Unified Field Equation in the Theory of Curved Space (T.C.S.): From Elementary Particles to Black Holes

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Содержание

Introduction: The Theory of Curved Space (T.C.S.) – a unified principle for organizing matter from elementary particles to black holes

This paper presents the Theory of Curved Space (T.S.C.) – a fundamentally new approach based on a revision of fundamental kinematic principles and offering a unified geometric description of all physical phenomena. The key foundations of the TCS are:

Fundamental Principles of the TCS

- **4+0 Geometry**: Spacetime is a four-dimensional Euclidean space with absolute time, where the light front forms a 3-sphere of radius ct
- **Two-way Speed of Light**: Only the two-way speed of light c has physical meaning as an invariant quantity, while the one-way speed is a conventional quantity dependent on clock synchronization
- **Projectional Nature of Physical Quantities**: Observed physical quantities are projection effects of 4D geometry:
  - **Mass** arises as oscillations perpendicular to the observed 3D slice
  - **The Electric Field** is global oscillation of the entire 4D space
  - **Gravity** manifests itself as secondary waves from the primary 4D oscillations
- **Homothety Conservation Law**: The fundamental condition $\det Q = 1$ expresses the conservation of the 4D phase volume and generates the universal law $\rho_{\text{hom}} R^2 = \text{const}$, where ρ_{hom} is the homothetic density as a measure of the phase filling of 4D space

Universal Homothety Law: From Atoms to Black Holes

TCS reveals the surprising universality of the homothety principle, which consistently manifests itself at all scales:

- **Atomic Scale**: For Bohr radii, the law $\rho_{\text{eff}} a_0^2 = \text{const}$ determines the characteristic sizes of atoms, deriving the value of the Bohr radius a_0 from geometric considerations.
- **Nuclear scale**: In atomic nuclei, the homothety law $\rho_{\text{hom}} \sim A^{-2/3}$ explains the shift of the stability valley and the existence of a stability attractor in the iron-nickel region ($A \approx 56 - 62$), which is traditionally interpreted as a result of the balance of nuclear forces.
- **Cosmological scale**: For black holes, the law $\rho_{\text{hom}} R_s^2 = \text{const}$ describes the structure of the event horizon, where ρ_{hom} is interpreted as the information packing density on the horizon.

Origin of fundamental constants

TCS offers a geometric explanation for the origin of fundamental physical constants:

- **Planck's constant** $\hbar$ arises as the amplitude of 4D oscillations: $A = \hbar/(mc)$, representing a natural quantum of phase volume circulation.

- **Gravitational constant** G arises as a measure of the coupling between primary oscillations, defined through the amplitude of secondary waves: $A = 2l_p^2/R$, where $l_p = \sqrt{\hbar G/c^3}$ is the Planck length.

Curved Space Theory does not contradict the experimentally establishedThe theory does not rely on the fundamental basis of special relativity and general relativity, but offers a reinterpretation of them within a more general framework that naturally resolves the aforementioned fundamental problems of modern physics. Thus, the TCS is not an alternative hypothesis in the narrow sense, but rather acts as a universal geometric framework in which the classical results of special relativity and general relativity emerge as particular projections. The most important distinction of the TCS is that it makes testable predictions: from the structure of nuclear stability to the properties of event horizons and large-scale cosmological effects. This allows it to be viewed not as a philosophical reconstruction, but as a physical theory in the strict sense.

1 Axioms and Kinematics (Two-Way Speed)

Axiom (Two-Way Speed). The TCS postulates the invariance of the two-way speed of light:

$$v_{\text{two-way}} = \frac{L_+ + L_-}{T} = c, \tag{1}$$

where L_+ and L_- are the forward and reverse path lengths along the same route, and T is the absolute time of the total journey, measured by the same clock.

This fixes c as a universal constant for all inertial frames of reference.

Absolute Time. The TCS uses a single time parameter t for all frames of reference. Transformations affect only spatial coordinates, while time remains absolute and is not mixed with spatial coordinates.

Space $4+0$. Space is described by four coordinates x^a, $a = 1, 2, 3, 4$, with a light front of radius ct:

$$(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = (ct)^2. \tag{2}$$

Thus, the light front in the TCS is a 3-sphere in 4-dimensional Euclidean space.

Conservation of volume. Fundamental condition of the TCS:

$$\det Q = 1, \tag{3}$$

where Q is the matrix of the homothetic transformation. This means that 4-dimensional volume is preserved under all transformations.

The propagation velocity of a light front is interpreted as the velocity of volume conservation in 4D space.

One-way velocity. In the TCS, the one-way speed of light has no independent physical meaning, since its definition always depends on the synchronization of distant clocks.

Any attempt to introduce a one-way velocity reduces to a two-way velocity, which is the fundamental observable quantity.

Thus, it is the two-way velocity c that acts as an invariant, consistent with all experiments.

Philosophical aspect (Einstein-Reichenbach). In classical relativistic literature, the "conventionality of synchronization"(Einstein, Reichenbach) has been long discussed: can one-way velocity be considered a conventional quantity, dependent on the choice of synchronization? The TCS fundamentally resolves this debate: since time is absolute, and only two-way velocity has physical meaning, the question of the conventionality of synchronization disappears.

Thus, the TCS provides not only physical but also philosophical clarity: c is not a convention, but a geometric invariant of a 4D volume.

Agreement with experiment. This description is completely consistent with relativistic experiments (Michelson-Morley, Kennedy-Thorndike, Ives-Stilwell), but has its own interpretation:

- c is fixed as an invariant two-way velocity, and not as a result of mixing time and space.
- Absolute time t is the same for all observers.
- Conservation of a 4-dimensional volume ($\det Q = 1$) guarantees consistency with the principle of the constancy of the speed of light.

For a detailed derivation of the basic kinematic formulas, the Lagrangian, and the dispersion equation, see 2.

2 Derivation, Physical Interpretation, and Verifiable Consequences (TCS)

2.1 Axioms and Geometric Construction

- Absolute time: $t' = t$, the full 4+0 interval is fixed by the front norm: $ds^2_{\text{full}} = c^2 dt^2$.
- Volume-preserving projection: the local projection is given by $Q(x)$ with $\det Q(x) = 1$ (preserving 4D volume).
- Invariance of the average light front velocity: a two-way (round-trip) operation yields the average velocity c in all inertial reference frames.

Geometry: A circle of radius v (isotropic relative velocity in the platform frame) is projected onto an inclined laboratory plane at an angle θ, where

$$\sin\theta = \frac{u}{c}, \qquad \cos\theta = \sqrt{1 - \frac{u^2}{c^2}}.$$

The projection of the circle yields an ellipse with semi-axes

$$a = v, \qquad b = v\sqrt{1 - \frac{u^2}{c^2}}, \qquad c_f = \sqrt{a^2 - b^2} = \frac{uv}{c}.$$

2.2 Detailed Derivation of Formulas

Decomposition of the relative velocity into components:

$$v_{\parallel} = v\cos\alpha, \qquad v_{\perp} = v\sin\alpha, \qquad v'_{\perp} = v_{\perp}\sqrt{1 - \frac{u^2}{c^2}}.$$

One-way resultant velocity:

$$W_{\text{one}}^2 = (u + v\cos\alpha)^2 + \left(v\sin\alpha\sqrt{1-\frac{u^2}{c^2}}\right)^2$$
$$= u^2 + v^2 + 2uv\cos\alpha - \frac{u^2v^2}{c^2}\sin^2\alpha.$$

The round-trip average velocity is:

$$W_{\pm}^2 = (u \pm v\cos\alpha)^2 + \left(v\sin\alpha\sqrt{1-\frac{u^2}{c^2}}\right)^2,$$
$$\langle w\rangle^2 = \frac{W_+^2 + W_-^2}{2} = u^2 + v^2 - \frac{u^2v^2}{c^2}.$$

2.3 Physical Interpretation of Projected Squaring

Why the transverse component is compressed, while the longitudinal one is not. The transverse component is measured in a plane inclined to the original plane by an angle θ; Its length geometrically decreases in the projection by a factor of $\cos\theta = \sqrt{1-u^2/c^2}$. The longitudinal component lies on the intersection line of the planes (the axis along $\boldsymbol{u}$) and therefore the unit scale is preserved under the projection. This reflects the anisotropy of the projection while preserving absolute time: there is no displacement of t, the scale along the axis of motion does not change, but it decreases across the axis.

Relation to 4D volume conservation ($\det Q = 1$). The operator Q specifies anisotropic along/across scales when projecting from 4D to 3D velocity subspace. The condition $\det Q = 1$ requires that the product of scales in mutually orthogonal directions be equal to 1 (volume conservation). The light-like norm of the front ($ds^2_{\text{full}} = c^2dt^2$) fixes the longitudinal scale to unity, then the transverse scale must be equal to $\sqrt{1-u^2/c^2}$ to match the slope $\sin\theta = u/c$; this yields an ellipse with semi-axis $b = v\sqrt{1-u^2/c^2}$.

2.4 Conformity with Experiments

Newtonian Limit for $u \ll c$. For small u/c, we have $\sqrt{1-u^2/c^2} \approx 1 - \frac{u^2}{2c^2}$ and

$$W_{\text{one}}^2 \approx (u + v\cos\alpha)^2 + v^2\sin^2\alpha = u^2 + v^2 + 2uv\cos\alpha,$$

that is, $W_{\text{one}} \approx \|\boldsymbol{u} + \boldsymbol{v}\|$ is the classical addition of velocities. For the two-way operation:

$$\langle w\rangle^2 \approx u^2 + v^2,$$

which also coincides with the classical result after symmetrization (the cross terms disappear with averaging).

Invariance of the two-way velocity and agreement with Michelson-Morley. The two-way operation (round-trip) cancels out the orientation dependence:

$$\langle w\rangle^2 = u^2 + v^2 - \frac{u^2v^2}{c^2},$$

is independent of α and is consistent with the invariance of the average front velocity c. This explains the null result of the Michelson-Morley experiment: in the two-way scheme (round-trip interferometric path), orientational anisotropy does not appear.

Comparison with STR

Parallel velocities. In STR, for parallel velocities,

$$W_{\text{SR}} = \frac{u+v}{1+\frac{uv}{c^2}}.$$

In TCS, a one-sided parallel composition yields

$$\alpha = 0: \quad W_{\text{one}} = u + v,$$

since there is no transverse residue. However, the two-sided (observable) quantity is physically relevant: in round-trip operations, the oblique anisotropy is compensated, and the front invariance is preserved. Thus, the difference between TCS and STR pertains to one-sided measurements; in two-sided tests (Michelson-Morley type), they agree on the invariance of the mean velocity.

Verifiable Implications

- **Anisotropy of one-sided measurements:** the formula

$$W_{\text{one}}^2 = u^2 + v^2 + 2uv\cos\alpha - \frac{u^2v^2}{c^2}\sin^2\alpha$$

  predicts angular dependence and transverse reduction at the level of one-sided velocities (ellipsoidal metric in projection).

- **Invariance of two-sided measurements:** forward/backward averaging yields

$$\langle w \rangle^2 = u^2 + v^2 - \frac{u^2v^2}{c^2},$$

  free of angle α, consistent with the null results of interferometric experiments and the invariant average front velocity c.

Projection Interval Substituting the resulting expression for $\langle w \rangle^2$ into the projection interval:

$$\begin{aligned} ds_{3D}^2 &= c^2dt^2 - \langle w \rangle^2 dt^2 \\ &= c^2dt^2\left(1 - \frac{u^2+v^2-\frac{u^2v^2}{c^2}}{c^2}\right) \\ &= c^2dt^2\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right). \end{aligned}$$

The factorized form demonstrates the multiplicative structure of the geometric corrections from each branch.

Coordinate transformations TCS The axiom of absolute time and the geometry of 4D→3D projection imply transformations between the projected (x', y', z', t') and observed (x, y, z, t) systems:

$$\boxed{\begin{array}{ll} x = x' + u\,t, & \text{(longitudinal phase drift)} \\ y = \dfrac{y'}{\gamma(u)}, & \text{(transverse projection squashing)} \\ z = \dfrac{z'}{\gamma(u)}, & \text{(isotropy of transverse directions)} \\ t = t', & \text{(absoluteness of time)} \end{array}}$$

where $\gamma(u) = \dfrac{1}{\sqrt{1 - u^2/c^2}}$ is the Lorentz factor arising from projection geometry.

Physical interpretation: The longitudinal shear reflects the phase evolution in absolute time, while the transverse compression is a consequence of the projection of the 4D geometry onto the observed 3D subspace.

Lagrangian of a free particle For a material particle, we adopt an action proportional to the length of the projection plane.new element:

$$S = -m \int \sqrt{\frac{ds_{3D}^2}{dt^2}}\, dt = -mc^2 \int \sqrt{1 - \frac{u^2}{c^2}} \sqrt{1 - \frac{v^2}{c^2}}\, dt.$$

The Lagrangian takes the form:

$$\boxed{L = -mc^2 \sqrt{1 - \frac{u^2}{c^2}} \sqrt{1 - \frac{v^2}{c^2}}.}$$

For the stationary geometric branch $v^2 = 2\Phi(x)$, where $\Phi(x)$ is the effective scalar projection potential, we introduce a geometric mask:

$$M(x) = \sqrt{1 - \frac{2\Phi(x)}{c^2}} = \sqrt{1 - \frac{v^2}{c^2}},$$

which yields:

$$L = -mc^2\, M(x) \sqrt{1 - \frac{u^2}{c^2}}.$$

Momentum, Energy, and Dispersion Relation The canonical momentum is defined as:

$$p = \frac{\partial L}{\partial u} = \frac{mu\, M}{\sqrt{1 - \frac{u^2}{c^2}}}.$$

Total energy (Hamiltonian):

$$E = pu - L = \frac{mc^2 M}{\sqrt{1 - \frac{u^2}{c^2}}}.$$

To calculate the dispersion relation, we find $E^2 - p^2c^2$:

$$\begin{aligned} E^2 - p^2c^2 &= \frac{m^2c^4M^2}{1 - \frac{u^2}{c^2}} - \frac{m^2u^2c^2M^2}{1 - \frac{u^2}{c^2}} \\ &= \frac{m^2c^2M^2(c^2 - u^2)}{1 - \frac{u^2}{c^2}} = m^2c^4M^2, \end{aligned}$$

since $\frac{c^2-u^2}{1-\frac{u^2}{c^2}} = c^2$.

Thus, the dispersion relation TCS is:

$$\boxed{E^2 = p^2c^2 + m^2c^4M^2.}$$

Relation to observable effects In the limit of weak fields $M(x) \approx 1 - \frac{\Phi(x)}{c^2}$, the dispersion relation approximately reproduces:

$$E \approx mc^2 + \frac{p^2}{2m} + m\Phi(x),$$

which corresponds to nonrelativistic mechanics with potential $\Phi(x)$.

Experimental Verifiability Deviations from the standard dispersion relation $E^2 = p^2c^2 + m^2c^4$ manifest themselves through:

- Modification of reaction thresholds in ultra-high-energy cosmic rays
- Phase shifts in interferometric experiments
- Anomalies in precision measurements of atomic transition frequencies

Note on Mathematical Rigor All conclusions are strictly derived from the axioms through:

- Explicit construction of 4D vectors V_i^μ and application of the chord formula on a local 3-sphere
- Use of antipodal symmetry for algebraic contraction of the scalar product
- Correct matching of normalizations when determining average shifts
- Variational Principle for Obtaining Dynamic Equations

The self-consistency of the theory is confirmed by reproducing known results in the corresponding limiting cases, while preserving the fundamental differences in strong fields and relativistic regimes.

3 Factorization: "There"/"Back"Branches

3.1 Factorization of Energy-Momentum and the Operational Contribution

We introduce the phase $\vartheta = S/\hbar$ and the phase factor in the interval at absolute time t:

$$ds = c\,dt\,e^{i\vartheta}, \qquad \vartheta = \frac{S}{\hbar}.$$

We define the one-way factors (red/blue)

$$\chi := \frac{E - pc}{mc^2}, \qquad \chi' := \frac{E + pc}{mc^2},$$

and their product as an operational invariant

$$\chi \cdot \chi' = M^2.$$

The modified variance in the TCS is written as

$$E^2 = c^2p^2 + m^2c^4M^2,$$

which yields an explicit factorization

$$\frac{E - pc}{mc^2} \cdot \frac{E + pc}{mc^2} = M^2.$$

Operational interpretation:

$$M = \sqrt{\chi \chi'}$$

is the "there"and "back"geometric mean and provides a purely external contribution to the system; only the product $\chi \cdot \chi'$ (or equivalently M) is fixed by round-trip measurements in the TCS, and individual χ and χ' are not directly observable without additional synchronization.

$$\text{Length element:} \qquad ds \;=\; ds'\, D(u)\, M(r)\, \exp\Big(\frac{i}{\hbar} S_{\mathrm{HJ}}(x,t)\Big),$$

where $S_{\mathrm{HJ}}(x,t)$ is the unit phase (action), and the operational factors are introduced as

$$D(u) = \sqrt{\frac{1 + u/c}{1 - u/c}} \quad \text{(real dilation, round-trip kinematics)},$$

$$M(r) = \exp\Big(\frac{S_2(r)}{\hbar}\Big) \quad \text{(geometric/amplitude mask)}.$$

Text Explanation:

- The kinematic correction is introduced multiplicatively through $D(u)$ (real, positive), and the geometric mask through $M(r)$ (an amplitude factor, generally complex).
- In TCS, all physical predictions must be formulated using round-trip operators; therefore, when applied to measurements, use combinations that yield invariants (for example, $\chi \cdot \chi' = M^2$ or M as the geometric mean).

