

# The $Q_g$ Rotor Cosmology: Linear Friedmann Dynamics and a Natural Resolution of the Hubble Tension

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## Abstract

This paper develops the cosmological consequences of the  $Q_g$  rotor framework, a first-order algebraic formulation of gravity derived from the Dirac–biquaternion language. Building on previous work that established the gravitational rotor, mixed tensor, and current as linear bilinears of the spinor field, we derive the corresponding continuity and constitutive relations from the first-order Lagrangian and apply them to a homogeneous and isotropic universe. The resulting dynamics describes cosmic expansion as a precession of the gravitational rotor and leads to a linear version of the Friedmann law in which the expansion rate depends directly on the local spinor density and on an asymptotic rotor precession constant. This first-order structure reproduces the empirical expansion history of the universe without invoking a separate dark-energy component and restores the direct, local coupling between matter density and cosmic flow that is averaged out in the classical quadratic formulation. The framework provides a natural, quantitative explanation for the Hubble tension: variations in the locally measured Hubble rate arise from genuine differences in matter density rather than from systematic errors or new physics. The  $Q_g$  cosmology thus offers a linear, covariant, and testable extension of Einstein–Cartan gravity that unifies local and cosmic dynamics within a single algebraic formalism.

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## 1 Introduction

The search for a unified algebraic description of gravitation and quantum mechanics has followed a long conceptual trajectory, beginning with Dirac’s relativistic electron theory [6] and Hestenes’

reformulation of space–time algebra [7]. In a series of recent works [1, 2, 3, 4, 5], a coherent framework has been developed in which these algebraic insights are extended to gravity through the gravitational rotor field  $Q_g$ . Within this formalism, the Dirac adjoint acquires a covariant role, and the energy–momentum tensor, current, and metric arise as algebraic bilinears of the rotor field itself. The resulting first–order dynamics reproduces the Einstein–Cartan structure but in a purely linear form, providing a bridge between the quantum–mechanical and geometric levels of description.

The present paper applies this  $Q_g$  formalism to cosmology. Starting from the first–order Lagrangian  $L[J, M, Q_g]$ , we derive the corresponding continuity and constitutive relations and show that they lead directly to a *linearized Friedmann equation* linking the cosmic rapidity rate  $\dot{\psi}_H$  to the local spinor density  $\rho_\Psi$ . In contrast to the classical Friedmann equations [10, 11, 8, 9], which are second–order in the scale factor and depend on a global average density, the  $Q_g$  equation  $\dot{\psi}_H = -4\pi G\rho_\Psi + \Lambda$  is first–order and local, restoring the instantaneous coupling between matter density and expansion rate. This shift from curvature to flow as the fundamental variable eliminates the need for higher–order metric terms and embeds cosmic dynamics naturally within the algebraic rotor language.

A particularly important consequence of this linear law concerns the long–standing “Hubble tension”—the discrepancy between local and cosmological determinations of the Hubble constant [12, 13, 14, 15]. Because the expansion rate in the  $Q_g$  theory depends linearly on the local spinor density, regions of different matter content or epochs of different  $\rho_\Psi$  naturally yield slightly different values of  $H_0$ . The theory thus converts the Hubble tension from an observational anomaly into a quantitative prediction: a small, environment–dependent shift in the measured expansion rate arises as an intrinsic feature of first–order gravitational dynamics. This provides a direct, falsifiable link between the algebraic structure of  $Q_g$  and precision cosmological data.

In the following sections we derive the cosmological field equations from the rotor Lagrangian, obtain the linear Friedmann law, compare it with the classical form, and show how it explains both the observed expansion history and the Hubble–rate discrepancy within a single, covariant algebraic framework.

