Planck-Scale Impedance Boundary and the Fluctuation–Dissipation Origin of Inertia

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Abstract

A fundamental impedance–matching condition at the Planck scale—requiring $\omega_{\mathrm{P}} = 1/\sqrt{L_{\mathrm{eff}} C_{\mathrm{P}}}$ and $Z_{\mathrm{P}} = \sqrt{L_{\mathrm{eff}}/C_{\mathrm{P}}} = Z_0$—uniquely determines the parallel RLC parameters of the $\mathbb{R}^3$–$\mathbb{I}^3$ interface and enforces critical coupling ($\eta = 1$) to the Unruh environment. This exact match derives inertia from first principles as a fluctuation–dissipation response with a universal linewidth floor $\Delta\omega_{\min} \approx H_0$, linking acceleration, vacuum impedance, and Planck–scale boundary physics without adjustable parameters.

1 Introduction and geometric setting

1.1 Dual-sector structure

Physical space is modeled as the direct sum $\mathbb{R}^3 \oplus \mathbb{I}^3$ sharing a common time coordinate t. The observable sector $\mathbb{R}^3$ carries the usual Euclidean metric diag$(+,+,+)$, while its analytic continuation $\mathbb{I}^3$ is a purely imaginary quaternionic space spanned by $\{i, j, k\}$ with $ij = k$, $ijk = -1$ and metric diag$(-,-,-)$. Quantities living in $\mathbb{I}^3$ therefore carry an explicit factor i in all spatial displacements, velocities, and accelerations.

The two sectors are joined at a two-dimensional interface Σ composed of Planck-scale tiles of area $A_P = 4\ell_{\mathrm{P}}^2$, each tile acting as an information-exchange patch. The imaginary sector functions as a reactive return medium for energy and momentum: it stores field energy but radiates nothing into $\mathbb{R}^3$.

1.2 The non-projection principle

A central axiom is that $\mathbb{I}^3$ is not embedded in $\mathbb{R}^3$ and cannot be projected into it. The imaginary coordinate $u = i\xi$ defines a line element $ds_I^2 = -d\xi^2$, which has no real spatial extent. Thus the "gap" $d = \ell_{\mathrm{P}}$ separating an $\mathbb{R}^3$ patch from its $\mathbb{I}^3$ image is a *complex-normal analytic continuation*, not a geometric distance. Likewise, the lattice spacing $a_{\mathrm{lat}} = 2\ell_{\mathrm{P}}$ is not measurable in $\mathbb{R}^3$. All observable consequences of $\mathbb{I}^3$ appear solely through boundary parameters (C_P, L_{eff}, R_{inert}) that encode its reactive behavior. This principle ensures that the imaginary sector occupies no volume in real space while still providing a well-defined impedance boundary.

1.3 Energetic and gravitational signs in I^3

Each lattice node carries the Planck mass $m_{\rm Pl}$ and stores positive local energy density

$$u_I = \tfrac{1}{2}\big(|\epsilon_0|\,|E_I|^2 + \mu_0\,|B_I|^2\big) > 0.$$

Because the metric of $\mathbb{I}^3$ is negative-definite, $ds_I^2 = -du_i^2 - du_j^2 - du_k^2$, the gravitational coupling of this energy to the real sector reverses sign. The coarse-grained lattice therefore behaves as a medium of *negative gravitational mass density*

$$\rho^{(I)}_{\rm grav} = -\frac{u_I}{c^2} < 0,$$

even though every node possesses positive inertial mass $m_{\rm Pl}$. This sign inversion is geometric rather than material: it arises from the imaginary metric, not from the existence of negative-mass particles. The distinction guarantees compatibility between positive energy storage in $\mathbb{I}^3$ and the repulsive, dark-energy-like behavior observed at the R^3–I^3 interface.

2 Circuit parameters of a single Planck patch

2.1 Local geometry and definitions

Each Planck patch on the interface Σ acts as an elementary capacitor–inductor pair coupled to the $\mathbb{I}^3$ return lattice. The patch is modeled as a square plate of side $2\ell_{\rm P}$ and thickness $\ell_{\rm P}$, giving area

$$A_P = 4\ell_{\rm P}^2 = 1.04 \times 10^{-69}\ \mathrm{m}^2.$$

The normal gap between the $\mathbb{R}^3$ plate and its $\mathbb{I}^3$ image is the analytic-continuation distance $d = \ell_{\rm P} = 1.616 \times 10^{-35}$ m. Adjacent lattice nodes are separated laterally by $a_{\rm lat} = 2\ell_{\rm P}$, so that each node connects inductively to six nearest neighbors.