3.2 Relationship between the Doppler factor D and the scalar mask M: Complete Derivation

Setup. Consider a point particle with an effective scalar mask of the medium $M(x)$ (including gravity, condensate, and topology) and standard relativistic kinematics. Let the Lagrangian

$$L \;=\; -\,mc^2\, M(x) \sqrt{1 - \frac{v^2}{c^2}}\,, \qquad \gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\,.$$

Canonical momentum. By definition

$$\mathbf{p} \;=\; \frac{\partial L}{\partial \mathbf{v}} \;=\; m\, M\, \gamma\, \mathbf{v}\,, \qquad p \equiv |\mathbf{p}| = m\, M\, \gamma\, v\,.$$

Energy. Hamiltonian energy $E = \mathbf{p} \cdot \mathbf{v} - L$:

$$E \;=\; (m\, M\, \gamma\, \mathbf{v}) \cdot \mathbf{v} + mc^2 M \sqrt{1 - \frac{v^2}{c^2}} \;=\; m\, M\, \gamma\, v^2 + mc^2 M\, \frac{1}{\gamma} \;=\; mc^2\, M\, \gamma\,.$$

Variance. We immediately obtain

$$E^2 = m^2c^4M^2\gamma^2 = m^2c^4M^2\left(1+\frac{v^2}{c^2}\gamma^2\right) = m^2c^4M^2 + m^2c^2M^2\gamma^2v^2 = m^2c^4M^2 + p^2c^2\,,$$

that is, the universal relation

$$E^2 = p^2c^2 + m^2c^4M^2\,.$$

Factorization of the "there/back"branches. Let's write the product of the factors

$$\left(\frac{E-pc}{mc^2}\right)\left(\frac{E+pc}{mc^2}\right) = \frac{E^2-p^2c^2}{m^2c^4} = M^2\,.$$

Introduce the branch notations

$$\chi \equiv \frac{E-pc}{mc^2} = D\,M\,, \qquad \chi' \equiv \frac{E+pc}{mc^2} = D^{-1}\,M\,,$$

so that

$$\chi\,\chi' = M^2, \qquad D\,D^{-1} = 1\,.$$

Interpretation of D and M. - $M(x)$ is the *scalar mask of the system*: includes the gravitational, condensate, and topological contributions; it is the same for both branches, so M^2 remains squared in the product. - D is the *generalized kinematic factor (Doppler)*: accumulates all directionally dependent shifts (blue/red Doppler, path anisotropy). For the out-and-back branches, it yields mutually inverse coefficients D and D^{-1} and therefore cancels out in the product.

Extension to interactions (minimal conjugation). When adding gauge fields, we replace $\mathbf{p} \to \mathbf{\Pi}$:

$$\mathbf{\Pi} = \mathbf{p} - \frac{q}{c}\,\mathbf{A}_{\text{em}} - \frac{g}{c}\,W_i^a\,T^a - \frac{g_s}{c}\,G_i^b\,t^b \quad \text{(without the homothetic contribution to } M\text{)},$$

and the variance takes the form

$$E^2 = c^2\,\mathbf{\Pi}^2 + m^2c^4\,M^2\,,$$

and the factorization is the same product of branches:

$$\left(\frac{E-c\,\Pi}{mc^2}\right)\left(\frac{E+c\,\Pi}{mc^2}\right) = M^2, \quad \chi = D\,M, \quad \chi' = D^{-1}\,M\,.$$

Conclusion. D is the single coefficient of directionally dependent (Doppler) shifts and anisotropy; M is the scalar mask of the medium. It is this decomposition that provides:

$$\text{objective energy through} \chi\chi' = M^2 \quad \text{and} \quad E^2 = c^2\Pi^2 + m^2c^4M^2.$$

3.3 Quantum operator form (first-order system TCS)

Definition of operators. In 4+0 geometry, we introduce the matrices α^A $(A = 1, \dots, 4)$ and β with the algebra

$$\{\alpha^A, \alpha^B\} = 2\delta^{AB} I, \qquad \{\alpha^A, \beta\} = 0, \qquad \beta^2 = I.$$

Covariant derivative. The generalized covariant derivative includes the minimum conjugate:

$$\mathcal{D}_\mu = \partial_\mu + i\frac{q}{\hbar c}A_\mu^{\text{em}} + i\frac{g}{\hbar c}W_\mu^a T^a + i\frac{g_s}{\hbar c}G_\mu^b t^b + i\frac{g_{\text{hom}}}{\hbar c}B_\mu^c T^c.$$

First-order equation. We define the diagonal mass-phase block

$$\mathbb{D}(x) = \text{diag}\big(\chi(x), \chi'(x), \chi(x), \chi'(x)\big), \qquad \chi = MD,\ \chi' = MD^{-1}.$$

Then the operator form of TCS is written as a first-order system

$$i\hbar\,\partial_t\Psi(x,t) = \Big[-\,i\hbar c\sum_{A=1}^{4}\alpha^A\,\mathcal{D}_A + mc^2\,\beta\,\mathbb{D}(x)\Big]\Psi(x,t),$$

where Ψ is a four-component spinor in 4+0.

4 Spinor form in 4+0

Branch expansion. First-order branch equations for two component states ψ_+ and ψ_-:

$$(i\hbar\partial_t - c\,\alpha\cdot\mathcal{D} - mc^2\,\beta\,\chi)\,\psi_+ = 0, \qquad (i\hbar\partial_t + c\,\alpha\cdot\mathcal{D} - mc^2\,\beta\,\chi')\,\psi_- = 0.$$

Collecting them into a 4-component spinor $\Psi = (\psi_+, \psi_-)^\mathsf{T}$, we obtain a matrix notation from the block equations (see the previous section) with diagonal mass block $\mathbb{D}(x)$.

Nonrelativistic limit. For $v \ll c$ and weak fields, we reduce to the generalized Pauli equation with contributions from all gauge fields (the Abelian part yields the familiar magnetic terms, while the non-Abelian ones yield additional spin-gauge interactions).

Spinor structure in TCS. The resulting TCS equation is formally reminiscent of the Dirac equation, but its geometric interpretation is different. Instead of the Lorentz group $SO(3,1)$, here we have the rotation group $SO(4)$ in four-dimensional Euclidean space with absolute time. The minimal spinor representation of SO(4) is realized as $SU(2) \times SU(2)$, and the four-component spinor TCS naturally decomposes into "left"and "right"SU(2) components.

In this context, spin 1/2 is treated as a two-valued representation of rotations in the (i,4) planes, where i=1,2,3.

In other words, the role of the "axis of rotation"is played by the distinguished fourth coordinate: the rotation by $2.\pi$ around it changes the sign of the wave function.

Thus, spin-$\frac{1}{2}$ in the TCS is not an internal quantum number, but a geometric property of rotations in 4-dimensional space, which provides the Dirac structure of the equation.

4.1 Component geometric mean and conditions for its applicability

Announcements: Let the base vector and energy for a given mode be denoted by Π and E, and let the unidirectional factors be given by

$$\chi(r,t) = D(r,t)\,M(r,t), \qquad \chi'(r,t) = D^{-1}(r,t)\,M(r,t).$$

Then

$$\Pi_+(r,t) = \Pi(r,t)\,\chi(r,t), \qquad \Pi_-(r,t) = \Pi(r,t)\,\chi'(r,t),$$

$$E_+(r,t) = E(r,t)\,\chi(r,t), \qquad E_-(r,t) = E(r,t)\,\chi'(r,t).$$

We define the component geometric mean for the vector and energy:

$$(\Pi_{\rm gm})_i(r,t) \;=\; \sqrt{(\Pi_+)_i(r,t)\,(\Pi_-)_i(r,t)} \;=\; \sqrt{\Pi_i^2(r,t)\,\chi(r,t)\chi'(r,t)}$$

and

$$E_{\rm gm}(r,t) \;=\; \sqrt{E_+(r,t)\,E_-(r,t)} \;=\; \sqrt{E^2(r,t)\,\chi(r,t)\chi'(r,t)}\,.$$

Using $\chi\chi' = M^2$, we obtain a simplified form:

$$(\Pi_{\rm gm})_i(r,t) \;=\; |\Pi_i(r,t)|\,M(r,t), \qquad E_{\rm gm}(r,t) \;=\; |E(r,t)|\,M(r,t).$$

With consistent signs, the components can be written without moduli:

$$(\Pi_{\rm gm})_i = \operatorname{sign}\big((\Pi_+)_i\big)\,\sqrt{(\Pi_+)_i(\Pi_-)_i}.$$

Full Wave via Component Geometric Mean

We introduce the slow amplitudes $A_\pm(r,t)$ and phase functions

$$S_+(r,t) = \Pi_+\cdot r - E_+t, \qquad S_-(r,t) = \Pi_-\cdot r - E_-t.$$

The component geometric mean of the phase vector and energy yields the effective phase

$$S_{\rm eff}(r,t) = \Pi_{\rm gm}\cdot r - E_{\rm gm}\,t.$$

The full wave in the form of a geometric average of components:

$$\Psi(r,t) = A(r,t)\,\exp\Big(\frac{i}{\hbar}S_{\rm eff}(r,t)\Big),$$

where the amplitude $A(r,t)$ can be taken as the symmetrized average of the branch amplitudes, for example

$$A(r,t) = \sqrt{A_+(r,t)A_-(r,t)}\;M(r,t).$$

Uniqueness Conditions and Selection of a Root Branch

For a component root to be correct, the arguments of the complex numbers must be consistent:

$$\mathrm{Arg}\big((\Pi_+)_i\big) = \mathrm{Arg}\big((\Pi_-)_i\big) \quad \forall i, \qquad \mathrm{Arg}(E_+) = \mathrm{Arg}(E_-).$$

If these conditions are met, the roots can be taken continuously and uniquely. Otherwise, it is necessary to explicitly fix the branch of the complex logarithm, for example by setting

$$\sqrt{z} = \exp\Big(\tfrac{1}{2}\mathrm{Log}z\Big), \qquad \mathrm{Log}z = \ln|z| + i\mathrm{Arg}z,$$

and specify the chosen interval for $\mathrm{Arg}z$.

Relation to Logarithmic Phase Averaging and Consistency Check

An alternative construction is logarithmic phase averaging:

$$\ln\Psi_{\text{full}} = \tfrac{1}{2}\big(\ln\psi_+ + \ln\psi_-\big),$$

which yields

$$A(r,t) = \sqrt{A_+A_-}\,\sqrt{\chi\chi'} = \sqrt{A_+A_-}\,M, \qquad S_{\text{full}} = \tfrac{1}{2}(S_+ + S_-).$$

This notation is equivalent to the component geometric mean, provided that the arguments of the components are consistent and the phase contributions from χ and χ' are taken into account.

Properties of the factors χ, χ'. We define

$$\chi(r,t) = D(r,t)\,M(r,t), \qquad \chi'(r,t) = D^{-1}(r,t)\,M(r,t), \qquad \chi\chi' = M^2.$$

1. Multiplicative Character and Regularity. The factors χ, χ' act as multiplicative operators. Correct application of differential operators requires at least continuous differentiability: $\chi, \chi' \in C^1(\Omega)$. Ideally, smoothness ($C^\infty$) or belonging to a suitable Sobolev space is assumed.

2. Leibniz's Rule. When the momentum operator $\hat{\Pi} = -i\hbar\nabla - \frac{q}{c}\mathbf{A}$ acts on the product, we obtain

$$\hat{\Pi}(\chi\psi) = (\hat{\Pi}\chi)\,\psi + \chi\,(\hat{\Pi}\psi).$$

The additional term $(\hat{\Pi}\chi)\psi$ introduces corrections to the effective gauge interactions and must be taken into account when deriving the nonrelativistic limit.

3. Zones by sign M^2. The mask $M(r,t)$ is by definition non-negative in the classical case, but three regimes are formally possible:

$M^2(r,t) > 0$	wave (hyperbolic) zone, effective mass $m_{\text{eff}} = mM$,
$M^2(r,t) = 0$	conversion boundary (horizon/threshold), boundary conditions analysis required,
$M^2(r,t) < 0$	evenscent (decaying, locally elliptical) zone.

It is important to emphasize: the region with $M^2 < 0$ does not imply physical "tachyonness but only a change in the type of equation and the nature of the solutions.

4. The Root and Logarithm Branch. For complex values, it is necessary to fix the branch:

$$\sqrt{z} = \exp\Bigl(\tfrac{1}{2}\,\mathrm{Log}\,z\Bigr), \qquad \mathrm{Log}\,z = \ln|z| + i\,\mathrm{Arg}z, \qquad \mathrm{Arg}z \in (-\pi, \pi].$$

This ensures uniqueness in determining $\sqrt{\chi\chi'}$ and in logarithmic averaging of the phases.

5. Physical patterns. As examples:

$$D(u) = \sqrt{\frac{1+u/c}{1-u/c}} \quad \text{(Doppler factor)},$$

$$M(r) = \sqrt{1 - \tfrac{2\Phi(r)}{c^2}} \quad \text{(gravity mask at } |\Phi| \ll c^2).$$

4.2 Spinor block structure with factors χ, χ'

The first-order branch equations for the components ψ_+, ψ_- are of the form

$$\bigl(i\hbar\partial_t - c\,\alpha\cdot\mathcal{D} - mc^2\beta\,\chi\bigr)\psi_+ = 0, \qquad \bigl(i\hbar\partial_t + c\,\alpha\cdot\mathcal{D} - mc^2\beta\,\chi'\bigr)\psi_- = 0.$$

Assembling them into a four-component spinor

$$\Psi(r,t) = \begin{pmatrix} \psi_+(r,t) \\ \psi_-(r,t) \end{pmatrix},$$

we obtain a block matrix system

$$\left[i\hbar\partial_t\,\mathbb{I}_4 - c\begin{pmatrix} \alpha\cdot\mathcal{D} & 0 \\ 0 & -\alpha\cdot\mathcal{D} \end{pmatrix} - mc^2\begin{pmatrix} \beta\,\chi & 0 \\ 0 & \beta\,\chi' \end{pmatrix} \right]\Psi(r,t) = 0.$$

Here $\mathbb{I}_4$ is the identity matrix of order 4, and the diagonal mass block

$$\mathbb{D}(x) = \begin{pmatrix} \beta\,\chi(r,t) & 0 \\ 0 & \beta\,\chi'(r,t) \end{pmatrix}$$

fixes the asymmetry between the branches. For $\chi = \chi' = 1$ (that is, $D = 1$, $M = 1$), the system reduces to the standard Dirac equation in a representation with a block-diagonal structure.

Nonrelativistic Limit

In the limit of $v \ll c$ and weak fields ($\chi, \chi' \approx 1$) the equation reduces to the generalized Pauli equation. The additional terms arise from the derivatives of $\nabla\chi, \nabla\chi'$ under the action of the operator $\mathcal{D}$ and provide corrections to the magnetic and spin-gauge interactions.

Geometric Interpretation

Formally, the structure of the equation is similar to the Dirac one, but the symmetry group differs: instead of $SO(3,1)$, $SO(4)$ acts with absolute time. The minimal spinor representation of $SO(4)$ is realized as $SU(2) \times SU(2)$, and the four-component spinor Ψ naturally decomposes into "left"and "right"components. The mask M acts as a geometric weight, and the factors χ, χ' ensure consistency between branches and determine the effective mass $m_{\text{eff}} = mM$.

4.3 Nonrelativistic Limit and Corrections from $\nabla\chi$

In the limit of low velocities $v \ll c$ and weak external fields ($\chi, \chi' \approx 1$) the equation for the four-component spinor Ψ reduces to the generalized Pauli equation. To do this, we isolate the "fast"phase $\exp(-imc^2t/\hbar)$ and decompose the spinor into large and small components.

Basic step. Substitution $\Psi(r,t) = e^{-imc^2t/\hbar}\,\Phi(r,t)$ gives

$$i\hbar\partial_t\Phi = \left[\frac{1}{2m}\big(\alpha\cdot\hat{\Pi}\big)^2 + V_{\text{eff}}(r,t)\right]\Phi + \mathcal{O}\left(\frac{v^2}{c^2}\right),$$

where $\hat{\Pi} = -i\hbar\nabla - \frac{q}{c}\mathbf{A}$.

Effective potential. The mask $M(r,t)$ and the factors χ, χ' introduce corrections of the form

$$V_{\text{eff}}(r,t) = q\varphi(r,t) + mc^2\left(M(r,t) - 1\right) + \Delta V_\chi(r,t),$$

where φ is the scalar potential, and ΔV_χ arises from the derivatives $\nabla\chi, \nabla\chi'$.

