A Critical Examination of the Infinite Decomposition Approach to Cantors Continuum Hypothesis:
Chang Hee Kims *"Cantors Continuum Hypothesis Is Proved Wrong"* in the Context of ZFC Set Theory

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Abstract

This paper presents a critical review of Chang Hee Kims 2025 preprint, *Cantors Continuum Hypothesis Is Proved Wrong* (ViXra:2505.0211v1). The work proposes an alternative approach to infinite cardinality by decomposing the set of natural numbers into infinitely many symmetric subsets. We examine the logical structure of Kims argument, the role of infinite decomposition, and its implications for Cantors diagonal method and the Continuum Hypothesis (CH). In particular, we demonstrate that Kims framework operates outside the standard ZermeloFraenkel set theory with the Axiom of Choice (ZFC), since it redefines core notions such as bijection, power set, and cardinality.

1 Introduction

Cantors diagonal argument and the Continuum Hypothesis (CH) constitute central results in modern set theory. Within the ZermeloFraenkel axiomatic framework (ZFC), the power set axiom guarantees that for any set A, the cardinality of its power set $P(A)$ strictly exceeds that of A:

$$|P(A)| > |A|.$$

From this, it follows that the set of real numbers $\mathbb{R}$ is uncountable and that $\mathbb{R}$ cannot be placed in one-to-one correspondence with $\mathbb{N}$. The Continuum Hypothesis (CH) concerns whether there exists a set X such that $|\mathbb{N}| < |X| < |\mathbb{R}|$. Gdel and Cohen showed that CH is independent of ZFCit can be neither proved nor disproved from its axioms.

Kims paper, however, rejects this independence and asserts that CH is *false*. He constructs what he calls an infinite decomposition of the natural numbers and claims that all infinite sets, including $\mathbb{R}$, are equinumerous with $\mathbb{N}$. The author also argues that Cantors diagonal argument contains a self-contradiction, arising from what he calls a non-symmetric matrix representation.

2 Summary of Kims Argument

2.1 Infinite Decomposition of $\mathbb{N}$

The central device of Kims paper is the definition of an infinite partition of $\mathbb{N}$ into disjoint arithmetic progressions:

$$S_{p,k} = \{b^p n + k \mid n \in \mathbb{N}\}, \quad \text{for integers } b > 1,\ p, k \geq 0.$$

Each $S_{p,k}$ is infinite, and the family $\{S_{p,k}\}$ forms a countably infinite collection of disjoint infinite subsets whose union is $\mathbb{N}$ itself. Kim refers to this as a *symmetric infinite matrix* representation, suggesting that rows and columns share the same infinite cardinality.

2.2 Critique of Cantors Diagonal Argument

Kim reinterprets Cantors diagonal array of real numbers as an asymmetric construction: the rows represent enumerated sequences, whereas the diagonal defines a new number external to that enumeration. He argues that this asymmetry arises from an incomplete enumeration, and that in a perfectly symmetric (infinitely decomposed) matrix, the diagonal element would already be contained within the set. Hence, no truly new number is produced, and the diagonal argument collapses.

2.3 Consequences for Cardinality

From this reformulation, Kim concludes that every infinite set can be symmetrically decomposed and mapped onto $\mathbb{N}$. Therefore, the classical hierarchy of infinite cardinalities $(\aleph_0, \aleph_1, \ldots)$ is dismissed as meaningless. In his view, all infinite sets are mutually equinumerous, and the continuum hypothesis is disproved.

3 Analysis within the ZFC Framework

3.1 Core Axioms of ZFC

The ZFC axioms relevant to this discussion include:

- The **Axiom of Infinity**, asserting the existence of $\mathbb{N}$.

- The **Power Set Axiom**, ensuring that $P(A)$ exists for all A and that $|P(A)| > |A|$.

- The **Axiom Schema of Replacement**, permitting the construction of new sets via definable functions.

- The definition of **cardinality** via bijective functions: $|A| = |B|$ if and only if there exists a bijection $f : A \to B$.

3.2 Conflict with ZFC Principles

Kims model conflicts with ZFC on several levels:

1. **Denial of the Power Set Axiom:** His claim that $|\mathbb{R}| = |\mathbb{N}|$ implies that $|P(\mathbb{N})| = |\mathbb{N}|$, which directly contradicts Cantors theorem and the power set axiom.

2. **Redefinition of Bijection:** Instead of a functional bijection, he interprets one-to-one correspondence as structural symmetry within an infinite decomposition. This notion lacks the function-based rigor required by ZFC.

3. **Rejection of Independence:** In ZFC, CH is independent. Claiming to disprove it outright means stepping outside the formal system in which that independence was proved.

4. **Introduction of a Non-Axiomatic Operator:** The infinite decomposition rule defines new sets without appeal to any ZFC comprehension axiom, effectively constituting an extension (or replacement) of the theory.

3.3 Philosophical Implication

By replacing Cantors asymmetry with a symmetric infinite structure, Kims proposal becomes a philosophical reinterpretation rather than a formal mathematical proof. It reflects a computational or algebraic intuition about infinity but does not conform to the axiomatic rigor of ZFC.

4 Discussion: Why It Lies Outside ZFC

To disprove CH within ZFC, one would need to derive a contradiction from the ZFC axioms themselves. Since Gdel (1940) and Cohen (1963) independently showed that ZFC + CH and ZFC + CH are both consistent (assuming ZFC is consistent), any proof that CH is false must either:

- adopt non-ZFC axioms (i.e., work in a different logical system), or

- redefine cardinality and bijection, thereby leaving the ZFC framework.

Kims infinite decomposition approach does both. It replaces functional bijections with combinatorial symmetry and abandons the power set principle. Hence, it constitutes a *non-axiomatic framework* a system of reasoning that cannot be formalized within standard set theory.

5 Conclusion

Chang Hee Kims paper provides an intriguing re-imagination of Cantors diagonal argument. Its central contribution lies in proposing a *symmetric reinterpretation* of infinite

structures through decomposition of $\mathbb{N}$. However, from the standpoint of formal logic, the framework diverges fundamentally from ZFC. The papers claim that all infinite sets are equinumerous amounts to rejecting the power set axiom and the classical definition of cardinality. Thus, rather than a proof against the Continuum Hypothesis, it should be understood as a philosophical or computational alternative to ZFC-based reasoning about infinity.

References

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