Collatz Conjecture model via Infinite Hotel Rearrangement

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October 19, 2025

Abstract

This paper presents an argument based on an analogy with an infinite hotel—where rooms correspond to positive integers—and a rearrangement of guests (also positive integers) following the inverse Collatz-tree rules. The construction aims to demonstrate an inconsistency that challenges the validity of the Collatz conjecture.

1. Introduction

The Collatz conjecture states that all positive integers eventually reach 1 under repeated application of the Collatz function:

$c(n) = n/2$, if n is even
$c(n) = (3n + 1)/2$, if n is odd.

2. Setup

Let R(i) denote the guest in room i, where i = 1, 2, 3, 4, ... , and initially R(i) = i.

For the Collatz inverse-tree construction, let guest 1 be the root of an infinite perfect binary tree. Guests are arranged along the edges of this tree as follows:

Consider a guest n. Let R(il) = gl and R(ir) = gr represent the left and right child guests of n, respectively.

- Swap R(il) with guest R(k1) = 2n, resulting in R(k1) = gl and R(il) = 2n.
- Similarly, swap R(ir) with guest R(k2) = (2n - 1)/3, giving R(k2) = gr and R(ir) = (2n - 1)/3.

Through these level-by-level swaps, guests are moved among rooms according to the inverse Collatz structure.

Let T denote the set of guests in the connected component of guest 1 under this rearrangement (the Collatz guest tree). Then:

T = { 1, 2, 4, 8, 16, 5, 32, 10, 3, 64, 21, 20, 6, ... }.

Let C be the set of rooms occupied by guests from T. Then:

C = { 1, 2, 4, 8, 16, 17, 32, 34, 35, 64, 65, 68, 70, ... }.

Hence, the mapping from T to C is one-to-one.

3. Key Observation

Since each room contains exactly one guest, and the construction proceeds explicitly level by level, the set C forms a proper subset of the natural numbers ℕ.

For instance, room 3 does not belong to C (i.e., $3 \notin C$). During the rearrangement process, as levels progress to infinity, the guest R(3) continues changing, and its limiting value cannot be definitively assigned. Therefore, $R(3) \notin T$.

4. Conclusion

Consequently, $T \neq \mathbb{N}$, meaning not all positive integers are contained within the Collatz tree. Under this framework, the Collatz conjecture would not hold unless guests not in C are all infinity.

References

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