Corrections from gradients. When the momentum operator acts on the product $\chi\psi$, we obtain

$$\hat{\Pi}(\chi\psi) = (\hat{\Pi}\chi)\psi + \chi(\hat{\Pi}\psi).$$

Accordingly, additional terms appear in the Hamiltonian

$$\Delta V_\chi(r,t) = -\frac{\hbar^2}{2m}\frac{\nabla^2\chi}{\chi} - \frac{\hbar}{m}\frac{\nabla\chi}{\chi}\cdot\hat{\Pi} + \cdots,$$

which are interpreted as effective gauge and magnetic interactions induced by the spacetime variation of χ.

Results Thus, the nonrelativistic limit of the equation with mask M and factors χ, χ' yields the generalized Pauli equation:

$$i\hbar\partial_t\Phi = \left[\frac{1}{2m}\big(\hat{\Pi}^2 - q\hbar\,\boldsymbol{\sigma}\cdot\mathbf{B}\big) + q\varphi + mc^2(M-1) + \Delta V_\chi\right]\Phi,$$

where $\mathbf{B} = \nabla\times\mathbf{A}$. The additional terms ΔV_χ capture the contribution from the spatiotemporal inhomogeneity of the mask and branch factors.

Covariance in 4+0 with absolute time and the group SL(4,$\mathbb{R}$). We work in the geometry of 4+0 with absolute time t (not mixed with space) and with the internal (homothetic) symmetry of SL(4,$\mathbb{R}$) under the volume conservation condition $\det Q(x) = 1$. This imposes the following structural restrictions on fields and operators: (i) Space and Time.

The spatial coordinates x^a ($a = 1, \ldots, 4$) belong to the Euclidean $\mathbb{R}^4$, and time t is a global evolution parameter:

$$x \in \mathbb{R}^4, \qquad t' \equiv t.$$

The derivative operators are separated as ∂_t (absolute in time) and ∂_a (in space).

(ii) Homothetic Covariant Derivative. SoThe voltage to SL(4,$\mathbb{R}$) is introduced through the homothetic field of constraints $B_a(x)$ with generators T^A (in the chosen representation):

$$\mathcal{D}_a \;=\; \partial_a \;+\; i\, g_{\rm hom}\, B_a^A(x)\, T^A \;+\; \dots, \qquad \mathcal{D}_t \;=\; \partial_t \quad \text{(time does not mix with space)}.$$

The field strength (curvature) of SL(4,$\mathbb{R}$) is defined as

$$F^A_{ab} \;=\; \partial_a B^A_b - \partial_b B^A_a + g_{\rm hom}\, f^A{}_{BC}\, B^B_a B^C_b\,,$$

while $\det Q = 1$ preserves the volume measure (Noether homothety current).

(iii) Spinor kinematics in 4+0. The kinetic block acts only on $\mathbb{R}^4$:

$$i\hbar\,\partial_t\,\Psi \;=\; \Big[-\,i\hbar c\,\alpha^a\,\mathcal{D}_a \;+\; mc^2\,\beta\,\mathbb{D}(x)\Big]\Psi, \qquad \mathbb{D}(x) = \mathrm{diag}\big(\chi(x),\chi'(x)\big).$$

Here α^a implement the spinor structure of 4D space (locally consistent with SO(4)), and β is the mass-phase block; the absoluteness of time excludes "mixing"∂_t with ∂_a.

(iv) Branch factors χ, χ' and their variations.

$$\chi(x,t) = D(x,t)\,M(x,t), \qquad \chi'(x,t) = D^{-1}(x,t)\,M(x,t), \qquad \chi\chi' = M^2.$$

When $\chi = \chi(x,t)$, new terms arise strictly within 4+0:

$$\hat{\Pi}_a(\chi\psi) = (\hat{\Pi}_a\chi)\,\psi + \chi\,(\hat{\Pi}_a\psi), \qquad i\hbar\,\partial_t(\chi\psi) = i\hbar\,(\partial_t\chi)\,\psi + \chi\, i\hbar\,\partial_t\psi,$$

where $\hat{\Pi}_a = -i\hbar\,\partial_a - \frac{q}{c}A_a - g_{\rm hom}B^A_a T^A - \dots$. Hence:

$$\Delta V_\chi \;=\; -\frac{\hbar^2}{2m}\frac{\partial_a\partial^a\chi}{\chi} \;-\; \frac{\hbar}{m}\,\frac{\partial^a\chi}{\chi}\,\hat{\Pi}_a, \qquad \Delta K_\chi \;=\; \frac{i\hbar}{\chi}\,\partial_t\chi,$$

which are interpreted as potential-like, gauge (spatial), and "pumping"(temporal) contributions.

(v) Variance and type of equation. Second order in 4D space with absolute time:

$$\Big(\hat{E}^2 - c^2\,\hat{\Pi}^2\Big)\psi \;=\; m^2c^4\,M^2(x,t)\,\psi, \qquad \hat{E} = i\hbar\,\partial_t, \quad \hat{\Pi}^2 = \hat{\Pi}_a\hat{\Pi}^a.$$

Regimes:

$M^2 > 0$ — hyperbolic (wave) zone, $\quad M^2 = 0$ — conversion boundary, $\quad M^2 < 0$ — evescent (damped, loca

This is a change in the type of equation in the spatial part $\mathbb{R}^4$ with constant absolute time evolution.

(vi) New classes of solutions in 4+0 for SL(4,$\mathbb{R}$).

- *Localized and waveguide modes* in $\mathbb{R}^4$ for quasi-static $M(x)$ profiles, including $M^2 = 0$ interfaces (conversion surfaces).
- *Evescent layers* and wave screening in $M^2 < 0$ regions (exponential attenuation in space, evolution in absolute t).
- *Floquet states* for periodic $\chi(t)$ (time is the absolute modulation parameter): quasi-spectra, dynamic gaps, parametric amplification.
- *Landau–Zener branch conversion* when passing through $M^2(t) = 0$ (mixing ψ_+, ψ_- in time without Lorentz "mixing"of coordinates).
- *Topological phase regimes* for vortex configurations Arg$\chi(x)$ in $\mathbb{R}^4$ (gauge equivalents via SL(4,$\mathbb{R}$) couplings).

(vii) Constraints and Branch Choices. The absoluteness of time means that all analytic continuations and branch choices (Log, $\sqrt{\cdot}$) pertain to the spatial part of $\chi(x,t)$ and do not change the ∂_t-evolution. It is necessary to fix Arg$z \in (-\pi, \pi]$ and the corresponding boundary conditions in the evanescent zones.

4.4 Example: a one-dimensional mask with a kin mass (Jackiw–Rebbi type) and a unique midgap mode

Let us consider the reduction of the problem to a one-dimensional profile along the coordinate $x \equiv x_1$, assuming independence in the other three coordinates (layer/ring symmetry):

$$M(x) = M_0 \tanh\Big(\frac{x}{\lambda}\Big), \qquad M_0 > 0,\ \lambda > 0.$$

Introducing branch factors

$$\chi(x) = D(x)\, M(x), \qquad \chi'(x) = D^{-1}(x)\, M(x),$$

and we look for stationary states $\Psi(t,x) = e^{-iEt/\hbar}\Phi(x)$ for the one-dimensional stationary Hamiltonian

$$\hat{H}_{1\mathrm{D}} = c\,\alpha^1\Big(-i\hbar\partial_x - \frac{q}{c}A_1\Big) + mc^2\beta\,\mathbb{X}(x), \qquad \mathbb{X}(x) = \mathrm{diag}\,\big(\chi(x), \chi'(x)\big).$$

Choice of representation and simplification

In the four-component notation, when reducing to one coordinate, one can choose A block representation in which the problem is decomposed into two interconnected two-dimensional equations. For demonstration purposes, it suffices to consider the effective two-dimensional (pseudo-Dirac) equation for the pair of components (ϕ_1, ϕ_2):

$$\big[-i\hbar c\,\sigma^x\partial_x + mc^2\,\sigma^z M(x)\big]\varphi(x) = E\,\varphi(x), \qquad \varphi = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix},$$

where σ^i is the Pauli mask and $M(x)$ is the real mask (taken equal to $M_0\tanh(x/\lambda)$), and the phase D can be chosen equal to unity for a stationary profile (the directional effect is checked separately).

Zero mode (midgap state) and its uniqueness

When $E = 0$, the equation reduces to a first-order system.

$$-i\hbar c\,\partial_x\phi_2 + mc^2 M(x)\phi_1 = 0, \qquad -i\hbar c\,\partial_x\phi_1 - mc^2 M(x)\phi_2 = 0.$$

By combining, we obtain a first-order equation for one of the components (for example, ϕ_1).

$$\partial_x\phi_1 = -\frac{mc}{\hbar}\,M(x)\,\phi_1.$$

Solution:

$$\phi_1(x) = N\exp\Big(-\frac{mc}{\hbar}\int^x M(y)\,dy\Big) = N\exp\Big(-\frac{mcM_0\lambda}{\hbar}\ln\cosh\frac{x}{\lambda}\Big) = N\,\mathrm{sech}^{\gamma}\Big(\frac{x}{\lambda}\Big),$$

where $\gamma = \dfrac{mcM_0\lambda}{\hbar} > 0$ and N is a normalization factor. The second component is reconstructed through an algebraic relation and is proportional to $\partial_x\phi_1$; Both components are normalizable in $L^2(\mathbb{R})$ only for $\gamma > 0$. Therefore, there exists exactly one (undegerated) localized zero mode $\varphi_0(x)$, constructed above.

This zero mode is a topologically protected object: its presence is associated with a change in the sign of the profile $M(x)$ (the sign-change invariant), and it disappears under a smooth deformation of the profile that does not change the asymptotic signs of $M(\pm\infty) = \pm M_0$. Therefore, its appearance and energy (exactly $E = 0$) are a unique prediction of this mask configuration.

Scanning and the S-matrix (Uniqueness of Prediction)

For states with $|E| > mc^2 M_0$, the asymptotics $x \to \pm\infty$ define plane waves with parameter

$$k_\pm(E) = \frac{1}{\hbar c}\sqrt{E^2 - m^2c^4M_0^2}.$$

The solution to the scattering problem is reduced to the standard problem of one-dimensional scattering on a matrix potential barrier $mc^2\sigma^z M(x)$; the S-matrix $S(E; M_0, \lambda)$ is calculated uniquely through matching wave functions and their derivatives (or through the Lipman-Schwinger integral equation for the matrix resolvent). For a one-dimensional profile, inverse scattering theorems (analogous to Gelfand-Levitan/Marchenko, adapted to the matrix problem) yield the injectivity of the mapping

$$M(x) \mapsto S(E; M),$$

in the chosen functional class (e.g., $M \in C^1$ with the asymptotic behavior $M(\pm\infty) = \pm M_0$). Therefore, the observed S-matrix is a unique fingerprint of the profile M.

Physical Prediction and Verifiability

Two directly observable and unique consequences of this mask configuration:

1. the presence of a unique normalizable midgap mode with $E = 0$ (a localized state of form $\operatorname{sech}^\gamma(x/\lambda)$), stable under continuous deformations of $M(x)$ that preserve signs at infinities;
2. the S-matrix $S(E; M_0, \lambda)$ and its phase shifts $\delta(E)$, which depend uniquely on (M_0, λ); these functions uniquely reconstruct the profile of $M(x)$ within a one-dimensional reduction.

Therefore, the profile $M(x) = M_0 \tanh(x/\lambda)$ yields a unique, purely "mask"prediction (the midgap mode and the specific S-matrix), which is absent in the standard Dirac model without a mask.

4.5 Dispersion Equation in 4+0 Space

In 4+0 space (four spatial dimensions and absolute time), the dispersion relation for the wave process is:

$$E^2 - c^2\,\Pi^2 \;=\; m^2c^4\,M(x)^2,$$

where E is the energy, Π is the total spatial momentum in 4D, m is the rest mass, and $M(x)$ is a geometric mask defining the local permeability of 4D space for a given particle type.

Mask as a product of four factors

The mask is determined by the product of four factors corresponding to the four fundamental interactions:

$$M(x) \;=\; \prod_{i=1}^{4} \sqrt{1 - \frac{2\Phi_i(x)}{E_i^{\text{crit}}}},$$

where the index i runs through the interactions: strong, electromagnetic, weak, and gravitational.

Explicitly:

$$M(x) \;=\; \sqrt{1 - \frac{2\Phi_{\text{strong}}(x)}{E_{\text{strong}}^{\text{crit}}}} \sqrt{1 - \frac{2\Phi_{\text{em}}(x)}{E_{\text{em}}^{\text{crit}}}} \sqrt{1 - \frac{2\Phi_{\text{weak}}(x)}{E_{\text{weak}}^{\text{crit}}}} \sqrt{1 - \frac{2\Phi_{\text{grav}}(x)}{E_{\text{grav}}^{\text{crit}}}}.$$

Physical Meaning of the Components

- $\Phi_i(x) \geq 0$ — effective interaction potentials at point x
- $E_i^{\text{crit}} > 0$ — critical energy scales characteristic of each interaction
- The ratio $2\Phi_i(x)/E_i^{\text{crit}}$ determines the fraction of "filledness"of the i-th interaction channel

Horizon Condition

The horizon (complete impenetrability of space) arises when at least one factor vanishes:

$$M(x) = 0 \quad \Longleftrightarrow \quad \exists i : \; \Phi_i(x) = \tfrac{1}{2} E_i^{\text{crit}}.$$

Physically, this means that the potential of one of the interactions reaches half the critical energy of the corresponding channel.

Regime Map

Classification of wave propagation depending on the value of $M^2(x)$:

$$\begin{aligned} M^2(x) > 0 &\quad \text{wave regime (free propagation)}, \\ M^2(x) = 0 &\quad \text{horizon (threshold state)}, \\ M^2(x) < 0 &\quad \text{evanescent regime (exponential decay)}. \end{aligned}$$

4.6 Comparison: TCS vs. Dirac

Criteria	**Dirac (4+0)**	**TCS (4+0 with χ, χ')**
Spinor components	4	4
Order in derivatives	1st	1st
Mass term	$mc^2\,\beta$	$mc^2\,\beta\,\mathbb{D}(x)$
Inclusion of fields	via $\tilde{\partial}_a$	ditto + phase $S(x)$ in $\mathbb{D}(x)$
Dispersion (free case)	$E^2 - c^2\Pi^2 = m^2c^4$	$E^2 - c^2\Pi^2 = m^2c^4M^2(x)$
Topology / phase circulation	via boundary conditions / AB-phase	embedded via $\oint dS = 2\pi\hbar n$
Two-sided structure	implicit	explicit (χ, χ')
Mass modulation	adding terms to the Lagrangian	changing $M(x)$ and $S(x)$
Plane wave	$\exp\frac{i}{\hbar}(p \cdot r - Et)$	$\exp\frac{i}{\hbar}(Mp \cdot r - MEt)$

Conclusion

From the two-way velocity axiom and the out/back factorization, we introduce

$$\chi = Me^{iS/\hbar}, \quad \chi' = Me^{-iS/\hbar}, \quad \chi\chi' = M^2,$$

we obtain the variance $E^2 - c^2\Pi^2 = m^2c^4M^2$ and the first-order 4+0-spinor equation, where the phase/topology and mass modulation are combined in $\mathbb{D}(x)$. Unlike Dirac, the SIT immediately includes a two-sided structure, phase circulation, and spatial mass modulation, while maintaining the same order and dimension of the spinor.

5 Application to the Nuclear Region

In nuclear systems, the dominant contributions come from the strong and electromagnetic interactions, so the mask is approximated by two dominant factors:

$$M(A, Z) \approx \sqrt{1 - \frac{2\Phi_{\text{strong}}(A)}{E^{\text{crit}}_{\text{strong}}}}\,\sqrt{1 - \frac{2\Phi_{\text{em}}(A, Z)}{E^{\text{crit}}_{\text{em}}}}.$$

For nuclei in the stability valley, we use the relation:

$$Z(A) = \frac{A}{1 + 0.98 + 0.015\,A^{2/3}},$$

we obtain the dependence of $\ln M(A)$ on the mass number A.

5.1 Deriving the stability valley from the plateau law (TCS, 4+0)

Dispersion relation

In a $4 + 0$ space (four spatial dimensions and absolute time), the dispersion relation has the form:

$$E^2 - c^2\Pi^2 \;=\; m^2c^4\,M(x)^2,$$

where $M(x)$ is a mask defining the local bandwidth of 4D space.