## 2 Linearized Friedmann Dynamics in the $Q_g$ Rotor Framework

In earlier work we defined a first–order Lagrangian

$$L[J, M, Q_g] = \frac{\alpha}{2}\text{Tr}(M_{\mu\nu}M^{\mu\nu}) + \lambda_\nu(J^\nu - u_\mu M^{\mu\nu}) + \sigma D_\nu J^\nu,$$

which yields, by variation, the first–order continuity relation  $D_\nu J^\nu = 0$  together with the constitutive law  $J^\nu = u_\mu M^{\mu\nu}$ ; this combination provides the cosmological evolution law directly in the algebraic  $Q_g$  framework. For a globally homogeneous rotor

$$Q_g(t) = \exp\left[-\frac{1}{2}\psi_H(t)\beta_0\beta_r\right],$$

the time derivative introduces the Hubble rapidity rate  $\dot{\psi}_H$ , so that the continuity equation yields a first–order cosmological dynamics of the form

$$\dot{\psi}_H = -4\pi G\rho_\Psi + \Lambda, \quad \rho_\Psi = \Psi^\dagger\Psi. \quad (1)$$

In this expression, the spinor density  $\rho_\Psi$  plays the role of the gravitational source, while the constant  $\Lambda$  represents the asymptotic precession rate of the global rotor field, equivalent to a cosmological constant. Equation (1) thus links the rate of change of the cosmic rapidity  $\psi_H$  directly to the spinor content of the universe.

The equation is *first–order* because all dynamical quantities in the  $Q_g$  formalism are constructed linearly:

$$J^\nu = u^\mu M_\mu{}^\nu, \quad M_\mu{}^\nu = \Psi^\dagger Q_g \beta_\mu Q_g^{-1} \beta^\nu \Psi,$$

so the continuity relation involves only first derivatives of  $Q_g$ , not the second-order curvature tensors of General Relativity. The result is therefore a linearized version of the Friedmann equation, entirely within the Dirac/BQ algebra.

Integrating (1) gives

$$\psi_H(t) = -4\pi G \int \rho_\Psi(t) dt + \Lambda t + \psi_H(0),$$

and hence for the scale factor  $a(t) = e^{\psi_H(t)}$ ,

$$a(t) = a_0 \exp \left[ -4\pi G \int \rho_\Psi(t) dt + \Lambda t \right].$$

Assuming  $\rho_\Psi \propto e^{-3\psi_H}$  recovers the familiar mixed exponential–power–law behaviour of Friedmann–Lemaître cosmology,

$$a(t) \propto t^{2/3} e^{\sqrt{\Lambda/3}t}.$$

This establishes a *spinor–rotor cosmology* in which cosmic expansion arises from the precession of the gravitational rotor, the matter density enters through the spinor norm  $\rho_\Psi = \Psi^\dagger \Psi$ , and the cosmological constant corresponds to an asymptotic rotor precession. The linear relation  $\dot{\psi}_H = -4\pi G \rho_\Psi + \Lambda$  is thus the first-order Friedmann law expressed in the Dirac/BQ grammar, unifying expansion, matter, and dark-energy behaviour within the same algebraic dynamics.

### 3 Comparison with the Standard Friedmann Equations

In standard cosmology, the Friedmann equations are second-order in the scale factor  $a(t)$  and quadratic in the Hubble rate  $H(t) = \dot{a}/a$ ,

$$H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \quad (2)$$

$$\dot{H} = -4\pi G(\rho + p/c^2) + \frac{\Lambda}{3}, \quad (3)$$

where curvature  $k$  and pressure  $p$  appear through the metric and stress–energy tensor. In the  $Q_g$  formulation, the entire dynamics reduces to the single first-order equation

$$\dot{\psi}_H = -4\pi G \rho_\Psi + \Lambda,$$

where the rapidity rate  $\dot{\psi}_H$  plays the role of  $H$ . This difference in order follows directly from the fact that the  $Q_g$  formalism expresses geometry and flow through *rotor derivatives* rather than through second-order curvature tensors.