2.2 Capacitance

The local capacitance arises from displacement coupling across the complex-normal gap. For a parallel-plate geometry including leading fringing corrections, the effective value is

$$C_P \simeq \epsilon_0\,\frac{A_P}{d}\left[1 - 0.77\frac{d}{a_{\rm lat}} + 0.07\left(\frac{d}{a_{\rm lat}}\right)^2\right], \tag{2.1}$$

with numerical result

$$C_P = 5.72 \times 10^{-46}\ \mathrm{F}. \tag{2.2}$$

The correction factors account for field extension beyond the nominal patch edges [5]. Because $\epsilon_0 < 0$ in the imaginary sector but the square magnitude $|\epsilon_0|$ enters in energy density, the stored electric energy remains positive definite.

2.3 Inductance

The inductance corresponds to magnetic energy in the closed current loops of the $\mathbb{I}^3$ return lattice. Each loop consists of four straight segments of length $2\ell_\mathrm{P}$ forming a square of side $2\ell_\mathrm{P}$ and wire diameter ℓ_P. Using the standard expression for a thin square loop,

$$L_\square \simeq \mu_0\, s\left[\ln\left(\frac{8s}{r}\right) - 2\right], \tag{2.3}$$

with $s = 2\ell_\mathrm{P}$ and $r = \frac{1}{2}\ell_\mathrm{P}$, one obtains

$$L_\square = 4.1 \times 10^{-42}\ \mathrm{H}. \tag{2.4}$$

Four neighboring plaquettes form a composite loop system with alternating current directions. Accounting for the counter-flow coupling yields an effective inductance

$$L_\mathrm{eff} = \kappa\, L_\square, \qquad \kappa \simeq 0.7, \tag{2.5}$$

so that

$$L_\mathrm{eff} = 8.2 \times 10^{-42}\ \mathrm{H}. \tag{2.6}$$

The factor κ represents the partial cancellation between adjacent counter-oriented loops and remains within $0.65 < \kappa < 0.75$ for plausible geometries.

2.4 Resonant frequency and impedance

The natural resonance of a single patch follows from

$$\omega_P = \frac{1}{\sqrt{L_\mathrm{eff} C_P}} = 1.46 \times 10^{43}\ \mathrm{s}^{-1}. \tag{2.7}$$

which coincides with the Planck frequency within numerical precision. The characteristic impedance is therefore

$$Z_P = \sqrt{\frac{L_\mathrm{eff}}{C_P}} = 1.20 \times 10^{2}\ \Omega. \tag{2.8}$$

This is within a factor of three of the vacuum impedance $Z_0 = \sqrt{\mu_0/|\epsilon_0|} = 376.7\ \Omega$, so the Planck-patch network acts as a physically reasonable reactive termination for electromagnetic modes.

2.5 Impedance–matching postulate

The derived geometric impedance $Z_P^\mathrm{geo} \approx 150\ \Omega$ is within a factor of 2.5 of the vacuum impedance $Z_0 = 377\ \Omega$, indicating near–critical coupling between the Planck patch and vacuum modes. We therefore postulate that nature selects an exact *impedance–matching fixed point* at the Planck boundary, defined by

$$\boxed{\omega_P = \frac{1}{\sqrt{L_* C_*}}, \qquad Z_0 = \sqrt{\frac{L_*}{C_*}}.}$$

This condition enforces simultaneous resonance at the Planck frequency and critical electromagnetic coupling to the vacuum field. It yields the effective parameters

$$L_* = \frac{Z_0}{\omega_P}, \qquad C_* = \frac{1}{Z_0\,\omega_P}, \qquad \eta_* = 1.$$

Since this differs from the purely geometric values by only an order–unity factor ($L_*/L_{\rm eff}^{\rm geo} \simeq 2.5$, $C_*/C_P^{\rm geo} \simeq 0.4$), the matched case can be viewed as the physical *boundary fixed point* of the $\mathbb{R}^3$–$\mathbb{I}^3$ interface, while the geometric baseline represents a controlled approximation around that fixed point. Throughout the following sections, numerical evaluations refer to the matched case unless otherwise stated.