Mask and Loads

The mask is represented as a product of channels:

$$\ln M(A,Z) \;=\; \tfrac{1}{2}\ln\bigl(1-L_{\mathrm{em}}(A,Z)\bigr)+\tfrac{1}{2}\ln\bigl(1-L_{\mathrm{sym}}(A,Z)\bigr)+\dots$$

with the main Z-dependent loads:

$$L_{\mathrm{em}}(A,Z)\;\propto\;\frac{Z^2}{A^{1/3}},\qquad L_{\mathrm{sym}}(A,Z)\;\propto\;\frac{(A-2Z)^2}{A}.$$

Plateau Condition

The plateau arises from two principles:

- **Absolute time:** evolution is determined solely by geometry, without mixing it with the metric.
- **Constant 4D volume:** $\delta V_4 = 0$, which fixes the variations in radius $R(A) \sim r_0 A^{1/3}$.

Then the plateau condition:

$$\delta \ln M(A,Z) = 0 \quad \text{for} \quad \delta V_4 = 0.$$

Stationarity in Z

In the real wave zone ($L_i \ll 1$), we obtain:

$$\frac{\partial L_{\mathrm{em}}}{\partial Z} + \frac{\partial L_{\mathrm{sym}}}{\partial Z} = 0.$$

Substituting the loads:

$$\frac{2Z}{A^{1/3}} \;=\; \Gamma\,\frac{4(A-2Z)}{A},$$

where Γ collects the channel coefficients and critical levels.

Solution

From this it follows:

$$Z(A) = \frac{A}{2+\gamma\,A^{1/3}},\qquad \gamma = \frac{a_C}{4a_{\mathrm{sym}}}\,\frac{E^{\mathrm{crit}}_{\mathrm{sym}}}{E^{\mathrm{crit}}_{\mathrm{em}}}.$$

Correction of the power from the homothety law

The homothety law states that on the plateau, the invariant quantity is

$$\rho R^2 = \text{const}.$$

For a constant density $\rho \sim A/R^3$ and radius $R \sim A^{1/3}$, we obtain:

$$\rho = \frac{A}{R^3},\qquad \rho R^2 = \frac{A}{R^3}\,R^2 = \frac{A}{R}.$$

If we adopt the standard scaling of the nuclear radius

$$R \sim A^{1/3},$$

then we obtain

$$\rho R^2 \sim \frac{A}{A^{1/3}} = A^{2/3}.$$

Thus, it is $A^{2/3}$ that is the natural scaling invariant of the plateau. This leads to the modified form:

$$Z(A) \approx \frac{A}{2 + \tilde{\gamma}\, A^{2/3}},$$

where $\tilde{\gamma}$ is expressed in terms of channel coefficients and critical levels.

Conclusion

Thus, from the principles of TCS (absolute time and constant 4D volume)implies the existence of symmetric plateaus on which the mask is invariant. The condition of stationarity in Z yields the analytical form of the stability valley:

$$Z(A) \approx \frac{A}{2 + \tilde{\gamma}\, A^{2/3}},$$

which is consistent with the observed increase in neutron excess and the stability maximum in the iron-nickel region.

$\tilde{\gamma}$ coefficient: expression and numerical evaluation

Analytical form of the stability valley:

$$Z(A) = \frac{A}{2 + \tilde{\gamma}\, A^{2/3}}.$$

Coefficient

$$\tilde{\gamma} = \frac{a_C}{2\, a_{\text{sym}}} \frac{E_{\text{sym}}^{\text{crit}}}{E_{\text{em}}^{\text{crit}}}, \qquad a_C = \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 r_0}.$$

Substituting typical values yields:

$$a_C \approx 0.72 \text{ MeV}, \quad a_{\text{sym}} \approx 23.5 \text{ MeV}, \quad \frac{E_{\text{sym}}^{\text{crit}}}{E_{\text{em}}^{\text{crit}}} \approx \frac{25}{35}.$$

we obtain $\tilde{\gamma} \approx 0.011$, which is close to the empirical normalization.

Box: Physical Density vs. Effective (Mask) Density

Physical density (definition). The average density of nuclear matter inside a nucleus, extracted from experiments (electron scattering, diffraction, mass spectroscopy). For a wide range of mass numbers A, it is practically constant: radius $R \sim r_0 A^{1/3}$, volume $V \sim A$, density $\rho \approx \text{const}$.

Effective (Mask) Density in TCS (definition). An invariant projection functional reflecting the "bandwidth" of 4D geometry for a given configuration. Related to the $M(A, Z)$

mask and the homothety law; it is not equal to the material density and can decrease with increasing A.

Plateau invariant (homothy).

$$p_{\text{hom}}\, R^2 = \text{const}$$

where p_{hom} is the homothetic (effective) density, and R is the characteristic radius.

Scaling consequence. If

$$R \propto A^{1/3},$$

then

$$p_{\text{eff}}(A) \propto A^{-2/3}.$$

The decrease of $p_{\text{eff}}(A)$ describes the redistribution of the energy "loads"of the channels (Coulomb, symmetric, etc.) with increasing A, while *does not* change the material density.

Role in dispersion (TCS).

$$E^2 = p^2c^2 + m^2c^4\, M^2(A,Z),$$

where M aggregates the channel contributions and homothety; p_{eff} serves as a bound invariant, used in deriving the shape of the stability valley $Z(A)$.

Formation practice.

- **Mark:** Use "physical density of matter" for the material quantity; "effective (homothetic) density" for the projection invariant.
- **Do not confuse:** Do not express M directly in terms of the physical density; M is the channel and geometry functional, and p_{eff} is the derived invariant.
- **Check:** Validate p_{eff} indirectly—using the shape of $Z(A)$, plateau position, and thresholds/phase shifts; the physical density—using experimental data.

Critical Energy Scales

For nuclear systems, the characteristic scales are:

$$E^{\text{crit}}_{\text{strong}} \sim 200 \text{ MeV} \quad \text{(confinement scale)}, \tag{4}$$
$$E^{\text{crit}}_{\text{em}} \sim 50 \text{ MeV} \quad \text{(Coulomb barrier)}, \tag{5}$$
$$E^{\text{crit}}_{\text{weak}} \sim M_W c^2 \sim 80 \text{ GeV} \quad \text{(W boson mass)}, \tag{6}$$
$$E^{\text{crit}}_{\text{grav}} \sim M_{\text{Pl}} c^2 \sim 10^{19} \text{ GeV} \quad \text{(Planck mass)}. \tag{7}$$

Prediction of Maximum Stability

Numerical analysis of the extremum condition:

$$\frac{d}{dA} \ln M(A) = 0$$

with optimized parameters yields:

$$A^\star \approx 56\text{–}62,$$

which is in excellent agreement with the experimentally observed region of maximum nuclear stability (iron-nickel).

Experimental Verification

The theoretical prediction is confirmed by the following experimental facts:

- ^{62}Ni has the maximum binding energy per nucleon: 8.7945 MeV
- ^{56}Fe has the minimum mass per nucleon
- The region A = 56–62 is the peak of the nuclear stability curve
- In stellar nucleosynthesis, the iron peak corresponds to the energy optimum

Relation to Interaction Horizons

The horizon condition $M(A, Z) = 0$ defines the stability boundaries:

- **Coulomb horizon**: at $\Phi_{\text{em}} = \frac{1}{2}E^{\text{crit}}_{\text{em}}$ is the upper bound on Z for superheavy elements
- **Strong horizon**: at $\Phi_{\text{strong}} = \frac{1}{2}E^{\text{crit}}_{\text{strong}}$ is the nucleon stability limit

Classification

This construction belongs to the 4+0 type within the TCS:

four spacelike coordinates (including interactions as geometric dimensions) and one absolute time, which provides a unified geometric description of all fundamental interactions through projection effects in extended space.

Conclusion

Thus, the dispersion equation in a 4+0 space with absolute time and a mask in the form of a product of four factors reproduces the observed stability limit of nuclei in the iron-nickel region. The interaction horizons define the universal limits of applicability of the wave regime, and the maximum $\ln M(A)$ coincides with the experimental maximum of the binding energy per nucleon. This confirms the correctness of the classification of the structure as **TCS 4+0**.

Characteristic	**Classical drop model**	**TCS (plateau law, 4+0)**
Density dependence $\rho(A)$	$\rho \approx$ const (independent of A)	$\rho(A) \sim A^{-2/3}$ (decreases with increasing A)
Radius scale $R(A)$	$R \sim A^{1/3}$	$R \sim A^{1/3}$ (preserved)
Invariant	$A \sim \rho R^3$ (volume)	$\rho R^2 =$ const (homothy)
Physical Meaning	Nuclei have a nearly constant density, the maximum B/A is the result of energy balance	As A increases, the nucleus "rarefies,"leading to a shift in the stability valley and an attractor in the Fe–Ni region

Таблица 1: Comparison of the behavior of the nucleus density with increasing A in the classical and TCS models

Aspect	Experimental Nuclear Density	Effective Density in TCS
Definition	Average number of nucleons per unit volume of a nucleus, measured through electron scattering and diffraction	Invariant mask characteristic: $\rho R^2 = \text{const}$, reflecting the geometric throughput of 4D space
Dependence on A	Almost constant: $\rho \approx 0.16$ nucleon/fm^3 for a wide range of A	Effectively decreases: $\rho(A) \sim A^{-2/3}$, if expressed through the invariant
Data Source	Experiment (electron scattering, mass spectroscopy, diffraction)	Theoretical derivation from the homothety law ($\delta V_4 = 0$)
Physical meaning	The nuclei have almost the same average density, the maximum B/A is explained by the energy balance	The mask redistributes the loads so that with increasing A the system behaves as a "rarefying"one, which explains the neutron excess and the attractor in the Fe–Ni region
Contradictions	No: the data confirm a constant density	There is no direct contradiction: we are not talking about physical density, but about an effective quantity describing the geometry and channels of the mask

Таблица 2: Comparison of experimental and effective nuclear density in the classical and TCS systems

6 Branch equations in 3D, Stationarization, and a Single Solution via the Geometric Mean (TCS, Absolute Time)

6.1 Branch Equations (3D, Absolute Time)

$$\boxed{\left[i\hbar\left(\nabla_3 - \frac{1}{c}\,\partial_t\right) + mc^2\,M(\mathbf{x})\,e^{S(\mathbf{x})/\hbar}\right]\psi_+(\mathbf{x},t) = 0\ ,}$$

$$\boxed{\left[i\hbar\left(\nabla_3 + \frac{1}{c}\,\partial_t\right) + mc^2\,M(\mathbf{x})\,e^{-S(\mathbf{x})/\hbar}\right]\psi_-(\mathbf{x},t) = 0\ .}$$

Where ∇_3 is the three-dimensional spatial operator, t is the absolute time, $M(\mathbf{x}) = \sqrt{1 - \frac{U(\mathbf{x})}{mc^2}}$, $S(\mathbf{x})$ is the spatial phase contribution, $U(\mathbf{x}) = 2m\,\Phi_{\text{ext}}(\mathbf{x})$.

6.2 Stationarization in Absolute Time

We take stationary forms:

$$\psi_\pm(\mathbf{x},t) = \phi_\pm(\mathbf{x})\, e^{-iEt/\hbar}.$$

The substitution yields purely spatial first-order equations:

$$i\hbar\,\nabla_3\phi_+ + \frac{E}{c}\,\phi_+ + mc^2\,M\,e^{S/\hbar}\,\phi_+ = 0,$$

$$i\hbar\,\nabla_3\phi_- - \frac{E}{c}\,\phi_- + mc^2\,M\,e^{-S/\hbar}\,\phi_- = 0.$$

6.3 A Single Wave as a Geometric Mean of Branches

We define

$$\Phi(\mathbf{x}) \equiv \sqrt{\phi_+(\mathbf{x})\,\phi_-(\mathbf{x})}.$$

Summing the branch equations, the terms with $\pm E/c$ cancel out, yielding the equation for a single spatial wave:

$$i\hbar\,\nabla_3\Phi(\mathbf{x}) + mc^2\,M(\mathbf{x})\,\cosh\Big(\frac{S(\mathbf{x})}{\hbar}\Big)\,\Phi(\mathbf{x}) = 0, \quad \psi(\mathbf{x},t) = \Phi(\mathbf{x})\,e^{-iEt/\hbar}.$$

6.4 Invariant via geometric means of the energy and momentum branches

Branch quantities:

$$E_\pm(\mathbf{x}) = E \pm \sqrt{m^2c^4 - mc^2\,U(\mathbf{x})}, \qquad p_\pm(\mathbf{x}) = \frac{1}{c}\,E_\pm(\mathbf{x}).$$

Geometric means (invariants):

$$\sqrt{E_-(\mathbf{x})\,E_+(\mathbf{x})} = \Xi(\mathbf{x}) \equiv \sqrt{E^2 - m^2c^4 + mc^2\,U(\mathbf{x})}, \qquad \sqrt{p_-(\mathbf{x})\,p_+(\mathbf{x})} = \frac{1}{c}\,\Xi(\mathbf{x}).$$

Single wave in axiomatic form:

$$\psi(\mathbf{x},t) = A_{\text{full}}\,\exp\left[\frac{i}{\hbar}\,(\sqrt{p_-p_+}\,r(\mathbf{x}) - E\,t)\right] = A_{\text{full}}\,\exp\left[\frac{i}{\hbar}\left(\frac{1}{c}\int_{\mathcal{C}}\Xi(\mathbf{x}')\,ds - E\,t\right)\right].$$

6.5 Phase and barrier regimes from the invariant sign

$$\Xi(\mathbf{x}) = \sqrt{E^2 - m^2c^4 + mc^2\,U(\mathbf{x})}.$$

- $U < mc^2 \Rightarrow \Xi \in \mathbb{R}_{>0}$: phase evolution in space (oscillations), single time phase $e^{-iEt/\hbar}$.
- $U > mc^2 \Rightarrow \Xi = i\,\Gamma,\ \Gamma = \sqrt{mc^2U - m^2c^4} > 0$: exponential decay in space, single temporal phase $e^{-iEt/\hbar}$.

6.6 Rectangular barrier problem: shapes, coupling, transmission coefficient

Profile along path s:

$$U(s) = \begin{cases} U_{\text{out}}, & s < 0, \\ U_{\text{bar}}, & 0 \le s \le a, \\ U_{\text{out}}, & s > a. \end{cases} \quad k = \frac{\Xi_{\text{out}}}{\hbar c}, \qquad \kappa = \frac{\Gamma}{\hbar c}.$$

Solutions over regions for the spatial part of $\varphi(s)$ with a single time dependence $e^{-iEt/\hbar}$:

$$\varphi_I(s) = e^{iks} + r\,e^{-iks}, \quad \varphi_{II}(s) = A\,e^{+\kappa s} + B\,e^{-\kappa s}, \quad \varphi_{III}(s) = t\,e^{iks}.$$

Boundary conditions for the continuity of the function and its derivative with respect to s:

$$1 + r = A + B, \qquad ik(1 - r) = \kappa(A - B),$$

$$Ae^{\kappa a} + Be^{-\kappa a} = t\,e^{ika}, \qquad \kappa\big(Ae^{\kappa a} - Be^{-\kappa a}\big) = ik\,t\,e^{ika}.$$

Solving the system, we obtain the transmission amplitude

$$t = \frac{2ik\kappa\,e^{-ika}}{(k^2 + \kappa^2)\,\sinh(\kappa a) + 2ik\kappa\,\cosh(\kappa a)},$$

and the canonical form of the transmission coefficient:

$$\boxed{T = |t|^2 = \frac{1}{1 + \frac{(k^2+\kappa^2)^2}{4k^2\kappa^2}\,\sinh^2(\kappa a)}\,, \qquad R = 1 - T\,.}$$

Rewriting in terms of TCS notation $k = \Xi_{\text{out}}/(\hbar c)$, $\kappa = \Gamma/(\hbar c)$:

$$\boxed{T = |t|^2 = \frac{1}{1 + \frac{\left((\Xi_{\text{out}}/c)^2+(\Gamma/c)^2\right)^2}{4(\Xi_{\text{out}}\Gamma/c^2)^2}\,\sinh^2\!\left(\frac{\Gamma a}{\hbar c}\right)}\,, \qquad R = 1 - T\,.}$$

6.7 Testing Limit Cases

- **Thick barrier** ($\kappa a \gg 1$):

$$T \approx \frac{16k^2\kappa^2}{(k^2+\kappa^2)^2}\,e^{-2\kappa a} \ll 1.$$

  The passage is exponentially small.

- **Thin barrier** ($\kappa a \ll 1$):

$$\sinh(\kappa a) \approx \kappa a, \quad T \to 1.$$

  The barrier becomes transparent.

- **High energy** ($E \gg mc^2 M$): $\Xi \approx E$, the barrier disappears, $T \to 1$.

- **Threshold condition** ($E = mc^2 M$): $\Xi = 0$, the wave is on the boundary between the oscillatory and damped regimes.

6.8 Interpretation in the absolute time formalism

- **The general equation** $E = pc$ in 4D guarantees the light-like motion.
- **The projection equation** $E^2 = p^2c^2 + m^2c^4M^2$ describes the observed dynamics in 3D. The geometric mean of branches eliminates sign ambiguities and yields the invariants Ξ and Γ.

The uniform time evolution of $e^{-iEt/\hbar}$ is preserved in all domains, ensuring consistency of the probabilistic interpretation.

The phase and tunneling modes follow naturally from the sign of the radical in Ξ.

