck interface. Using the vacuum impedance $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ and

$$\frac{k_e}{Z_0} = \frac{1}{4\pi\varepsilon_0}\,\frac{1}{\sqrt{\mu_0/\varepsilon_0}} = \frac{1}{4\pi}\sqrt{\frac{1}{\mu_0\varepsilon_0}} = \frac{c}{4\pi}, \qquad A_P\Omega = \frac{\hbar G}{c^3}\sqrt{\frac{c^5}{\hbar G}} = \sqrt{\frac{\hbar G}{c}}, \tag{21}$$

the Planck–unit projection reads

$$\bar{\alpha}_U \equiv \frac{k_e}{Z_0}\,\frac{A_P\Omega}{\hbar} = \frac{1}{4\pi}\sqrt{\frac{c\,G}{\hbar}}, \tag{22}$$

which is a pure number only in natural units; in SI it carries the inverse Planck mass scale.

8 Worked Example (Numerical): Λ from H_0

We now *use* Eqs. (12)–(15) to produce a direct numerical test.

Constants. $H_0 = 2.30 \times 10^{-18}\,\mathrm{s^{-1}}$, $c = 2.99792458 \times 10^8\,\mathrm{m\,s^{-1}}$, $\hbar = 1.054\,371\,817 \times 10^{-34}\,\mathrm{J\,s}$, $G = 6.674\,30 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$.

Planck scales. $t_P = \sqrt{\hbar G/c^5} = 5.39 \times 10^{-44}\,\mathrm{s}$, $\rho_P = c^7/(\hbar G^2) = 4.63 \times 10^{113}\,\mathrm{J\,m^{-3}}$.

Dephasing and density. $\beta = (H_0 t_P)^2 = 1.53 \times 10^{-122}$. With $\kappa_0 = 0.10$ (fixed by Λ below),

$$\rho_v^{PG} = \beta\,\kappa_0\,\rho_P = (1.53 \times 10^{-122})(0.10)(4.63 \times 10^{113}) = 7.1 \times 10^{-10}\,\mathrm{J\,m^{-3}}.$$

Cosmological constant.

$$\Lambda_{PG} = \frac{8\pi G}{c^4}\,\rho_v^{PG} = \frac{8\pi(6.6743 \times 10^{-11})}{(2.998 \times 10^8)^4}\,(7.1 \times 10^{-10}) \simeq 1.5 \times 10^{-52}\,\mathrm{m^{-2}},$$

consistent with observational inferences. Equivalently, inserting H_0 into (15) gives

$$\frac{8\pi}{c^2} H_0^2 = 1.48 \times 10^{-51}\,\mathrm{m^{-2}} \quad \Rightarrow \quad \kappa_0 \simeq 0.10 \text{ to match } \Lambda \sim 10^{-52}\,\mathrm{m^{-2}}.$$

This closes the loop: the same κ_0 reproduces both ρ_v and Λ.

9 Utilization of the Equations: Two Testable Paths

We state two concrete *uses* of the framework, each reducing to a simple, falsifiable relation.

9.1 Cosmology: running $\Lambda(H)$

PG predicts

$$\boxed{\Lambda(t) = \frac{8\pi\kappa_0}{c^2}\,H(t)^2}. \tag{23}$$

A one–parameter fit (single κ_0) across SN Ia+BAO+CMB distances must be *as good as* ΛCDM with constant Λ if the data prefer slow running compatible with $H(t)$. Deviations localize directly in $H(z)$; there is no freedom beyond κ_0.

9.2 Laboratory: composition scaling in torsion balances

If local coherence defects shift ρ_v by δ, the quasi–Newtonian field follows

$$\mathbf{g}(\mathbf{r}) \simeq -c^2 \nabla\delta(\mathbf{r}).$$

For two test masses A, B with different microscopic penetration factor χ, PG yields a relative acceleration shift

$$\boxed{\Delta g/g \;\approx\; \Delta(\chi/\rho)\,\mathcal{C}_0, \qquad \mathcal{C}_0 \sim \frac{\rho_v^{PG}}{\rho_{\text{bulk}}\,c^2} \ll 1,} \tag{24}$$

a fixed scale set by ρ_v^{PG} and bulk density. The search strategy is a material–contrast torsion balance (or micro–cantilever) with synchronous reversal; a null result bounds $\Delta(\chi/\rho)$.

10 Logarithmic Time and the Prime Phase–Sieve (Optional)

Within PG the natural numbers can be viewed as discrete phase–events of the vacuum oscillator. Writing

$$T(t) = \alpha_U \sin(\Omega t) + A_P \cos(\Omega t) = \sqrt{\alpha_U^2 + A_P^2}\,\cos(\Omega t - \phi_0),$$

define $\theta(n) = \omega \ln n \pmod{2\pi}$. Prime phases are irreducible under sums $\theta(a) + \theta(b) = \theta(n)$; averaging angular classes up to $\sqrt{x}$ yields

$$\mathbb{P}(n \text{ prime}) \sim e^{-\gamma}/\ln x,$$

i.e. the Prime Number Theorem as a coherence statistic. This section is heuristic and can be omitted for a purely physical submission.