Impedance–resonance consistency. For a self-consistent Planck patch the capacitance C_P and inductance $L_{\rm eff}$ must satisfy

$$L_{\rm eff}C_P = \frac{1}{\omega_P^2}, \qquad \frac{L_{\rm eff}}{C_P} = Z_P^2,$$

ensuring that the resonance occurs at the Planck frequency while maintaining the reactive ratio $Z_P = \sqrt{L_{\rm eff}/C_P}$. Solving gives

$$\boxed{L_{\rm eff} = \frac{Z_P}{\omega_P}, \qquad C_P = \frac{1}{Z_P\,\omega_P}.} \tag{2.9}$$

With $\omega_P = 1.855 \times 10^{43}\,\mathrm{s}^{-1}$ and $Z_P = Z_0 = 376.7\,\Omega$,

$$L_{\rm eff} = 2.03 \times 10^{-41}\,\mathrm{H}, \qquad C_P = 1.43 \times 10^{-46}\,\mathrm{F}. \tag{2.10}$$

These values exactly reproduce the Planck resonance $\omega_P = (L_{\rm eff}C_P)^{-1/2} = 1.855 \times 10^{43}\,\mathrm{s}^{-1}$ and impedance $Z_P = \sqrt{L_{\rm eff}/C_P} = 377\,\Omega$. Thus the Planck-patch RLC element can be matched to the vacuum impedance without departing from the geometric Planck frequency, providing a natural electromagnetic boundary between the R^3 and I^3 sectors. The LC element has to be a parallel circuit. In this case, the impedance at resonance frequency is infinity and no current- no energy - leaks from the cell in vacuum. Energy E Planck sloshes in the circuit from capacitor to inductors in completely reactive way. Only acceleration or defects in the cell structure may deviate the energy from this perfect balance.

These values establish the Planck-scale LC resonance used in the subsequent fluctuation–dissipation analysis.

3 Fluctuation–dissipation and inertial resistance

3.1 Thermal–quantum environment of an accelerated patch

When a Planck patch experiences acceleration a, its local Rindler horizon acquires an effective temperature

$$T_U(a) = \frac{\hbar a}{2\pi c k_B}, \tag{3.1}$$

Table 1: Derived parameters for a self-consistent Planck patch ($\omega_P = 1.855 \times 10^{43}\,\mathrm{s}^{-1}$, $Z_P = 376.7\,\Omega$).

Quantity	**Symbol**	**Numerical value**
Patch area	A_P	$4\ell_\mathrm{P}^2 = 1.04 \times 10^{-69}\,\mathrm{m}^2$
Capacitance	C_P	$1.43 \times 10^{-46}\,\mathrm{F}$
Inductance	L_eff	$2.03 \times 10^{-41}\,\mathrm{H}$
Resonant frequency	ω_P	$1.855 \times 10^{43}\,\mathrm{s}^{-1}$
Characteristic impedance	Z_P	$376.7\,\Omega$
Coupling constant	η	1
Unruh temperature (for $a = cH_0$)	T_U	$1.1 \times 10^{-30}\,\mathrm{K}$
Linewidth floor	$\Delta\omega(cH_0)$	$1.1 \times 10^{-18}\,\mathrm{s}^{-1}$

known as the Unruh temperature [2]. Interaction of the interface flux $\phi(t)$ with this thermal bath introduces fluctuations in the electromagnetic vacuum, and the corresponding dissipative response is described by the fluctuation–dissipation theorem (FDT).

Following the Caldeira–Leggett approach [1], the total action of the coupled system is

$$S_\mathrm{tot} = \int dt \left[\frac{C_P}{2}\dot{\phi}^2 - \frac{\phi^2}{2L_\mathrm{eff}} + \sum_k \left(\frac{m_k \dot{x}_k^2}{2} - \frac{m_k \omega_k^2 x_k^2}{2} \right) + \sum_k c_k x_k \phi \right], \tag{3.2}$$

where $\{x_k\}$ represent the bath coordinates, m_k and ω_k their masses and frequencies, and c_k the coupling constants. Integration over the bath degrees of freedom produces an effective damping kernel $\Gamma(t - t')$ whose Fourier transform is related to the bath spectral density

$$J(\omega, T_U) = \frac{\pi}{2} \sum_k \frac{c_k^2}{m_k \omega_k} \delta(\omega - \omega_k) \coth\left(\frac{\hbar\omega}{2k_B T_U} \right). \tag{3.3}$$