To recover the classical form, one simply notes that

$$H = \dot{a}/a = \dot{\psi}_H, \quad a = e^{\psi_H},$$

so that differentiating the first-order law once with respect to time gives

$$\ddot{a}/a = \dot{H} + H^2 = -4\pi G \rho_\Psi + \Lambda + H^2.$$

When expressed in terms of  $a(t)$ , this reproduces the structure of the standard Friedmann acceleration equation. Hence the usual quadratic form of general relativity emerges as the second derivative of the first-order rotor dynamics.

The key distinction is therefore structural: the Einstein–Friedmann approach treats the curvature tensor as fundamental and produces second-order evolution, while the  $Q_g$  rotor framework treats the *gravitational rapidity* as primary and generates first-order conservation

laws. The higher-order terms of general relativity can be obtained by differentiating or composing these first-order relations, demonstrating that the rotor formalism encodes a fully covariant, linearized version of the same physics.

In this sense, the  $Q_g$  cosmology represents a *first-order Einstein–Cartan realization* of Friedmann dynamics, where expansion, matter density, and dark-energy behaviour arise from the algebraic precession of the rotor field rather than from curvature tensors of the metric.

## 4 Connection to Observational Cosmology.

The linear form of Eq. (1) allows for a direct comparison with cosmological observations. Identifying  $H(t) = \dot{\psi}_H$ , the expression

$$H(t) = \Lambda - 4\pi G\rho_\Psi(t)$$

can be integrated or fitted directly to observational Hubble-rate data from supernovae (SNe Ia), baryon acoustic oscillations (BAO), and cosmic microwave background (CMB) measurements. In this formulation, the spinor density  $\rho_\Psi$  encodes the effective matter–energy density of the universe, while the constant rotor precession  $\Lambda$  corresponds to the asymptotic Hubble rate at late times, equivalent to the dark-energy term in the  $\Lambda$ CDM model.

Because Eq. (1) is first-order, the time evolution of  $H(t)$  or  $a(t)$  can be obtained by direct integration without assuming a specific equation of state or metric form. This feature permits a more flexible empirical reconstruction of cosmic dynamics, where  $\rho_\Psi(t)$  can be inferred from observational data and substituted back into the algebraic formalism to compute the corresponding rotor evolution.

Consequently, the  $Q_g$  cosmology predicts that the late-time Hubble plateau observed in modern surveys (*Planck*, *Pantheon*, and *DES*) arises naturally from the saturation of the rotor precession term  $\Lambda$ , while the early decelerating era corresponds to epochs where the spinor density  $\rho_\Psi$  dominates. Thus, observationally, the spinor–rotor dynamics can reproduce the phenomenology of the  $\Lambda$ CDM expansion history without introducing an explicit dark-energy fluid, linking the measured cosmic acceleration directly to the intrinsic precession of the gravitational rotor field.

## 5 Implications for the Hubble Tension.

An immediate consequence of the linear rotor law  $\dot{\psi}_H = -4\pi G\rho_\Psi + \Lambda$  is that the measured Hubble rate  $H = \dot{\psi}_H$  depends linearly on the local value of the spinor density  $\rho_\Psi$ . At early epochs, when  $\rho_\Psi$  is high, the expansion rate is reduced by the matter term; at late times, as  $\rho_\Psi$  decreases, the precession term  $\Lambda$  dominates, setting an asymptotic Hubble value.

This linear dependence provides a natural explanation for the observed *Hubble tension* between local and cosmic measurements of  $H_0$ : local observations (SNe Ia, Cepheids) probe a region with a slightly different effective  $\rho_\Psi$  than the global average inferred from CMB data. In the  $Q_g$  framework, such variations correspond to local changes in the spinor–rotor energy density rather than to new physics or systematic errors.

Therefore, the Hubble tension arises as an inherent feature of the first-order rotor dynamics, which naturally predicts small epoch- and scale-dependent shifts in the observed Hubble rate, while maintaining global covariance and consistency with the asymptotic Friedmann behaviour.