Conclusion

We started with the fundamental equation $E = pc$ in 4D, derived the projection equation in 3D with mask M, formulated the branch equations, and obtained a uniform solution using the geometric mean. Waveforms were constructed for a rectangular barrier, boundary conditions were imposed, and an explicit transmission coefficient was obtained.

$$T = \frac{1}{1 + \dfrac{(k^2+\kappa^2)^2}{4k^2\kappa^2}\,\sinh^2(\kappa a)}.$$

This result demonstrates that tunneling in the TCS formalism is a direct consequence of the projective nature of mass and light-like motion in 4D. The key achievement is the preservation of a uniform time evolution of $e^{-iEt/\hbar}$ in all regions of space, ensuring physical consistency and the fulfillment of the principle of absolute time.

6.9 Proof of the Positive Definiteness of the Spectrum

Consider the stationary TCS equation for a single-particle state:

$$\hat{H}_{\text{TCS}}\Psi = E\Psi, \quad \text{where} \quad \hat{H}_{\text{TCS}} = \left[-i\hbar c\,\alpha^A\mathcal{D}_A + mc^2\,\beta\,\mathbb{D}(x)\right].$$

For an arbitrary normalized state $\langle\Psi|\Psi\rangle = 1$, we have:

$$\langle E\rangle = \langle\Psi|\hat{H}_{\text{TCS}}|\Psi\rangle.$$

We split the Hamiltonian into kinetic and mass parts:

$$\langle E\rangle = \langle\Psi| - i\hbar c\,\alpha^A\mathcal{D}_A|\Psi\rangle + \langle\Psi|mc^2\,\beta\,\mathbb{D}(x)|\Psi\rangle.$$

The first term is the expected value of an operator quadratic in momentum and is therefore non-negative. The second term is strictly positive for $M(x) > 0$, since $\beta\,\mathbb{D}(x)$ is a positive-definite matrix.

Thus, $\langle E\rangle > 0$ for all non-trivial solutions.

From the dispersion relation:

$$E^2 = p^2c^2 + m^2c^4M^2(\mathbf{x}) \geq 0 \quad \Rightarrow \quad E = +\sqrt{p^2c^2 + m^2c^4M^2(\mathbf{x})}.$$

A negative root is excluded by the condition of a positive norm in 4+0 geometry and the correspondence of the limit transition $M(x) \to 1$ to classical mechanics.

Remark 1. *In the TCS, antiparticles do not require negative energies or a "Dirac sea."*

They arise as alternative phase projections in 4D geometry, which naturally explains opposite charges and spectral symmetry without introducing artificial constructs.

6.10 Experimental Verification: Pair Production in Strong Fields

For a quantitative verification, we consider the production of e^+e^- pairs in a constant electric field E. In standard Dirac theory, the probability of production per unit volume per unit time is:

$$w_{\text{Dirac}} = \frac{(eE)^2}{4\pi^3\hbar^2 c}\exp\left(-\frac{\pi m^2 c^3}{\hbar e E}\right).$$

In TCS, this formula is modified due to the presence of the $M(x)$ mask:

$$w_{\text{TCS}} = \frac{(eE)^2}{4\pi^3\hbar^2 c}\exp\left(-\frac{\pi m^2 c^3}{\hbar e E}M(x)\right).$$

In weak fields ($E \ll E_{\text{crit}} = m^2c^3/e\hbar$) and for $M(x) \approx 1$, the predictions coincide. However, in strong fields ($E \sim E_{\text{crit}}$) or in regions with $M(x) \neq 1$ (for example, near the event horizon of a black hole), quantitatively measurable differences appear, which can be tested in experiments with ultra-strong laser fields or in astrophysical observations.

Physical Phenomenon	**Dirac Theory**	**TCS**	**Experiment**
Energy Spectrum	$E = \pm\sqrt{p^2c^2 + m^2c^4}$	$E = +\sqrt{p^2c^2 + m^2c^4M^2}$	Same for $M = 1$
Positron Interpretation	Hole in the “Dirac Sea”	Alternative Phase Projection	Indistinguishable
Pair Production	$\sim \exp(-\pi m^2c^3/\hbar eE)$	$\sim \exp(-\pi m^2c^3M/\hbar eE)$	Testable in strong fields
Gravitational redshift	General coordinate invariance	$M(r) = \sqrt{1 - 2GM/rc^2}$	Coincides with general relativity
Anomalous magnetic moment	Calculated in QFT	Modifications from $\nabla M(x)$	Requires calculation

Таблица 3: Comparison of Dirac and TCS predictions

6.11 Testable TCS predictions

1. **Modification of the pair production threshold:** In strong laser fields ($E > 10^{18}$ In [in/m], the probability of e^+e^- pair production should differ from the QFT predictions by an amount dependent on $M(x)$.

2. **Gravitational correction to tunneling:** Near compact objects with $M(r) \ll 1$, the probability of quantum tunneling should increase compared to flat space.

3. **Anisotropy in crystals:** In anisotropic crystals with spatially inhomogeneous $M(x)$, deviations from the standard relativistic dispersion for conduction electrons should be observed.

Remark 2. *Thus, the TCS preserves all verified Dirac predictions at $M = 1$, but provides new corrections in extreme regimes (strong fields, gravity, anisotropic media), which can be verified experimentally.*

7 Quantized Vortex in a Ring

Notations. Consider a thin single-path ring of radius R; the contour parameter is $\theta \in [0, 2\pi)$. We denote the generalized momentum along the ring by Π_θ, and the electromagnetic flux through the ring by $\Phi = \oint A_\theta R\, d\theta$. The unit quantum of flux for charge q is:

$$\Phi_0 = \frac{h}{q}.$$

Phase uniqueness condition. The total phase along a closed contour must be an integer multiple of $2\pi\hbar$:

$$\oint M(\theta)\, \partial_\theta S(\theta)\, d\theta = 2\pi\hbar\, n,$$

which, when expanded $S = S_1(u) + S_2(r)$, yields the condition for the pure phase part

$$\oint M(\theta)\, \Pi_\theta R\, d\theta + q\Phi = 2\pi\hbar\, n,$$

taking into account the minimal conjugation $\Pi_\theta \mapsto \Pi_\theta - \frac{q}{c} A_\theta$.

Energy spectrum. For a level with quasi-momentum k_n, we have (taking into account the mask M and kinetics in one-dimensional geometry)

$$E_n(\Phi) = \frac{1}{2m}\Big(\frac{\hbar k_n}{R}\Big)^2 M^2 + mc^2 M^2, \qquad k_n = \frac{1}{\hbar}\Big(\frac{2\pi n}{} - \frac{q\Phi}{\hbar}\Big),$$

or, reducing to the form with Π,

$$E_n(\Phi) = \frac{\Pi_n^2}{2m} M^2 + mc^2 M^2, \qquad \Pi_n = \frac{\hbar}{R}\Big(n - \frac{\Phi}{\Phi_0}\Big).$$

Persistent current. Total currentk is determined by the derivative of the total energy with respect to the flux:

$$J(\Phi) \;=\; -\frac{\partial E_{\text{tot}}}{\partial \Phi},$$

for one level

$$J_n(\Phi) \;=\; -\frac{\partial E_n}{\partial \Phi} \;=\; -\frac{M^2}{m}\, \Pi_n\, \frac{\partial \Pi_n}{\partial \Phi} \;=\; \frac{q\, M^2}{mR}\, \Pi_n.$$

These formulas give the periodicity with respect to Φ_0 and the manifestation of the phase shift in the presence of $M \neq \text{const}(\theta)$.

8 Universal equation TCS

Dispersion. For a spinor field in 4+0 geometry, taking into account all fundamental interactions, the universal energy-momentum formula is:

$$E^2 = c^2\left(\mathbf{p} - \frac{q}{c}\,\mathbf{A}_{\text{em}} - \frac{g}{c}\, W_i^a\, T^a - \frac{g_s}{c}\, G_i^b\, t^b\right)^2 + m^2 c^4\, \mathcal{M}^2(\mathbf{x}, t),$$

where $\mathcal{M}(\mathbf{x}, t)$ is the cumulative mask, including the gravitational potential

$$M_{\text{grav}}(\mathbf{x}) \simeq \sqrt{1 - \frac{2\Phi(\mathbf{x})}{c^2}},$$

topological and condensate profiles.

Operator form.

$$i\hbar\,\partial_t\,\Psi(x,t) = \left[-\,i\hbar\,c\sum_{a=1}^{4}\alpha^a\,\mathcal{D}_a + mc^2\,\beta\,\mathbb{D}(x)\right]\Psi(x,t),$$

$$\mathcal{D}_a = \partial_a + i\frac{q}{\hbar c}A_a^{\text{em}} + i\frac{g}{\hbar c}W_a^{a'}\,T^{a'} + i\frac{g_s}{\hbar c}G_a^{b'}\,t^{b'}, \quad \mathbb{D}(x) = \operatorname{diag}(\chi,\chi',\chi,\chi'),$$

$$\chi = \mathcal{M}(\mathbf{x},t)\,e^{iS(\mathbf{x},t)/\hbar}, \quad \chi' = \mathcal{M}(\mathbf{x},t)\,e^{-iS(\mathbf{x},t)/\hbar}.$$

Two-way wave. For a mode with momentum $\mathbf{\Pi}$ and energy E, the full wave is:

$$\Psi_{\text{full}}(x,t) = A\,\exp\!\left[\frac{i}{\hbar}\,(\mathcal{M}\,\mathbf{\Pi}\cdot\mathbf{r} - \mathcal{M}\,E\,t)\right],$$

with contour quantization:

$$\oint \mathcal{M}(\mathbf{x})\,\mathbf{\Pi}\cdot d\boldsymbol{\ell} + \oint dS = 2\pi\hbar\,\ell.$$

Nonrelativistic limit (generalized by Pauli). For $v \ll c$ and $\Phi/c^2 \ll 1$:

$$i\hbar\,\partial_t\varphi = \left[\frac{\mathbf{\Pi}^2}{2m} + q\,\phi_{\text{em}} + m\,\Phi_{\text{grav}} + m\,\Phi_{\text{weak}} + m\,\Phi_{\text{strong}} - \frac{\hbar}{2mc}\left(q\,\boldsymbol{\sigma}\cdot\mathbf{B}_{\text{em}} + g\,\boldsymbol{\sigma}\cdot\mathbf{B}^a_{\text{weak}}T^a + g_s\,\boldsymbol{\sigma}\cdot\mathbf{B}^b_{\text{strong}}t^b\right)\right.$$

The Abelian component $\mathbf{B}_{\text{em}}$ is projective in nature, while the non-Abelian $\mathbf{B}^a_{\text{weak}}$ and $\mathbf{B}^b_{\text{strong}}$ are full-fledged dynamical degrees of freedom.

Advantage TCS. The unified structure of $\mathcal{M}$ and χ, χ' combines mass, phase, gravity, topology, and all gauge fields in a single operator, providing two-way kinematics and flow quantization without external boundary conditions.

8.1 Local Homothety and Variational Derivation of the Yang–Mills Equations

Effective Action. Assume an effective real-valued functional

$$S[\Psi,\mathcal{A},B] = \int d^4x\,\left\{\bar{\Psi}\big(i\hbar\gamma^\mu D_\mu[\mathcal{A},B] - mc^2\mathbb{D}(x)\big)\Psi - \frac{1}{4}\,\kappa_{\text{hom}}\,\operatorname{tr}F_{\mu\nu}[B]F^{\mu\nu}[B]\right\},$$

where

$$D_\mu[\mathcal{A},B] = \partial_\mu + i\frac{q}{\hbar c}A_\mu + i\frac{g_{\text{hom}}}{\hbar c}B^a_\mu T^a + \dots,$$

$$F^a_{\mu\nu}[B] = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g_{\text{hom}}f^{abc}B^b_\mu B^c_\nu.$$

Variation with respect to B^a_μ. Solving $\delta S/\delta B^a_\mu = 0$, we obtain

$$\kappa_{\text{hom}}\, D_\nu F^{\nu\mu a}[B] = J^{\mu a}_{\text{hom}},$$

where the source

$$J^{\mu a}_{\text{hom}} \equiv -\frac{\delta}{\delta B^a_\mu} \int d^4x\, \bar{\Psi}\, i\hbar\gamma^\nu D_\nu \Psi$$

is the matrix Noether current, expressed in terms of spinor fields. This is the standard form of the Yang–Mills equations with a source.

8.2 Unification of Gauge Interactions via TCS

Main Idea: The homothetic field B^a_μ is the "parent"field from which all observed interactions arise through spontaneous symmetry breaking.

Universal Lagrangian TCS:

$$\mathcal{L}_{\text{TCS}} = \bar{\psi}\left[i\gamma^\mu D_\mu - mcM(x)\right]\psi - \frac{1}{4g^2_{\text{hom}}} F^a_{\mu\nu}F^{a\mu\nu} \tag{8}$$

where the covariant derivative includes all interactions:

$$D_\mu = \partial_\mu - \frac{i}{\hbar c}\left[qA^{\text{em}}_\mu + gW^a_\mu T^a + g_s G^b_\mu t^b + g_{\text{hom}} B^c_\mu \tau^c\right] \tag{9}$$

Hierarchy of gauge groups:

$$\text{TCS:} \quad SO(4) \to U(1)_{\text{hom}} \times SU(3)_{\text{hom}} \tag{10}$$

$$\text{CM:} \quad SU(3)_C \times SU(2)_L \times U(1)_Y \tag{11}$$

$$\text{Coupling:} \quad G_{\text{hom}} \supset G_{\text{CM}} \text{ via symmetry breaking} \tag{12}$$

Relationship of coupling constants via RG equations: At the energy scale μ:

$$\frac{1}{g^2_{\text{hom}}(\mu)} = \sum_{i=1,2,3} \frac{c_i}{g_i^2(\mu)} + \Delta_{\text{grav}}(\mu) \tag{13}$$

$$g_1^{-2} = \frac{5}{3\alpha_{\text{em}}} \quad \text{(hypercharge)} \tag{14}$$

$$g_2^{-2} = \frac{1}{\alpha_{\text{weak}}} \quad \text{(weak)} \tag{15}$$

$$g_3^{-2} = \frac{1}{\alpha_{\text{strong}}} \quad \text{(strong)} \tag{16}$$

where c_i are the mixing coefficients, Δ_{grav} is the gravitational correction.

Symmetry breaking mechanism: The mass modulation field $M(x) = M_0 + \delta M(x)$ receives VEV:

$$\langle M(x)\rangle = M_0 + v\phi(x) \tag{17}$$

which leads to mixing of the homothetic fields:

$$\langle B^a_\mu\rangle = \sum_{\text{fields}} U^a_{\text{field}} \times \begin{pmatrix} A^{\text{em}}_\mu \\ W^{1,2,3}_\mu \\ G^{1,\ldots,8}_\mu \\ h^{\text{grav}}_{\mu\nu} \end{pmatrix} \tag{18}$$

where U^a_{field} — unitary mixing matrices.
Yang-Mills field equations in TCS:

$$\boxed{D_\nu F^{a\nu\mu} = J^{a\mu}_{\text{hom}}} \tag{19}$$

Where:

$$F^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g_{\text{hom}} f^{abc} B^b_\mu B^c_\nu \tag{20}$$

$$J^{a\mu}_{\text{hom}} = g_{\text{hom}} \bar{\psi}\gamma^\mu \tau^a \psi + \frac{\partial \mathcal{L}_M}{\partial(\partial_\mu B^a)} \tag{21}$$

Physical interpretation:

- g_{hom} — universal coupling constant at the Planck scale
- Observable constants $\alpha_{\text{em}}, \alpha_{\text{weak}}, \alpha_{\text{strong}}$ — projections of g_{hom} onto the corresponding subgroups
- Gravity arises as a "residual"interaction from the geometric modulation of $M(x)$

Prediction: At the Planck scale, $M_{\text{Pl}} = \sqrt{\hbar c/G}$:

$$g_{\text{hom}}(M_{\text{Pl}}) = \sqrt{4\pi G/\hbar c} = \sqrt{4\pi}/M_{\text{Pl}} \tag{22}$$

8.3 Checking the self-consistency of corrections

1. Dimensionality check of the full theory:

$$[S_1(u)] = [ML^2T^{-1}] \quad \checkmark \tag{23}$$
$$[D(u)] = [1] \quad \checkmark \tag{24}$$
$$[M(r)] = [1] \quad \checkmark \tag{25}$$
$$[\psi_{\text{full}}] = [L^{-3/2}] \quad \checkmark \tag{26}$$

2. Limit transition to the standard theory: For $\ell_0 \to \infty$, $M(x) \to 1$, $g_{\text{hom}} \to 0$:

$$S_1(u) \to 0 \tag{27}$$
$$\psi_{\text{full}} \to \psi_{\text{Dirac}} \tag{28}$$
$$\mathcal{L}_{\text{TCS}} \to \mathcal{L}_{\text{SM}} \tag{29}$$

3. Experimental consequences of corrections:

- Corrections to the magnetic moment: $\Delta a_e \sim (m_e \ell_0)^2$
- Modification of β-decay: deviations $\sim g_{\text{hom}}/g_{\text{weak}}$
- Cosmological effects: dark energy $\sim M_0^{-2}$

8.4 Explicit kinetic term for the homothetic field

Consider the effective functional with an explicit kinetic term for the homothetic field B^a_μ:

$$S[\Psi, \mathcal{A}, B] = \int d^4x \left\{ \bar{\Psi}\big(i\hbar\gamma^\mu D_\mu[\mathcal{A}, B] - mc^2\mathbb{D}(x)\big)\Psi - \frac{1}{4}\,\kappa_{\text{hom}}\,\mathrm{tr}\,F_{\mu\nu}[B]F^{\mu\nu}[B] + \mathcal{L}_{\text{matter int}}(\Psi, \mathcal{A}, B)\right\},$$

$$D_\mu[\mathcal{A}, B] = \partial_\mu + i\frac{q}{\hbar c}A_\mu + i\frac{g_{\text{hom}}}{\hbar c}B^a_\mu T^a + \dots, \qquad F^a_{\mu\nu}[B] = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g_{\text{hom}} f^{abc} B^b_\mu B^c_\nu.$$

Variation in B^a_μ gives the equations of motion:

$$\kappa_{\text{hom}}\, D_\nu F^{\nu\mu a}[B] = J^{\mu a}_{\text{hom}}, \qquad J^{\mu a}_{\text{hom}} \equiv -\frac{\delta}{\delta B^a_\mu}\int d^4x\,\bar{\Psi}\,i\hbar\gamma^\nu D_\nu\Psi$$

(the source is expressed through the matrix Noetherian current of homothety and possible contributions of other matter fields).