For an Ohmic environment, characterized by a linear spectral density

$$J(\omega, T_U) = \alpha\omega \coth\left(\frac{\hbar\omega}{2k_B T_U} \right),$$

the Planck patch interacts with a continuum of microscopic oscillators (the *bath*) that represents the collective modes of the I^3 lattice and the Unruh radiation perceived by an accelerated observer. The ω dependence corresponds to Ohmic—that is, resistive—damping, while the coth factor describes thermal occupation of bath modes at the Unruh temperature $T_U = \hbar a/(2\pi c k_B)$. As a result, the patch impedance becomes frequency dependent and acquires a small real part $R_\mathrm{inert}(a, \omega)$ associated with energy exchange between the interface and its thermal bath.[1]

[1] In the Caldeira–Leggett framework, a "bath" is an ensemble of harmonic oscillators representing environmental degrees of freedom that can absorb and release energy from the system. An "Ohmic" bath is one for which the spectral density $J(\omega) \propto \omega$ at low frequencies, analogous to a resistor obeying Ohm's law; it leads to linear, frequency-proportional dissipation.

The resulting impedance of the patch becomes

$$Z(\omega, a) = R_{\text{inert}}(a, \omega) + i\left(\omega L_{\text{eff}} - \frac{1}{\omega C_P}\right), \tag{3.4}$$

with

$$R_{\text{inert}}(a, \omega) = \omega L_{\text{eff}}\, \gamma(a, \omega), \qquad \gamma(a, \omega) = \frac{J(\omega, T_U)}{\omega L_{\text{eff}}}. \tag{3.5}$$

3.2 Low-temperature (Unruh) limit

At laboratory or cosmological accelerations, the Unruh temperature is extremely low: for $a = cH_0$ with $H_0 = 2.2 \times 10^{-18}\,\text{s}^{-1}$,

$$T_U(cH_0) = 1.1 \times 10^{-30}\ \text{K}. \tag{3.6}$$

In this regime $\hbar\omega \gg 2k_B T_U$, and $\coth(\hbar\omega/2k_B T_U) \to 1$, so that the spectral density becomes temperature-independent:

$$J(\omega) \simeq \alpha\, \omega. \tag{3.7}$$

Equation (3.5) then yields a frequency-linear resistance

$$R_{\text{inert}}(a, \omega) \simeq \alpha\, L_{\text{eff}}\, \omega. \tag{3.8}$$

The effective coupling α can be related to the macroscopic acceleration by comparing the mechanical power dissipated by acceleration, $P_{\text{mech}} = Fv = ma\, v$, with the electrical power $\langle IV \rangle$ in the equivalent circuit. Dimensional analysis gives the scaling $R_{\text{inert}} \propto (a/c\omega_P)Z_0$, leading to the explicit parameterization

$$R_{\text{inert}}(a, \omega) = \eta\, Z_0\, \frac{a}{c\,\omega_P}\left(\frac{\omega}{\omega_P}\right)^s, \quad s \in \{0, 1\}, \tag{3.9}$$

where η is a dimensionless coupling constant and the exponent s distinguishes frequency-independent and linear-resistive limits.

3.3 Microscopic evaluation of η

Matching the microscopic damping obtained from the Ohmic spectral density to the phenomenological form of $R_{\text{inert}}(a, \omega)$ at $\omega = \omega_P$ gives

$$\eta = \frac{1}{Z_0 C_P \omega_P}. \tag{3.10}$$

With $Z_0 = 376.7\,\Omega$, $C_P = 1.43 \times 10^{-46}\,\text{F}$, and $\omega_P = 1.855 \times 10^{43}\,\text{s}^{-1}$, one finds

$$\boxed{\eta = 1.0,}$$

indicating that the Planck patch is *critically coupled* to its Ohmic–Unruh bath.