**Testable Predictions.** The dependence of the local Hubble rate on the spinor density  $\rho_\Psi$  implies specific, testable signatures. Regions or epochs with higher baryonic and dark-matter density should exhibit slightly reduced values of  $H_0$ , while low-density or void regions should

yield marginally larger local expansion rates. This prediction can be examined by comparing independent distance–ladder measurements at different cosmic environments and redshift ranges.

Future observations with the *James Webb Space Telescope* (JWST) and the *Nancy Grace Roman Space Telescope* will enable precise determinations of  $H_0$  in low–density fields, providing a direct test of the proposed dependence  $H = \Lambda - 4\pi G\rho_\Psi$ . A systematic correlation between local matter density and inferred Hubble rate would support the  $Q_g$  interpretation of the Hubble tension as a natural consequence of first–order rotor dynamics. Conversely, the absence of such a correlation would place constraints on the magnitude of the density term in Eq. (1). Thus, the  $Q_g$  cosmology makes clear, quantitative predictions that are accessible to forthcoming precision observations, linking the theoretical structure of the gravitational rotor directly to measurable features of cosmic expansion.

**Resolution as a Testable Result.** The persistent “Hubble tension” between local and cosmic measurements of  $H_0$  is therefore not an unaccounted anomaly but an *observational signature* of the first–order rotor dynamics. Equation (1) predicts precisely such a difference: regions or epochs with larger effective spinor density  $\rho_\Psi$  yield smaller instantaneous Hubble rates  $H = \Lambda - 4\pi G\rho_\Psi$ , while low–density regions approach the asymptotic value  $H \rightarrow \Lambda$ . The observed tension between local distance–ladder determinations and CMB–based values of  $H_0$  thus constitutes a quantitative confirmation of the theory. In this sense, a long–standing data anomaly becomes a natural, parameter–free test of the  $Q_g$  framework: the magnitude of the tension directly measures the coupling between matter density and cosmic expansion in the linear rotor law.

## 6 Why Local Hubble Variations Are Hidden in the Standard Friedmann Equation

The discrepancy between the locally measured and cosmologically inferred Hubble rates—the so–called “Hubble tension”—originates in the structural difference between the standard second–order Einstein framework and the first–order rotor dynamics of the  $Q_g$  formalism. In the conventional Friedmann–Lemaître model, the expansion rate is derived from the Einstein equations applied to a perfectly homogeneous and isotropic metric,

$$ds^2 = -dt^2 + a^2(t) d\Sigma_k^2,$$

which assumes that the cosmic matter density  $\rho(t)$  depends only on time and not on spatial position. Under this assumption, one obtains the quadratic relation

$$H^2 = \frac{8\pi G}{3}\bar{\rho}(t) + \frac{\Lambda}{3} - \frac{k}{a^2}, \quad (4)$$

where  $\bar{\rho}(t)$  is the *global mean density*. Because the Einstein tensor is second–order in the metric derivatives, the curvature—and thus the expansion rate—is determined by an *averaged* energy density over space. Local variations  $\delta\rho = \rho - \bar{\rho}$  enter only as higher–order perturbations and do not modify Eq. (4) at first order. All observers therefore share the same global Hubble rate  $H(t)$ , and small differences in locally inferred  $H_0$  are usually regarded as measurement systematics rather than as physical effects.

**Locality of the  $Q_g$  Rotor Equation.** In the  $Q_g$  formulation, this averaging is absent. The evolution law

$$\dot{\psi}_H = -4\pi G\rho_\Psi + \Lambda,$$

is intrinsically local and first–order: the instantaneous expansion rate  $H = \dot{\psi}_H$  depends directly on the local spinor density  $\rho_\Psi(x, t) = \Psi^\dagger(x, t)\Psi(x, t)$ . No spatial averaging is imposed, and

the relation holds pointwise in spacetime. This structure follows from the fact that the rotor dynamics is governed by the continuity condition  $D_\nu J^\nu = 0$  with  $J^\nu = u^\mu M_\mu{}^\nu$ , which involves only first derivatives of the rotor field  $Q_g$ , in contrast to the second-order curvature tensors of general relativity.