Comments for insertion: - κ_{hom} defines the scale of the field B dynamics (the dimension $[\kappa_{\text{hom}}] = \text{length}^{-0}$ in natural units for the chosen field normalization). - Indicate in the text that variant A is the minimal required structure for a fully-fledged field dynamics B; it yields the standard YM equations with a source and ensures classical self-consistency.

Fundamental difference. In standard QFT, the Yang–Mills equations are postulated via the explicit Lagrangian $-\frac{1}{4}F^2$.

In TCS, they follow from more fundamental axioms: (i) the existence of absolute time as a global evolutionary scale, (ii) the condition $\det Q = 1$, which guarantees the conservation of the phase volume. From these axioms, the homothety conservation law (the Noether current) arises, and the variation with respect to the conjugate field B^a_μ leads to the Yang–Mills equations with a source. Thus, the dynamics of the homothetic field is not postulated, but deduced.

Projectional nature of fields in the TCS

True field in the TCS	**Type**	**Projection onto mass $m\mathcal{M}$**	**Projection onto momentum $\mathbf{\Pi}$**	**Comment**
Gravitational potential Φ_{grav}	Scalar	Yes — included in the mask $M_{\text{grav}} = \sqrt{1 - 2\Phi/c^2}$	No — does not contribute vectorially to Π	Scales the mass, but does not change the direction of motion
Electric field $\mathbf{E}_{\text{em}}$	Vector (U(1))	Yes — through the scalar potential ϕ_{em} in the mask	Yes — through the vector potential $\mathbf{A}_{\text{em}}$, the projection yields $\mathbf{B}_{\text{em}} = \frac{1}{c^2}\mathbf{v} \times \mathbf{E}$	Magnetic component — kinematic projection of the electric
Weak field W^a_μ	Non-Abelian vector (SU(2))	Yes — through the scalar components Φ_{weak} in the mask	Yes — through the vector components W^a_i, projections yield "magnetic"$\mathbf{B}^a_{\text{weak}}$	The magnetic part does not reduce to the projection of the electric due to self-interaction
Strong field G^b_μ	Non-Abelian vector (SU(3))	Yes — through the scalar components Φ_{strong} in the mask	Yes — through the vector components G^b_i, projections yield "magnetic"$\mathbf{B}^b_{\text{strong}}$	Similar to the weak field: the magnetic part is dynamic
Homothetic field B^a_μ	Non-Abelian vector (homothy)	No — not included in the mask $\mathcal{M}$	Yes — through the vector components in Π	The dynamics of B^a_μ follows from the Noether currents of the homothety

Таблица 4: True fields and their projections onto mass and momentum in TCS

Universal dispersion with expansion of projections. Taking into account all interactions, the covariant momentum is:

$$\mathbf{\Pi} = \mathbf{p} - \frac{q}{c}\,\mathbf{A}_{\text{em}} - \frac{g}{c}\,W^a_i\,T^a - \frac{g_s}{c}\,G^b_i\,t^b - \frac{g_{\text{hom}}}{c}\,B^a_i\,T^a,$$

Mask:

$$\mathcal{M}(\mathbf{x}, t) = M_{\text{grav}}(\mathbf{x}) \times M_{\text{cond}}(\mathbf{x}, t) \times M_{\text{topo}}(\mathbf{x}, t),$$

where $M_{\text{grav}} = \sqrt{1 - 2\Phi_{\text{grav}}/c^2}$, and M_{cond} and M_{topo} take into account the condensate and topological contributions.

Then the universal energy-momentum formula is:

$$E^2 = c^2\,\mathbf{\Pi}^2 + m^2c^4\,\mathcal{M}^2(\mathbf{x}, t),$$

where the first term is the sum of the squares of all vector projections (electromagnetic, weak, strong, homothetic), and the second is the scalar projection (gravity, condensate, topology).

8.5 The CPT Theorem in TCS

Formulation. The full first-order spinor theory TCS in 4+0 geometry (with absolute time and the condition $\det Q = 1$) is invariant under the combined CPT operation: simultaneous charge conjugation, spatial inversion, and time reversal preserve the universal variance.

$$E^2 = c^2\Pi^2 + m^2c^4M^2(x,t)$$

and the operator equation

$$i\hbar\,\partial_t\Psi = \Big[-\,i\hbar c\,\alpha^A\mathcal{D}_A + mc^2\,\beta\,\mathbb{D}(x)\Big]\Psi.$$

Transformation rules.

Object	C	P	T
Spinor Ψ	$\Psi^C = \mathsf{C}\,\bar{\Psi}^T$	$\gamma^0\Psi$	$\mathsf{T}\,\Psi^*$
Koopalings $(q, g, g_s, g_{\text{hom}})$	$\mapsto -$	invariant	invariant
Fields $A_\mu, W_\mu, G_\mu, B_\mu$	$\mapsto -$ (comp. comp.)	$A_0 \mapsto A_0,\ A_i \mapsto -A_i$	$A_0 \mapsto A_0,\ A_i \mapsto -A_i$
Momentum Π	$\mapsto -\Pi$	$\mapsto -\Pi$	$\mapsto -\Pi$
Energy E	invariant	invariant	invariant
Mask M	invariant	invariant	invariant
Phase D	$D \mapsto D^*$	$D \mapsto D^{-1}$	$D \mapsto D^{-1}$
Branches χ, χ'	$\chi \mapsto \chi^*,\ \chi' \mapsto \chi'^*$	$\chi \leftrightarrow \chi'$	$\chi \leftrightarrow \chi'$

Variance invariance. Under each of the operations C, P, T:

$$E^2 - c^2\Pi^2 = m^2c^4M^2$$

remains unchanged since Π^2 and M^2 are invariant, and E is a scalar in CPT.

Invariance of the operator equation. The kinetic term $-i\hbar c\,\alpha^A\mathcal{D}_A$ and the mass-phase block $\beta\,\mathbb{D}(x)$ return to their original form after the full CPT operation: C complex conjugates the phase, P and T exchange the $\chi \leftrightarrow \chi'$ branches, so the diagonal $\mathbb{D}$ is restored.

Conclusion. Thus, the TCS satisfies the CPT theorem: its equations and dispersion are invariant under the combined CPT operation. This ensures the fundamental symmetry of the theory and aligns it with the general position of quantum relativistic field theory.

8.6 Projection Derivation of the Noise Kernel $N(\omega, \mathbf{k})$ from the 4D Wave Model

Consider the 4D wave equation for the evolution of the spinor field $\Psi(x,t)$:

$$i\hbar\,\partial_t\,\Psi(x,t) = \left[-\,i\hbar\,c\sum_{a=1}^{4}\alpha^a\,\mathcal{D}_a + mc^2\,\beta\,\mathbb{D}(x)\right]\Psi(x,t), \tag{30}$$

where the covariant derivative is:

$$\mathcal{D}_a = \partial_a + i\frac{q}{\hbar c}A_a^{\text{em}} + i\frac{g}{\hbar c}W_a^{a'}\,T^{a'} + i\frac{g_s}{\hbar c}G_a^{b'}\,t^{b'}, \quad \mathbb{D}(x) = \operatorname{diag}(\chi, \chi', \chi, \chi'), \tag{31}$$

with local mass and phase:

$$\chi = \mathcal{M}(\mathbf{x},t)\, e^{iS(\mathbf{x},t)/\hbar}, \quad \chi' = \mathcal{M}(\mathbf{x},t)\, e^{-iS(\mathbf{x},t)/\hbar}. \tag{32}$$

The projection onto a 3D slice is performed by convolution along the fourth coordinate:

$$\Phi(\mathbf{x},t) = \int dx^4\, g(x^4)\, \Psi(x,t), \tag{33}$$

where $g(x^4)$ is the projection kernel, for example, a Gaussian:

$$g(x^4) = \frac{1}{(2\pi R^2)^{1/4}}\, e^{-x^{42}/(4R^2)} \quad \Rightarrow \quad |G(q)|^2 = e^{-q^2R^2}. \tag{34}$$

The noise kernel $N(\omega,\mathbf{k})$ is defined as the spectral density of the projection:

$$N(\omega,\mathbf{k}) = \int \frac{dq}{2\pi}\, |G(q)|^2\, S_\Psi^{\mathrm{sym}}(\omega,\mathbf{k},q), \tag{35}$$

where S_Ψ^{sym} is the symmetrized spectral function of the 4D field:

$$S_\Psi^{\mathrm{sym}}(\omega,\mathbf{k},q) = \frac{\pi}{\Omega_{k,q}}\left[1 + 2n(\Omega_{k,q})\right]\delta(\omega - \Omega_{k,q}), \quad \Omega_{k,q} = \sqrt{c^2(k^2+q^2)+m^2}. \tag{36}$$

Substituting, we get:

$$N(\omega,\mathbf{k}) = \int \frac{dq}{2\pi}\, e^{-q^2R^2}\, \frac{\pi}{\Omega_{k,q}}\left[1 + 2n(\Omega_{k,q})\right]\delta(\omega - \Omega_{k,q}). \tag{37}$$

Solving the δ-function for q, we denote:

$$q_0^2 = \frac{\omega^2}{c^2} - k^2 - \frac{m^2}{c^2}, \quad q_0 > 0.$$

Then:

$$N(\omega,\mathbf{k}) = \frac{1 + 2n(\omega)}{2c^2 q_0}\, e^{-q_0^2R^2}. \tag{38}$$

Comparison with Quantum FDT

Quantum theory yields:

$$N_{\text{quantum}}(\omega) = \frac{1}{2\omega}\coth\left(\frac{\hbar\omega}{2k_BT}\right). \tag{39}$$

In the projection model:

$$N_{\text{projection}}(\omega,\mathbf{k}) = \frac{1 + 2n(\omega)}{2c^2 q_0}\, e^{-q_0^2R^2}. \tag{40}$$

Feature	TCS (projection)	Quantum theory
Source of fluctuations	Information loss in 4D geodesy projection	Virtual particles, quantum superposition
Noise kernel $N(\omega, \mathbf{k})$	Hidden mode integral: depends on projection shape $g(x^4)$ and spectrum $\Omega_{k,q}$	Proportional to $\coth(\hbar\omega/2k_B T) \cdot \operatorname{Im} D_R(\omega)$
Presence of zero-point noise at $T \to 0$	Absent for the classical ensemble	Present: $\lim_{T\to 0} \coth(\hbar\omega/2k_B T) = 1$
UV convergence	Provided by the kernel shape $g(x^4)$: suppression of $\|G(q)\|^2$ as $q \to \infty$	Requires regularization and counter-terms
Renormalization	Redefinition of projection and hidden spectrum parameters	Redefinition of physical constants and fields
Physical interpretation of counter-terms	Geometric: smoothing parameters, measures, ensemble	Quantum: elimination of virtual contribution infinities
Signatures in experiments	FDT violation, dependence of noise on Geometries, Nonstandard Spectra	Universal Quantum Responses, Topological Protections

Таблица 5: Comparison of the Projection TCS and Quantum Theory in Noise Structure and Renormalization

Conclusion

The noise kernel $N(\omega, \mathbf{k})$ in the TCS: - arises from the information loss during the projection of 4D deterministic dynamics; - has a finite form and depends on the projection geometry (via R); - differs from quantum FDT in structure and scale dependence; - allows local approximation and renormalization of the projection parameters.

Thus, the TCS allows the construction of a renormalizable effective 3D theory without virtual particles, where fluctuations are geometric consequences, not quantum superpositions.

8.7 Specification of the phase mass via the geometry of $Q(x)$

Let $\chi = \mathcal{M}(x,t)\, e^{iS(x,t)/\hbar}$ be the local complex mass depending on the cutoff geometry. Then:

$$\boxed{\mathcal{M}(x,t) = M_0 \left[\det(Q_{3\times 3}(x)) \right]^{\gamma}} \tag{41}$$

where M_0 is the scaling constant and γ is the homothety index.

The phase $S(x,t)$ is defined as the integral over the homothetic flow:

$$\boxed{S(x,t) = \int_{\Sigma}^{\mathbf{x}} d\xi^{\mu}\, \partial_{\mu} \ln \det(Q_{3\times 3}(\xi))} \tag{42}$$

Thus, $\mathcal{M}$ and S are geometric functions generated by the local structure of the matrix $Q(x)$ and describe the density and phase current as projected onto a 3D slice.

8.8 Deriving the Noise Kernel $N(\omega, \mathbf{k})$ from a 4D Projection

Consider the projection of the 4D wave field $\Psi(x,t)$ onto a 3D slice:

$$\Phi(\mathbf{x}, t) = \int dx^4\, g(x^4)\, \Psi(x,t), \tag{43}$$

where $g(x^4)$ is the projection kernel, for example, a Gaussian kernel:

$$g(x^4) = \frac{1}{(2\pi R^2)^{1/4}}\, e^{-x^{42}/(4R^2)} \quad \Rightarrow \quad |G(q)|^2 = e^{-q^2 R^2}. \tag{44}$$

The noise kernel is defined as:

$$N(\omega, \mathbf{k}) = \int \frac{dq}{2\pi}\, |G(q)|^2\, S_\Psi^{\rm sym}(\omega, \mathbf{k}, q), \tag{45}$$

where is the spectral density:

$$S_\Psi^{\rm sym}(\omega, \mathbf{k}, q) = \frac{\pi}{\Omega_{k,q}} \left[1 + 2n(\Omega_{k,q})\right] \delta(\omega - \Omega_{k,q}), \quad \Omega_{k,q} = \sqrt{c^2(k^2 + q^2) + m^2}. \tag{46}$$

Solving the δ-function for q, we obtain:

$$\boxed{N(\omega, \mathbf{k}) = \frac{1 + 2n(\omega)}{2c^2 q_0}\, e^{-q_0^2 R^2}, \quad q_0^2 = \frac{\omega^2}{c^2} - k^2 - \frac{m^2}{c^2}} \tag{47}$$

8.9 Phase current from geometric phase

Let $\chi = \mathcal{M}\, e^{iS/\hbar}$ be the complex mass. Then the phase current is defined as:

$$\boxed{\delta J^\mu_{\rm phase} = \frac{1}{\hbar}\, \mathcal{M}^2\, \partial^\mu S(x,t)} \tag{48}$$

Taking into account the specification $S(x,t)$ through $Q(x)$:

$$\partial^\mu S(x,t) = \partial^\mu \int_\Sigma^{\mathbf{x}} d\xi^\nu\, \partial_\nu \ln\det(Q_{3\times3}(\xi)) = \partial^\mu \ln\det(Q_{3\times3}(x)) \tag{49}$$

Total:

$$\boxed{\delta J^\mu_{\rm phase} = \frac{1}{\hbar}\, \mathcal{M}^2\, \partial^\mu \ln\det(Q_{3\times3}(x))} \tag{50}$$

8.10 Magnetic deviation from a phase vortex

Let the phase phase $S(x)$ contain a vortex defect:

$$S(\theta) = \hbar\, n\, \theta, \quad n \in \mathbb{Z} \tag{51}$$

Then the phase current:

$$\delta J^\theta_{\rm phase} = \frac{1}{\hbar}\, \mathcal{M}^2\, \partial_\theta S = n\, \mathcal{M}^2 \tag{52}$$

By the magnetic deviation according to the Biot–Savart law:

$$\boxed{\delta B(r) = \frac{\mu_0}{4\pi} \oint \frac{\delta J^\theta_{\rm phase}\, dl}{r^2} = \frac{\mu_0\, n\, \mathcal{M}^2}{2\pi r}} \tag{53}$$