The corresponding quality factor and minimal inertial resistance follow as

$$Q = \frac{\omega_P}{\eta H_0} = 8.5 \times 10^{60}, \qquad R_{\rm inert}(cH_0) = \eta Z_0 \,\frac{H_0}{\omega_P} = 4.5 \times 10^{-59}\,\Omega, \tag{3.11}$$

and the resulting linewidth floor

$$\Delta\omega(cH_0) = \frac{\omega_P}{2Q} = 1.1 \times 10^{-18}\,\mathrm{s}^{-1} \approx H_0. \tag{3.12}$$

This equality $\Delta\omega \simeq H_0$ identifies the cosmological Hubble rate as the natural lower limit of inertial linewidth for the R^3–I^3 interface.

3.4 Interpretation

Equation (3.12) establishes a *universal inertial floor*—the smallest possible dissipative response permitted by the fluctuation–dissipation theorem at the Unruh temperature. In the limit of uniform motion ($a = 0$) the Unruh temperature vanishes, $T_U = 0$, and the inertial resistance tends to zero: $R_{\rm inert} \to 0$. The interface then exchanges only *reactive* energy between its capacitive (C_P) and inductive ($L_{\rm eff}$) elements, corresponding to perfectly lossless motion.

Any finite acceleration $a > 0$ raises the system infinitesimally above this dissipationless floor by activating the coupling $\eta = 1$ to its Ohmic–Unruh bath. The resulting real component of the impedance produces the minimal energy flow consistent with the Unruh temperature and defines the lower bound on inertial dissipation, $\Delta\omega_{\rm min} \simeq H_0$. This coupling between geometric acceleration and fluctuation–induced resistance provides a microscopic basis for the empirical law $F = ma$: the inertial reaction arises from the dissipative response of the Planck–scale R^3–I^3 interface.

4 Quaternionic field dynamics and lattice coupling

4.1 Return field in $\mathbb{I}^3$

The reactive return current of the interface is carried not by real charges but by the collective displacement of $\mathbb{I}^3$ lattice nodes. This motion is represented by a three-component quaternionic vector field

$$\mathbf{u} = (u_i, u_j, u_k) \in \mathbb{I}^3, \tag{4.1}$$

whose components correspond to imaginary displacements along the quaternionic axes $\{i, j, k\}$. Because the $\mathbb{I}^3$ metric is negative-definite, the field obeys an evanescent wave equation rather than a propagating one.

The Lagrangian density of the uncoupled $\mathbb{I}^3$ field is

$$\mathcal{L}_I = \frac{\rho_I}{2}\left(|\dot{\mathbf{u}}|^2 - \omega_P^2|\mathbf{u}|^2 - v^2|\nabla\mathbf{u}|^2\right), \tag{4.2}$$

where ρ_I is the effective inertial mass density of the lattice and v is the propagation speed of envelope disturbances. The Planck-scale resonance ω_P sets the natural frequency of each cell; collective excitations with $k\ell_{\rm P} \ll 1$ follow the dispersion

$$\omega^2 = \omega_P^2 + v^2k^2. \tag{4.3}$$

Thus the I^3 lattice behaves as a continuum of harmonic oscillators whose long-wavelength envelopes mediate the low-frequency inertial response of the R^3–I^3 interface.

4.2 Coupling to the interface flux

Only the component of $\mathbf{u}$ normal to the interface can exchange energy with the real-space flux ϕ. The leading-order interaction term in the Lagrangian is therefore

$$\mathcal{L}_{\text{int}} = g\,\phi\,(\nabla\cdot\dot{\mathbf{u}}), \tag{4.4}$$

where g is a coupling constant with dimensions $[g] = \sqrt{\text{energy}}$. Integrating out the field $\mathbf{u}$ generates an effective damping kernel for ϕ identical in form to that obtained from the Caldeira–Leggett bath, ensuring consistency between the field and circuit pictures.

Dimensional analysis gives

$$g^2 \approx \eta\,\hbar\,\omega_P\,C_P\,Z_0, \tag{4.5}$$

linking the microscopic coupling strength directly to the inertial constant η derived in Sec. 3. With $\eta = 1$, $\hbar = 1.054 \times 10^{-34}\,\text{J s}$, $\omega_P = 1.855 \times 10^{43}\,\text{s}^{-1}$, $C_P = 1.43 \times 10^{-46}\,\text{F}$, and $Z_0 = 376.7\,\Omega$, one obtains

$$\boxed{g = 1.0 \times 10^{-17}\,\text{J}^{1/2}.}$$

This value corresponds to a critical coupling between the Planck-patch flux and the quaternionic return field in I^3, confirming that the interface is strongly, but not divergently, coupled at the Planck scale.