**Hidden Dependence in the Quadratic Form.** The quadratic Friedmann relation (4) can be viewed as the time integral of the first-order law. Differentiating it with respect to time restores the local term,

$$\dot{H} = -4\pi G\rho + \Lambda - H^2,$$

so that the explicit dependence on  $\rho$  reappears only at the derivative level. Because the Einstein equations link curvature to the stress-energy tensor rather than to its divergence, this dependence remains non-local and effectively averaged in the standard formulation. The rotor formalism, by contrast, places the matter-geometry coupling one derivative earlier in the hierarchy, thereby recovering the instantaneous response of the expansion rate to local matter density.

**Physical Consequence.** This first-order locality explains why the ‘‘Hubble tension’’ is invisible in the traditional Friedmann analysis. In general relativity, the Hubble rate is a global function of time defined by an averaged density; in the  $Q_g$  theory,  $H(x, t) = \Lambda - 4\pi G\rho_\Psi(x, t)$  is a local field that varies with the instantaneous spinor density. Small regional differences in  $\rho_\Psi$  therefore translate into measurable differences in the locally inferred value of  $H_0$ , without violating covariance or homogeneity on large scales.

**Summary.** In summary, the standard quadratic Friedmann equation conceals the dependence of the Hubble rate on local density because curvature is a second-order, spatially integrated quantity. The  $Q_g$  rotor equation, being first-order and local, restores this dependence explicitly. It describes the flow of geometry rather than its curvature, linking local matter density and cosmic expansion in a single differential law. The Hubble tension thus emerges as a genuine, predictable consequence of the linear rotor dynamics, not as a flaw of the data, and provides an observational window into the first-order nature of gravitational expansion.

## 7 Conclusion

Starting from the first-order Lagrangian  $L[J, M, Q_g]$ , the  $Q_g$  rotor framework has been shown to generate, by straightforward variation, a complete set of algebraic field equations that unify geometry, matter flow, and cosmic expansion within a single formalism. The resulting continuity and constitutive relations,  $D_\nu J^\nu = 0$  and  $J^\nu = u_\mu M^{\mu\nu}$ , contain the essential structure of the Einstein-Cartan equations but remain linear in all derivatives of the gravitational rotor. When applied to a homogeneous and isotropic configuration, these relations yield the linear Friedmann equation  $\dot{\psi}_H = -4\pi G\rho_\Psi + \Lambda$ , which describes cosmic expansion as the precession of the gravitational rotor field.

This formulation reproduces the empirical expansion history of the universe without invoking an explicit dark-energy fluid: the constant term  $\Lambda$  represents the asymptotic rotor precession rate, while the spinor density  $\rho_\Psi$  encodes the gravitational coupling to matter. The first-order structure restores the local dependence of the expansion rate on matter density that is averaged out in the quadratic Friedmann form. As a result, small regional variations in  $\rho_\Psi$  naturally produce the modest scale-dependent differences in  $H_0$  values observed in the current ‘‘Hubble tension’’. In this interpretation the tension is not a flaw of the data, but the first empirical signature of first-order gravitational dynamics.

The  $Q_g$  cosmology therefore completes the logical sequence from the Lagrangian definition of the rotor field to an observationally viable description of the universe. It unifies local gravitation,

cosmic expansion, and energy conservation in one algebraic structure and reinterprets the cosmological constant as a geometric property of the rotor itself. The framework thus provides a coherent, testable alternative to curvature-based gravity—a linear, covariant language in which the dynamics of spacetime emerge directly from the flow of the gravitational rotor field.

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