8.11 Renormalization and suppression of UV fluctuations

The noise kernel $N(\omega, \mathbf{k})$ contains exponential suppression:

$$N(\omega, \mathbf{k}) \propto e^{-q_0^2 R^2}, \quad q_0^2 = \frac{\omega^2}{c^2} - k^2 - \frac{m^2}{c^2} \tag{54}$$

This means: - High-momentum contributions are suppressed for $R > 0$ - The influence kernel is approximated by local operators - All counterterms are geometric: redefinition of R, γ, M_0 - The theory is renormalizable in the projection sense

$$\boxed{\text{UV convergence is ensured:} \quad N(\omega, \mathbf{k}) < \infty \quad \forall\, \omega, k} \tag{55}$$

Component	**Specification and Relationship**
Matrix $Q(x)$	Geometry of a 3D slice; defines local deformation, volume measure $J_3 = \det(Q_{3\times 3})$
Scalar $\phi(Q)$	Logarithm of the volume measure: $\phi = \ln J_3(Q)$; phase flow source
Mass amplitude $\mathcal{M}(x,t)$	Geometric density: $\mathcal{M} = M_0\, J_3^\gamma$
Phase $S(x,t)$	Homothetic potential: $S = \int d\xi^\mu\, \partial_\mu \ln J_3(Q)$
Complex mass χ, χ'	$\chi = \mathcal{M}\, e^{iS/\hbar}$, $\chi' = \mathcal{M}\, e^{-iS/\hbar}$; is included in $\mathbb{D}(x)$
Projection onto 3D	$\Phi(\mathbf{x}, t) = \int dx^4\, g(x^4)\, \Psi(x,t)$; kernel $g(x^4)$ defines smoothing
Noise kernel $N(\omega, \mathbf{k})$	$N \propto \int dq\, \lvert G(q)\rvert^2\, S_\Psi^{\rm sym}$; depends on the shape of $g(x^4)$ and the spectrum of $\Omega_{k,q}$
Phase current $\delta J^\mu_{\rm phase}$	$\delta J^\mu = \frac{1}{\hbar}\, \mathcal{M}^2\, \partial^\mu \ln J_3(Q)$
Magnetic deviation $\delta B(r)$	$\delta B(r) = \frac{\mu_0\, n\, \mathcal{M}^2}{2\pi r}$ at vortex phase $S = \hbar n \theta$
Counterterms and renormalization	All corrections are geometric: redefinition of R, γ, M_0; UV convergence is ensured by

Таблица 6: Relationship between $Q(x)$ geometry, phase mass, noise, and observable effects in TCS

9 General derivation of the one–loop kinetic term via the heat–kernel

Objective. Show how the operator

$$-\frac{1}{4}\, \kappa_{\rm hom}^{\rm eff}\, \mathrm{tr}\, F_{\mu\nu}[B] F^{\mu\nu}[B]$$

arises in the effective action for the homothetic coupling field B_μ^a when integrating the fast degrees of freedom (one–loop) using the heat–kernel (the method of proper thermal kernels).

9.1 Setup: Operator and Functional Determinant

Consider a form quadratic in the matter fields (for definiteness, *fermions* in the R representation):

$$S_{\rm matter} = \int d^4x\; \bar{\Psi}\Big(i\hbar\gamma^\mu D_\mu[B] - M_{\rm heavy}\Big)\Psi, \qquad D_\mu[B] = \partial_\mu + i\frac{g_{\rm hom}}{\hbar c} B_\mu^a T^a,$$

where T^a are the generators of the inner homothety group in the R representation, M_{heavy} is the characteristic mass of the fast sector.

Integrating over Ψ:

$$S_{\text{eff}}[B] \;=\; -i\hbar \ln\det\Big(i\hbar \not{D}[B] - M_{\text{heavy}}\Big) + \text{const.}$$

It's convenient to switch to a positive definite operator:

$$\mathcal{O} \;\equiv\; \Big(i\hbar\not{D} - M_{\text{heavy}}\Big)^{\dagger}\Big(i\hbar\not{D} - M_{\text{heavy}}\Big) \;=\; -\hbar^2 D^2 \;+\; \frac{\hbar^2}{4}[\gamma^\mu,\gamma^\nu]F_{\mu\nu} \;+\; M_{\text{heavy}}^2,$$

where $F_{\mu\nu} = F^a_{\mu\nu}T^a$, $F^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g_{\text{hom}} f^{abc} B^b_\mu B^c_\nu$.

9.2 Heat–kernel representation

We use the standard formula:

$$\ln\det\mathcal{O} \;=\; \operatorname{Tr}\ln\mathcal{O} \;=\; -\int_0^\infty \frac{ds}{s}\;\operatorname{Tr}\, e^{-s\,\mathcal{O}} \;+\; \text{const.}$$

The heat kernel extension for small s in four-dimensional Euclidean space:

$$\operatorname{Tr}\, e^{-s\mathcal{O}} \;\sim\; \frac{1}{(4\pi s)^2}\sum_{k=0}^{\infty} s^k \int d^4x\, \operatorname{tr}_{\text{int}}\, a_k(x),$$

where $\operatorname{tr}_{\text{int}}$ is the trace over internal indices (spin/color), and the local coefficients a_k (Seeley–DeWitt) are expressed in terms of fields and their derivatives. In our case:

$$a_0 = I_R, \qquad a_1 = 0 \quad (\text{in flat space}), \qquad a_2 \;\supset\; \alpha\, \operatorname{tr} F_{\mu\nu}F^{\mu\nu} \;+\; \cdots\,,$$

where α is a numerical coefficient depending on the content of the fast fields and their representation R.

9.3 Extraction of F^2 and Regularization

Substituting the expansion into the integral over s and introducing ultraviolet regularization (e.g., a hard cutoff Λ or $\overline{\text{MS}}$), we obtain the contribution

$$\Delta S^{(1)}[B] \;\supset\; \frac{1}{(4\pi)^2}\,\ln\frac{\Lambda^2}{\mu^2}\int d^4x\, \operatorname{tr} c_1(R)\, F_{\mu\nu}F^{\mu\nu},$$

where $c_1(R)$ takes into account the spin/internal traces and normalizations of the generators T^a in the representation of R:

$$c_1(R) \;\sim\; \hbar\, g_{\text{hom}}^2\, C(R), \qquad C(R) \equiv \frac{\operatorname{tr}(T^aT^b)}{\delta^{ab}}.$$

So, the effective coefficient of the kinetic term is:

$$\kappa_{\text{hom}}^{\text{eff}} \;\simeq\; \hbar\,\frac{g_{\text{hom}}^2}{16\pi^2}\, C(R)\, \ln\frac{\Lambda^2}{\mu^2},$$

with an exact numerical factor determined by the chosen regularization and the set of fast fields.

9.4 Final effective dynamics and YM–equations

The resulting one–loop term adds to the action the contribution

$$S_{\text{eff}}[B] \supset -\frac{1}{4}\kappa^{\text{eff}}_{\text{hom}} \int d^4x \, \text{tr}\, F_{\mu\nu}[B]F^{\mu\nu}[B],$$

and variation over B^a_μ gives the Yang–Mills equations with a source (homothetic Noether current $J^{\mu a}_{\text{hom}}$ from matter):

$$\kappa^{\text{eff}}_{\text{hom}} D_\nu F^{\nu\mu a}[B] = J^{\mu a}_{\text{hom}}, \qquad J^{\mu a}_{\text{hom}} \equiv -\frac{\delta \mathcal{L}_{\text{matter}}}{\delta B^a_\mu}.$$

9.5 Assumptions, Checks, and Controls

- **Composition of fast fields:** Explicitly specify which fields are integrated (fermions/bosons), their representation R, and masses M_{heavy}.
- **Regularization:** Choose a scheme (hard cutoff, $\overline{\text{MS}}$, Pauli–Villars) and ensure covariance preservation.
- **Anomalies:** Check for the absence of local homothety anomalies for the selected set of fields, or indicate counterterms/mechanism for their compensation.
- **Sign of $\kappa^{\text{eff}}_{\text{hom}}$:** Ensure that the sign ensures stable dynamics (positive kinetic term).
- **Dimensions and Normalizations:** Consistency between the normalization of T^a, the definition of tr, and units (natural/SI) throughout the text.
- **Agreement with Scales:** Interpret Λ (UV–cutoff) and μ (renormalization group/physical scale), compare the order of magnitude of $\kappa^{\text{eff}}_{\text{hom}}$ with phenomenological constraints.

9.6 Heat-kernel calculations for TCS

Heat kernel method

The effective action for homothetic fields in curved space is defined through the functional determinant:

$$S_{\text{eff}}[\mathcal{B}] = -i \ln \det(\mathcal{O}[\mathcal{B}]) + \text{const}, \tag{56}$$

where $\mathcal{O}[\mathcal{B}]$ is the operator of motion in the presence of the background field $\mathcal{B}^a_\mu$. The Schwinger–DeWitt proper time representation is used [13, 14]:

$$\ln \det \mathcal{O} = -\int_0^\infty \frac{ds}{s} \, \text{Tr}\, e^{-s\mathcal{O}}, \qquad K(s) \equiv \text{Tr}\, e^{-s\mathcal{O}}. \tag{57}$$

Minimal operator in TCS

For 4+0 geometry:

$$\mathcal{O} = -D_\mu D^\mu + \mathcal{V}(x), \qquad D_\mu = \partial_\mu - i g_{\text{hom}} \mathcal{B}^a_\mu T^a, \tag{58}$$

$$\mathcal{V}(x) = m^2 M(x)^2 + i g_{\text{hom}} F^a_{\mu\nu} T^a \sigma^{\mu\nu} + \mathcal{R}_{\text{eff}}(x). \tag{59}$$

Seeley–DeWitt expansion

Asymptotics as $s \to 0^+$:

$$K(s) = \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g(x)} \sum_{n=0}^{\infty} a_n(x)\, s^n, \qquad d = 4. \tag{60}$$

Coefficient	**View in TCS**
$a_0(x)$	$\mathrm{tr}\,\mathbf{1} = N_{\text{comp}}$
$a_1(x)$	$N_{\text{comp}} m^2 M(x)^2 + \mathrm{tr}[ig_{\text{hom}} F^a_{\mu\nu} T^a \sigma^{\mu\nu}] + N_{\text{comp}} \mathcal{R}_{\text{eff}}(x)$
$a_2(x)$	$\mathrm{tr}\big[\frac{1}{6}\mathcal{R}\mathbf{1} + \frac{1}{2}\mathcal{V}^2 + \frac{1}{6} D_\mu D^\mu \mathcal{V} + \frac{1}{12} F^a_{\mu\nu} F^{a\mu\nu}\mathbf{1}\big]$

Таблица 7: Seeley–DeWitt coefficients for TCS

For SL(4,ℝ): $\mathrm{tr}[F^a_{\mu\nu} T^a \sigma^{\mu\nu}] = C_2 F^a_{\mu\nu} F^{a\mu\nu}$, where $C_2 = 5$.

Mask $M(x)$ and gradient contributions

$$\nabla^2[m^2 M^2] = m^2 \left[2(\nabla M)^2 + M \nabla^2 M\right], \tag{61}$$

$$\nabla_\mu \ln M = \frac{1}{M} \nabla_\mu M. \tag{62}$$

For the factorized mask $M(x) = \prod_i \sqrt{1 - 2\Phi_i / E_i^{\text{crit}}}$:

$$\frac{\partial \ln M}{\partial \Phi_i} = -\frac{1}{E_i^{\text{crit}}(1 - 2\Phi_i / E_i^{\text{crit}})}. \tag{63}$$

One-loop action and β-functions

Divergent part:

$$S^{(1)}_{\text{eff}} = \frac{1}{(4\pi)^2} \int d^4 x \sqrt{g} \left[\tfrac{a_2(x)}{\epsilon} + a_2(x) \ln \Lambda^2 + \text{finite parts}\right]. \tag{64}$$

One-loop coefficient for SL(4,ℝ):

$$b_0^{\text{hom}} = \frac{55 - 4n_f - n_s}{6}, \qquad \beta_{g_{\text{hom}}} = \frac{b_0^{\text{hom}}}{16\pi^2} g^3_{\text{hom}}. \tag{65}$$

Asymptotic freedom condition: $n_f + n_s/4 < 13.75$.

Renormalization of the mask and critical energies

$$M_0(x) = Z_M M(x), \qquad Z_M = 1 + \frac{\delta Z_M}{16\pi^2 \epsilon}, \tag{66}$$

$$\gamma_M = -\frac{m^2 M(x)^2}{16\pi^2}, \qquad \beta_{E_i} = -\gamma_M E_i^{\text{crit}}. \tag{67}$$

Physical consequences

The final part of the Lagrangian:

$$\mathcal{L}_{\text{eff}} = \tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \tfrac{\alpha_1}{4}M^2 F^a_{\mu\nu}F^{a\mu\nu} + \tfrac{\alpha_2}{2}M(\nabla M)^2 + \dots \tag{68}$$

$$\alpha_1 = \tfrac{5}{48\pi^2}, \qquad \alpha_2 = \tfrac{N_{\text{comp}}m^2}{12\pi^2}. \tag{69}$$

Comments

The heat-kernel technique [10, 11, 12] shows that TCS is renormalizable in the one-loop approximation: all divergences are eliminated by a finite set of counterterms compatible with SL(4,$\mathbb{R}$) symmetry. For higher loops, generalization of the methods is required (see Barvinsky–Vilkovisky).

One-loop Lagrangian in TCS

The effective action at one loop level is of the form

$$\Gamma^{(1)} = -i\ln\det D(x) = \frac{i}{2}\int_0^\infty \frac{ds}{s}\,\mathrm{Tr}\, e^{-sD^\dagger D},$$

where

$$D(x) = i\gamma^\mu\partial_\mu - m\,M(x).$$

Using the heat-kernel decomposition and Seeley–DeWitt coefficients, we obtain

$$\mathcal{L}^{(1)}_{\text{eff}} = \frac{1}{32\pi^2}\left\{m^4M^4(x)\left[\ln\frac{m^2M^2(x)}{\mu^2} - \tfrac{3}{2}\right] + m^2(\partial_\mu M)^2\left[\ln\frac{m^2M^2(x)}{\mu^2} - 1\right] + \tfrac{1}{6}R\,m^2M^2(x)\right\}.$$

Modification of β-functions in TCS

In the standard theory

$$\mu\frac{dg}{d\mu} = -\frac{b}{16\pi^2}g^3,$$

where b depends on the field spectrum. In TCS, the loop integrals contain a factor of $M^2(x)$, so

$$\beta_{TCS}(g) = -\frac{b}{16\pi^2}M^2(x)\,g^3.$$

Group	Standard $\beta(g)$	В TCS (with mask $M(x)$)
$U(1)_Y$	$\frac{41}{96\pi^2}g^3$	$\frac{41}{96\pi^2}M^2(x)\,g^3$
$SU(2)_L$	$-\frac{19}{96\pi^2}g^3$	$-\frac{19}{96\pi^2}M^2(x)\,g^3$
$SU(3)_c$	$-\frac{7}{16\pi^2}g^3$	$-\frac{7}{16\pi^2}M^2(x)\,g^3$

Таблица 8: Comparison of standard β functions and those modified in TCS.

A One-loop derivation of superconductivity equations from TCS

A.1 Starting equation

The basic TCS equation for the spinor is

$$i\hbar\,\partial_t\Psi = \Big[-\,i\hbar c\,\alpha^A\mathcal{D}_A + mc^2\,\beta\,\mathbb{D}(x)\Big]\Psi,$$

where $\mathcal{D}_A = \partial_A + i\frac{q}{\hbar c}A_A + \ldots$, and $\mathbb{D} = \mathrm{diag}(\chi, \chi', \chi, \chi')$ with $\chi = MD$, $\chi' = MD^{-1}$.

A.2 Nambu Representation and Pairing Field

Introducing the Nambu Spinor

$$\Psi_N = \begin{pmatrix}\psi \\ \psi^\dagger\end{pmatrix}, \qquad \mathcal{H}_N = \begin{pmatrix} h(\mathcal{D}) & \Delta(x) \\ \Delta^*(x) & -h^T(\mathcal{D})\end{pmatrix},$$

and using the Hubbard–Stratonovich transform, we define the complex pairing field

$$\Delta(x) = |\Delta(x)|\,e^{i\theta(x)}.$$

A.3 Effective Functional

Integration over fermionic degrees of freedom at one loop level yields the Ginzburg–Landau functional:

$$\mathcal{F}_{\text{eff}}[\Delta, A] = \int d^3x \left\{a\,|\Delta|^2 + \tfrac{b}{2}\,|\Delta|^4 + K\left|\big(\nabla - i\tfrac{q}{\hbar c}\mathbf{A}\big)\Delta\right|^2\right\} + \frac{1}{8\pi}\int d^3x\,|\mathbf{B}|^2.$$

The coefficients $a, b, and K$ depend on the spectrum and the mask $M(x)$; for anisotropic M, the value K becomes the tensor K_{ij}.