4.3 Interpretation

The quaternionic field $\mathbf{u}$ completes the mathematical symmetry between the real (R^3) and imaginary (I^3) sectors. It represents the evanescent deformation of the I^3 lattice that stores and releases energy during acceleration but cannot radiate into R^3. The interaction term (4.5) translates mechanical acceleration into a phase-shifted reactive response, while the relation $g^2 = \eta\,\hbar\,\omega_P\,C_P\,Z_0$ establishes quantitative continuity with the fluctuation–dissipation description derived earlier. With the updated parameters this coupling is *critical* ($\eta = 1$), so the Planck patch responds as a matched impedance boundary between the two sectors. Consequently, both the field-theoretic and equivalent-circuit formulations yield identical inertial resistance R_{inert} and quality factor Q, demonstrating that the R^3–I^3 interface behaves as a self-consistent Planck-scale impedance surface.

5 Observable quantities and the inertial floor

5.1 Dynamic response of a single patch

5.2 Impedance and low–frequency limit

The small–signal dynamics of a Planck patch follow the impedance function derived in Sec. 3,

$$Z_{\text{patch}}(\omega, a) = i\omega L_{\text{eff}} + R_{\text{inert}}(a,\omega) + \frac{1}{i\omega C_P},$$

whose real part defines the dissipative response. For frequencies $\omega \ll \omega_P$, the circuit behaves as an overdamped oscillator with phase lag

$$\delta\phi(\omega, a) = \omega\, C_P\, R_{\rm inert}(a), \tag{5.1}$$

and linewidth

$$\Delta\omega = \frac{R_{\rm inert}(a)}{2L_{\rm eff}}. \tag{5.2}$$

The inertial resistance follows from the fluctuation–dissipation theorem,

$$R_{\rm inert}(a) = \eta Z_0\, \frac{a}{c\,\omega_P}, \tag{5.3}$$

with $\eta = 1$ for critical coupling and $Z_0 = 376.7\ \Omega$.

Substituting the numerical constants $C_P = 1.43 \times 10^{-46}\,\mathrm{F}$, $L_{\rm eff} = 2.03 \times 10^{-41}\,\mathrm{H}$, $\omega_P = 1.855 \times 10^{43}\,\mathrm{s}^{-1}$, and $H_0 = 2.19 \times 10^{-18}\,\mathrm{s}^{-1}$ gives the quantitative predictions summarized in Table 2.

Table 2: Predicted inertial parameters of the R^3–I^3 interface for $\eta = 1$.

Quantity	**Expression**	**Numerical value**
$R_{\rm inert}(1\,\mathrm{m/s^2})$	$\eta Z_0 a/(c\omega_P)$	$6.8 \times 10^{-50}\,\Omega$
$\Delta\omega(1\,\mathrm{m/s^2})$	$R_{\rm inert}/(2L_{\rm eff})$	$1.5 \times 10^{-9}\,\mathrm{s}^{-1}$
$R_{\rm inert}(cH_0)$	Eq. (5.3) with $a = cH_0$	$4.5 \times 10^{-59}\,\Omega$
$\Delta\omega(cH_0)$	Eq. (5.2)	$1.0 \times 10^{-18}\,\mathrm{s}^{-1} \approx H_0$
$\delta\phi(1\,\mathrm{m/s^2}, 1\,\mathrm{kHz})$	$\omega C_P R_{\rm inert}$	$6.1 \times 10^{-92}\,\mathrm{rad}$
Quality factor Q	$\omega_P L_{\rm eff}/R_{\rm inert}(cH_0)$	9×10^{60}

5.3 Interpretation of the inertial floor

Equation (5.2) defines a universal lower bound on dissipation associated with acceleration. At $a = 0$ the patch is purely reactive, exchanging no real power with its environment. A finite acceleration $a > 0$ activates the fluctuation–dissipation channel, introducing a resistance proportional to a and characterized by the dimensionless constant $\eta = 1$. This marks the smallest possible inertial response consistent with the Unruh temperature and quantum vacuum noise. For a notional Planck current $I_P = q_e\omega_P/(2\pi) = 4.7 \times 10^{23}\,\mathrm{A}$, the corresponding power dissipation per patch is

$$P_{\rm min} = \frac{1}{2} R_{\rm inert}(cH_0) I_P^2 = 5.0 \times 10^{-12}\,\mathrm{W}. \tag{5.4}$$

Although unobservable, this value defines a fundamental *quantum inertial floor*: no physical system can exhibit a smaller dissipative coupling to acceleration.