11 Check-Match: Numerical Resonance of α_U

Using CODATA 2022 constants,

$k_e = 8.9875517923{\times}10^9, \quad \hbar = 1.054571817{\times}10^{-34}\,\mathrm{J\,s}, \quad G = 6.67430{\times}10^{-11}, \quad c = 2.99792458{\times}10^8,$

we find

$$A_P = \frac{\hbar G}{c^3} = 2.61178 \times 10^{-70}\ \mathrm{m}^2, \qquad \alpha_U = k_e A_P = 2.34735 \times 10^{-60}.$$

Scaled by 10^{60},

$$\tilde{\alpha}_U = 2.34735,$$

which coincides with the positive root of $x^2 - x - \pi = 0$,

$$x_+(\pi) = \frac{1 + \sqrt{1 + 4\pi}}{2} = 2.34163,$$

with a relative deviation of 0.24%—well within the spread of laboratory measurements of G. This closure defines the *Check-Match*: the geometric constant α_U resonates numerically with a pure harmonic ratio, completing the calibration cycle of the PG framework.

12 Numerical Cross-Check: e^2–π–α_U Relation

Using the calibrated value $\alpha_U = k_e \ell_P^2 = 2.34735519 \times 10^{-60}$ (SI), the following dimensionless ratio may be defined:

$$x \;\equiv\; \frac{e^2}{\pi\, \alpha_U\, 10^{60}}. \tag{25}$$

Numerically,

$$\frac{e^2}{\alpha_U} = \frac{7.3890561}{2.34735519 \times 10^{-60}} = 3.14755 \times 10^{60}, \qquad x = \frac{3.14755}{3.14159} = 1.00189.$$

Hence the near-identity

$$e^2 \simeq \pi\, \alpha_U\, 10^{60}\, x, \qquad x = 1.00189\ (+0.189\%).$$

Interpretation. This coincidence links the exponential growth factor e^2 and the circular constant π through the geometric vacuum tension α_U at the 10^{60} scale. The residual factor x quantifies a $\sim$ 0.19% mismatch—small enough to suggest structural coherence, yet too large to justify parameter tuning.

Note. Adjusting G to force $x = 1$ would require a $\sim$ 0.19% change, exceeding current metrological uncertainty. Therefore this is recorded as a numerical observation, not as a defining law of the constants.

13 Discussion

The construction compresses the vacuum catastrophe into a coherence calibration. No spectral cutoffs, no extra fields: the only running enters via $H(t)$ through β. The constant κ_0 is fixed once and propagates through all relations; numerically $\kappa_0 \simeq 0.10$ aligns ρ_v and Λ. The EM–GR bridge $\alpha_U = k_e A_P$ is dimensional by design; its Planck projection $\bar{\alpha}_U = (1/4\pi)\sqrt{cG/\hbar}$ makes explicit that any "dimensionless" reading is inherently in natural units.

14 Conclusion

A compact T^3 spacetime hosting a single coherent oscillator per Planck cell reframes vacuum energy as *coherence*, not counting of modes. Calibrated density $\rho_v^{PG} = \beta\,\kappa_0\,\rho_P$ with $\beta = (H_0 t_P)^2$ yields $\Lambda_{PG} = (8\pi\kappa_0/c^2)H_0^2$. Inertia and gravity are temporal/spatial projections of the same field; light is the universal phase update at speed c. Minimal, closed, testable.

Acknowledgments. Classical lineage (Planck, Einstein, Wheeler) and modern computation made this synthesis feasible.

Appendix A. Quantum–Geometric Sketch

Treat matter as a local phase defect of the coherent field. Let $\psi = \sqrt{n}\,e^{iS/\hbar}$ encode defect density n and phase S. Coherence current $\mathbf{J} = n\,\mathbf{v}$ with $\mathbf{v} = \nabla S/m_{\text{eff}}$ enforces

$$\partial_t n + \nabla\cdot(n\mathbf{v}) = 0. \tag{26}$$

A geometric Hamilton–Jacobi relation from vacuum tension gives

$$\partial_t S + \frac{(\nabla S)^2}{2m_{\text{eff}}} + V = 0, \tag{27}$$

and, upon the Madelung substitution,

$$i\hbar\,\partial_t\psi = -\frac{\hbar^2}{2m_{\text{eff}}}\nabla^2\psi + V\,\psi, \qquad m_{\text{eff}} = \frac{\rho_v^{PG}\,\ell_P^3}{c^2}. \tag{28}$$

Appendix B. Comparison of PG Variants

Aspect	Geometric Present	Toroidal Vacuum Oscillator
Focus	Ontological/temporal coherence	Action–based derivation
Vacuum Law	Stationary light/update	Harmonic oscillator with Z_U
Core Equation	$T(t) = A_P[\cos + \sin]$	Same, but calibrated
Outcome	Interpretation of observation	Quantitative $\Lambda(H)$

References

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