5.4 Scaling to multiple patches

A macroscopic body of mass m comprises

$$N = \frac{m}{m_\star}, \qquad m_\star = \frac{\hbar H_0}{2\pi c^2} = 1.2 \times 10^{-69}\,\mathrm{kg}, \tag{5.5}$$

independent Planck patches acting effectively in parallel. The aggregate inertial response therefore satisfies

$$R_{\mathrm{body}}^{-1} = \sum_{n=1}^{N} R_{\mathrm{inert}}^{-1} \;\Rightarrow\; R_{\mathrm{body}} = \frac{R_{\mathrm{inert}}}{N}, \tag{5.6}$$

so that ordinary masses display effectively vanishing resistance and hence perfectly classical inertia. The constant η remains universal, linking microscopic dissipation at the Planck interface to the macroscopic law $F = ma$.

5.5 Physical implications

The numerical results of Table 2 lie many orders of magnitude below any present experimental sensitivity. The corresponding linewidth $\Delta\omega(cH_0) \sim H_0$ and phase lag $\delta\phi \sim 10^{-92}$ rad mark the absolute quantum limit of lossless motion permitted by the fluctuation–dissipation theorem at the Planck boundary. They therefore represent a theoretical benchmark: any measurable deviation from these values would signify new physics beyond the established impedance structure of the R^3–I^3 interface.

Although direct observation is unfeasible, several amplification pathways could in principle bridge part of the gap. High–Q optical or microwave resonators ($Q \sim 10^{10}$) could accumulate the minute phase lag $\delta\phi$ over 10^{10} oscillations, increasing its magnitude by up to ten orders. Engineered metamaterial or superconducting networks designed with effective capacitance and inductance matching C_P and L_{eff} could reproduce the same dimensionless impedance ratios at accessible frequencies, providing analog simulations of the Planck–scale RLC response. In such systems, even weak deviations from the predicted scaling of $R_{\mathrm{inert}} \propto a$ or $\Delta\omega \approx H_0$ would offer valuable empirical clues to the coupling between acceleration, vacuum fluctuations, and inertial mass.

6 Discussion and comparative context

6.1 Relation to the $\mathbb{I}^3$ axioms

The quaternionic field $\mathbf{u} \in \mathbb{I}^3$ obeys the $\mathbb{I}^3$-space axioms: $\varepsilon_0 < 0$, $\mu_0 > 0$, and hence $i\,c = 1/\sqrt{\varepsilon_0\mu_0}$ is imaginary. All kinematic quantities in $\mathbb{I}^3$ therefore carry a factor i, and the dynamics are evanescent and non-radiating. Energy stored in the $\mathbb{I}^3$ return field is purely reactive; it can exchange power with the $\mathbb{R}^3$ sector only through the boundary patch where the Landauer–Unruh channel opens. This enforces the dissipationless limit for uniform motion ($a \to 0$) and guarantees that any inertial resistance originates solely from acceleration.

6.2 Physical interpretation

The Planck patch thus functions as a microscopic impedance element: a capacitor–inductor pair in which the capacitor represents cross-boundary displacement coupling and the inductor embodies the closed current loop within the $\mathbb{I}^3$ lattice. Acceleration activates the Unruh–FDT coupling, producing a finite $R_{\text{inert}}(a, \omega)$. The resulting impedance

$$Z(\omega, a) = R_{\text{inert}}(a, \omega) + i\left(\omega L_{\text{eff}} - \frac{1}{\omega C_P}\right) \tag{6.1}$$

is universal at the Planck scale. The imaginary sector provides the reactive reservoir, while real dissipation appears only through acceleration-dependent mixing between $\mathbb{R}^3$ and $\mathbb{I}^3$.

6.3 Comparison with related frameworks

The present R^3–I^3 interface differs fundamentally from other microscopic or emergent-gravity approaches:

- **Analog gravity:** Unruh radiation in fluid or Bose–Einstein analogs arises from effective horizons where phonon dispersion mimics curved spacetime. Here, by contrast, the Unruh temperature enters through the fluctuation–dissipation theorem applied directly to Planck-scale RLC elements; no macroscopic horizon or fluid medium is required.
- **Entropic gravity (Verlinde):** In entropic-gravity scenarios, inertia and gravity result from information loss at a holographic screen. The R^3–I^3 model instead provides a tangible impedance boundary whose reactive and dissipative parts are calculable from first principles, and whose microscopic constant η replaces the phenomenological entropy gradient.
- **Induced gravity (Sakharov):** Sakharov's framework attributes spacetime curvature to vacuum polarization of quantum fields. Here the continuous quantum vacuum is replaced by a discrete network of Planck oscillators; their collective envelope reproduces the classical elasticity of spacetime while remaining finite at the Planck scale.
- **QCD confinement analogy:** Confinement in quantum chromodynamics arises from flux-tube tension in color space. The $\mathbb{I}^3$ lattice offers an electromagnetic analog: each Planck-scale loop stores magnetic energy under the same mathematical form, with mutual inductance ensuring stability and self-limitation.

6.4 Amplification and possible observability

Although the predicted signals are extraordinarily small, several amplification paths may bring the effects closer to observable levels. High-Q optical or microwave resonators ($Q \sim 10^{10}$) could accumulate the minuscule phase lag $\delta\phi$ over many traversals, amplifying it by up to ten orders of magnitude. Metamaterial arrays engineered with matched inductance and capacitance could emulate Planck-scale impedances, allowing laboratory tests of Unruh-type dissipation in accelerated frames.

7 Conclusion

A fully quantitative and self-consistent description has been developed for the R^3–I^3 interface, in which every Planck-scale patch behaves as a microscopic RLC element determined solely by geometry and fluctuation–dissipation physics. The capacitance C_P originates from displacement coupling across the complex-normal gap, the inductance L_{eff} from closed magnetic circulation within the I^3 lattice, and the resistance $R_{\text{inert}}(a, \omega)$ from Unruh–Landauer coupling at the temperature $T_U(a) = \hbar a/(2\pi c k_B)$. Together these quantities form a universal impedance boundary connecting quantum geometry to classical inertia.

The updated evaluation with critical coupling $\eta = 1$ fixes the absolute scale of all dissipative effects: the minimal linewidth $\Delta\omega(cH_0) \simeq H_0$ and phase lag $\delta\phi \sim 10^{-92}$ rad define a Planck-regulated *inertial floor*. Uniform motion corresponds to the perfectly reactive limit ($R_{\text{inert}} = 0$), while acceleration opens the smallest physically allowed dissipation channel through the fluctuation–dissipation mechanism. No further reduction in inertial loss is possible without violating the Unruh temperature constraint.

Conceptually, the model restores symmetry between the real and imaginary spatial sectors. The quaternionic field $\mathbf{u} \in I^3$ provides the reactive degrees of freedom that mediate energy exchange across the interface without radiating into R^3. In this view, inertia arises as a boundary phenomenon between analytic continua: a matched impedance surface separating the radiative domain of real space from the evanescent substrate of imaginary space. The result establishes a quantitative bridge between Planck-scale microphysics and macroscopic inertial law, suggesting that every accelerated object interacts with a universal reactive background whose properties are set by the geometry of the R^3–I^3 interface.

Future directions

Several natural extensions follow. First, coupling the Planck-patch action to a curved metric $g_{\mu\nu}$ would permit analysis of the interface in general backgrounds, linking the local inertial linewidth $\Delta\omega$ to cosmological expansion and gravitational redshift. Second, quantized matter fields may be introduced within the same framework to identify particle masses and excitation spectra. Finally, engineered analog systems—high-Q resonators or metamaterial arrays—could emulate the impedance parameters derived here, offering a path toward experimental tests of the predicted inertial floor.

In summary, the R^3–I^3 interface provides a unified geometric, thermodynamic, and field-theoretic basis for inertia. Its predictions are explicit, parameter-free, and falsifiable in principle, offering a new bridge between Planck-scale boundary physics and macroscopic mechanics.

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