A.4 Supercurrent and London's Equations

Expanding $\Delta = |\Delta|e^{i\theta}$, we obtain an expression for the supercurrent:

$$\mathbf{J}_s = \frac{qK}{\hbar}\,|\Delta|^2\left(\hbar\nabla\theta - \frac{q}{c}\mathbf{A}\right).$$

For a fixed $|\Delta|$, this leads to the London equations:

$$\nabla\times\mathbf{J}_s = -\frac{c}{4\pi\lambda^2}\,\mathbf{B}, \qquad \lambda^2 = \frac{m^*c^2}{4\pi q^2 K|\Delta|^2}.$$

Here λ is the field penetration depth, m^* is the effective mass renormalized by the mask M.

A.5 Flux Quantization and Persistent Currents

The uniqueness of the phase implies the condition

$$\oint\left(\hbar\nabla\theta - \tfrac{q}{c}\mathbf{A}\right)\cdot d\ell = 2\pi\hbar n,$$

which yields magnetic flux quantization

$$\Phi = n\,\Phi_0, \qquad \Phi_0 = \frac{h}{2e}.$$

Thus, persistent currents in superconducting circuits are interpreted in the TCS as a projection of the homothetic current conservation law onto a 3D slice.

The Role of the M Mask and Homothety

The M mask changes the density of states and effective masses, making the parameters K, λ, and ξ anisotropic. The homothetic current enhances phase rigidity and ensures the topological stability of the supercurrent.

B Two Solutions for a Black Hole in TCS: General Equation, Projection, and Radial Dynamics

B.1 1. General equation and projection form (absolute time)

In the TCS formalism (4+0, absolute time t), the full 4D dynamics is light-like:

$$\boxed{E = p\,c}$$

When projected onto observable 3D space (radial geometry), a mask $M(r)$ arises:

$$\boxed{E^2 = p^2c^2 + m^2c^4\,M^2(r)}, \qquad M(r) = \sqrt{1 - \frac{U(r)}{mc^2}}.$$

For the stationary level E, we introduce a local invariant

$$\Xi(r) \equiv \sqrt{E^2 - m^2c^4 + mc^2U(r)} = \sqrt{E^2 - m^2c^4\,M^2(r)}.$$

Then the branch momenta and energies yield unambiguous geometric means:

$$\sqrt{p_-(r)\,p_+(r)} = \frac{\Xi(r)}{c}, \qquad \sqrt{E_-(r)\,E_+(r)} = \Xi(r).$$

B.2 2. Radial Statement: Branch Equations and Stationarity

Three-dimensional (radial) branch with absolute time:

$$\boxed{\left[i\hbar\left(\nabla_3 - \frac{1}{c}\,\partial_t\right) + mc^2\,M(r)\,e^{S(r)/\hbar}\right]\psi_+(r,t) = 0\ ,}$$

$$\boxed{\left[i\hbar\left(\nabla_3 + \frac{1}{c}\,\partial_t\right) + mc^2\,M(r)\,e^{-S(r)/\hbar}\right]\psi_-(r,t) = 0\ .}$$

Stationarity in absolute time:

$$\psi_\pm(r,t) = \phi_\pm(r)\,e^{-iEt/\hbar}.$$

Summing the branches yields the equation for a single spatial wave:

$$i\hbar\,\nabla_3\Phi(r) + mc^2\,M(r)\,\cosh\!\left(\frac{S(r)}{\hbar}\right)\Phi(r) = 0, \qquad \psi(r,t) = \Phi(r)\,e^{-iEt/\hbar},$$

or equivalently (in the invariant notation along the path r)

$$\psi(r,t) = A_{\text{full}}\, \exp\left[\frac{i}{\hbar}\left(\frac{1}{c}\int^{r} \Xi(r')\, dr' - E\, t\right)\right].$$

B.3 3. Local dispersion and regimes in the black hole potential

Local dispersion follows from the projection form:

$$\boxed{E^2 = (\hbar c\, k(r))^2 + m^2 c^4\, M^2(r)}.$$

Regimes:

- **Phase regime** ($E > mc^2 M(r)$): $\Xi(r) \in \mathbb{R}$, $k(r) = \Xi(r)/(\hbar c)$, oscillations.
- **Tunnel regime** ($E < mc^2 M(r)$): $\Xi(r) = i\Gamma(r)$, $\kappa(r) = \Gamma(r)/(\hbar c)$, evescent solutions.

Typical black hole mask (radial profile), compatible with the horizon:

$$M(r) \xrightarrow[r \to r_s]{} 0 \quad \text{(the mass disappears at the horizon, the wave is free)},$$
$$M(r) \xrightarrow[r \to 0]{} \infty \quad \text{(the effective mass increases, the wave decays)}.$$

B.4 4. Two solutions: a phase branch at the horizon and an evanescent branch inside

Radial forms for the stationary level E:

$$\Phi_{\text{phase}}(r) = C_1\, \exp\left(+i\int^{r} k(r')\, dr'\right) \;+\; C_2\, \exp\left(-i\int^{r} k(r')\, dr'\right), \quad k(r) = \frac{\Xi(r)}{\hbar c} \in \mathbb{R},$$

$$\Phi_{\text{evan}}(r) = A\, \exp\left(+\int^{r} \kappa(r')\, dr'\right) \;+\; B\, \exp\left(-\int^{r} \kappa(r')\, dr'\right), \quad \kappa(r) = \frac{\Gamma(r)}{\hbar c} > 0.$$

Full wave with absolute time:

$$\psi_{\text{phase}}(r,t) = \Phi_{\text{phase}}(r)\, e^{-iEt/\hbar}, \qquad \psi_{\text{evan}}(r,t) = \Phi_{\text{evan}}(r)\, e^{-iEt/\hbar}.$$

B.5 5. Boundary conditions and the physical picture

- **At the horizon** $r = r_s$: $M(r_s) = 0 \Rightarrow k(r_s) = E/(\hbar c)$, the wave is locally free (phase), the incoming/outgoing component is selected according to the physical problem.
- **Inside** $r < r_s$ with increasing $M(r)$: transition to the evescent regime, the wave decays toward the center ($r \to 0$), the center is *dynamically empty* (no probability accumulation).
- **Docking**: continuity of ψ and its radial derivative at separating radii (for example, at $r = r_s + \epsilon$ and $r = r_s - \epsilon$), which fixes the coefficients $C_{1,2}$, A, B.

B.6 6. Connection with the spinor-matrix equation and mask

For the four-component form (analog of the quasi-Dirac form with mask):

$$E\Psi = \Big[- i\hbar c \sum_{A=1}^{4} \alpha^A \mathcal{D}_A + mc^2 \beta\, \mathbb{D}(r) \Big] \Psi,$$

$$\mathbb{D}(r) = \mathrm{diag}\big(M(r)D(r),\, M(r)D^{-1}(r),\, M(r)D(r),\, M(r)D^{-1}(r)\big).$$

Component separation:

$$\psi_- = -\frac{i\hbar c}{E + mc^2 M(r)} (\boldsymbol{\sigma} \cdot \mathbf{p})\, \psi_+,$$

$$\Big[E - mc^2 M(r) - \frac{\hbar^2 c^2}{E + mc^2 M(r)}\, \mathbf{p}^2\Big] \psi_+ = 0,$$

gives the same local variance

$$E^2 = c^2 \hbar^2 k^2 + m^2 c^4 M^2(r),$$

coinciding with the projection form of the TCS and the modes via $\Xi(r)$ and $\Gamma(r)$.

B.7 7. Final unified solution and interpretation

Unified wave via the geometric mean of branches:

$$\boxed{\psi(r,t) = A_{\text{full}}\, \exp\Big[\frac{i}{\hbar}\Big(\frac{1}{c}\int^r \Xi(r')\, dr' - E\, t\Big)\Big]}\ .$$

Picture:

- At the horizon $M(r_s) \to 0$: mass vanishes, *phase-free* dynamics (locally $E = pc$), both incoming and outgoing solutions are possible.
- Inside $M(r) \to \infty$: $\Xi = i\Gamma$, *evescent* dynamics, exponential decay $\Rightarrow$, empty center.
- Time remains absolute: $e^{-iEt/\hbar}$ in all regions, ensuring a uniform temporal evolution.

B.8 8. Particle inside the horizon and the gravitational tunneling effect

If we consider a particle located *inside* the horizon ($r < r_s$), then the mask $M(r)$ increases, and the invariant

$$\Xi(r) = \sqrt{E^2 - m^2 c^4 + mc^2 U(r)}$$

becomes purely imaginary: $\Xi(r) = i\Gamma(r)$, where $\Gamma(r) > 0$. This means that the spatial part of the wave takes on an evanescent form:

$$\Phi_{\text{in}}(r) \sim \exp\Big(-\int^r \kappa(r')\, dr'\Big), \qquad \kappa(r) = \frac{\Gamma(r)}{\hbar c}.$$

Formalization of the gravitational tunneling effect. For a Schwarzschild black hole with $r_s = 2GM/c^2$, the mask has the form:

$$M(r) = \sqrt{1 - \frac{r_s}{r}}.$$

Near the horizon ($r \approx r_s$), we expand:

$$M(r) \approx \sqrt{\frac{2\kappa_H}{c^2}(r - r_s)} + O((r - r_s)^{3/2}),$$

where $\kappa_H = c^4/(4GM)$ is the surface gravity.

Tunneling coefficient through the horizon:

$$T_{\text{grav}} \sim \exp\left(-2\,\mathrm{Im}\int p_r\,dr\right) = \exp\left(-2\int_{r_1}^{r_2} \kappa(r)\,dr\right),$$

where $r_1 < r_s < r_2$ are turning points.

Comparison with Hawking radiation. For massless particles ($m = 0$), we obtain:

$$T_{\text{grav}} \approx \exp\left(-\frac{2\pi E}{\hbar\kappa_H}\right) = \exp\left(-\frac{E}{k_B T_H}\right),$$

where $T_H = \frac{\hbar\kappa_H}{2\pi k_B} = \frac{\hbar c^3}{8\pi GMk_B}$ is the Hawking temperature.

For massive particles, tunneling is suppressed by an additional factor:

$$T_{\text{grav}} \sim \exp\left(-\frac{2\pi E}{\hbar\kappa_H} - \frac{\pi m^2 c^3}{\hbar\kappa_H E}\right).$$

Physical interpretation.

- Inside the horizon, the wave does not oscillate, but decays exponentially toward the center. This means that the center of the black hole is dynamically empty: the probability of finding a particle at $r \to 0$ disappears.
- At the boundary $r = r_s$, the evescent solution merges with the phase solution outside. This is the gravitational tunneling effect: the part of the amplitude located inside the horizon has a nonzero probability of tunneling outward.
- In the absolute time formalism, the temporal phase $e^{-iEt/\hbar}$ is preserved, so the tunneling process is described as a *purely spatial* transition through the barrier $M(r)$.
- The resulting distribution $\exp(-E/k_B T_H)$ coincides with the Hawking distribution for blackbody radiation.

Conclusion. Thus, a particle located inside the horizon does not "fall into a singularity,"but is described by a decaying wave that can tunnel outward. This is the natural mechanism of the gravitational tunneling effect in TCS:

- the center is empty (attenuation),
- the horizon is semitransparent (junction of the phase and evescent branches),
- absolute time ensures a single phase $e^{-iEt/\hbar}$ for the entire process,
- the tunneling coefficient exactly reproduces the Hawking distribution.

The TCS formalism provides a microscopic description of the Hawking mechanism as a projection tunneling effect in 4+0 geometry with absolute time.

B.9 Homothety conservation law

Continuity equation:

$$\partial_t \rho_{\text{hom}} + \nabla \cdot \mathbf{J}_{\text{hom}} = 0.$$

In a static spheric:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 J^r_{\text{hom}}\right) = 0 \quad \Rightarrow \quad r^2 J^r_{\text{hom}}(r) = C.$$

Boundary condition at the center: finite energy requires $C = 0$,

$$J^r_{\text{hom}}(0) = 0.$$

Therefore, the center is mute (no flux), and at the horizon $M(r) \to 0$ the current and density are concentrated.

Joint conclusion

Both approaches yield the sameArtin:

- The center of a black hole in TCS: a zone of exponential decay, without a singularity.
- Mass and energy are localized near the horizon $r = r_s$.
- Observable consequence: echo signals and a modified ringdown.

C Phase charge and electric field as an oscillation of 4D space in TCS

Fundamental principle. In the Theory of Curved Space (TCS), the electric field is treated as a *global oscillation of the entire four-dimensional space* relative to the internal oscillation. Complex interval

$$ds^{\mathbb{C}} = c\,dt\,e^{i\vartheta}, \qquad \vartheta = \tfrac{S}{\hbar},$$

describes the phase of this oscillation. The global $U(1)$ symmetry of the interval ensures the discreteness of charges as stable phase orientations.

Discrete phase channels. Four orientations of the tilt of the 4D trajectory to the observed 3D slice are stable:

$$\vartheta = -\tfrac{\pi}{2} \quad \Rightarrow \quad q = -q_0 \qquad \text{(electron)} \tag{70}$$
$$\vartheta = +\tfrac{\pi}{2} \quad \Rightarrow \quad q = +q_0 \qquad \text{(positron)} \tag{71}$$
$$\vartheta = 0 \quad \Rightarrow \quad q = 0 \qquad \text{(neutrino)} \tag{72}$$
$$\vartheta = \pi \quad \Rightarrow \quad q = 0 \qquad \text{(antineutrino)} \tag{73}$$

The difference between neutrinos and antineutrinos is determined by phase and helicity, not charge.

Field masks and mass. The oscillation of 4D space manifests itself in a 3D slice through field masks:

$$M_1 = \sqrt{1 - 2q\,\phi_{\text{eff}}}, \qquad M_2 = \sqrt{1 + 2q\,\phi_{\text{eff}}},$$

where ϕ_{eff} is constructed from the homothetic invariants $F_{\mu\nu}F^{\mu\nu}$ and the homothetic current. For neutrinos ($q = 0$), $M = 1$, and the mass is minimal. For an electron ($q = \pm q_0$), $M < 1$, which reduces the effective interval and increases the observed mass:

$$m_{\text{eff}} = \frac{m_0}{M(\vartheta)}.$$

Consistency and Volume Conservation. The condition $\det Q = 1$ for the homothetic transformation matrix ensures multiplicative compensation:

$$M_1(\phi)\, M_2(\phi) = \sqrt{1 - (2q_0\phi)^2} \;\approx\; 1 - 2(q_0\phi)^2 + O((q_0\phi)^4),$$

which conserves the homothetic charge and guarantees global consistency without time warping.

Physical Interpretation.

- The electric field is an oscillation of the entire 4D space, observed as a gradient of the phase potential.
- Charged particles ($\vartheta = \pm\pi/2$) resonate with this oscillation, have a linear response, and large mass.
- Neutral particles ($\vartheta = 0, \pi$) do not have a linear response, $M = 1$, and their mass is minimal.
- The product of the masks $M_1 M_2$ is related to $\det Q$ and ensures conservation of volume.

Consequences.

- The discreteness of the charges is derived from the $U(1)$ symmetry of the interval and the stability of the phase orientations.
- The electron and neutrino are the same channel: the only difference is that the neutrino has M=1, while the electron has M<1.
- The electron's mass is greater than the neutrino's precisely because of the charge-related mask.
- Experimental verification is possible through interference phase measurements and comparison of the masses of charged and neutral leptons.

D Magnetism as a Projection in the TCS

Fundamental Interpretation In the TCS, the magnetic field is not an independent source, but arises as a projection of spatial oscillations onto the observed three-dimensional slice.

The electric field reflects the direct component of the oscillations, while the magnetic field reflects the phase projection along the additional coordinate ϑ, which describes the phase state of space in 4D.

Mathematical Formulation of Projection In the TCS formalism, the projection relationship between the electric and magnetic fields is expressed through the projection operator $\mathcal{P}_{\text{TCS}}$:

$$\mathbf{B}_{\text{TCS}} = \mathcal{P}_{\text{TCS}}\left[\frac{\partial \mathbf{E}_{4\text{D}}}{\partial \theta}\right],$$

where θ is the phase coordinate in 4D space. Equivalently, through the effective potential:

$$\mathbf{B}_{\text{TCS}} = \nabla_\theta \times \mathbf{A}_{\text{eff}}.$$

Thus, the magnetic field is a phase derivative of the electric field, not an independent source.

Relation to classical electrodynamics In the limit of small fluctuations, the projection equations of the TCS reproduce Maxwell's equations:

$$\nabla \cdot \mathbf{B} = 0 \quad \text{(a topological consequence of the projection nature)},$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(a consequence of phase coherence)}.$$

The source equations arise as projections of the homothetic current J^μ_{hom}:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \delta \mathbf{J}_{\text{phase}},$$

where $\delta \mathbf{J}_{\text{phase}}$ is the additional phase current that arises when phase symmetry is broken (e.g., in superconductors or time-domain crystals)ah). Under normal conditions, $\delta \mathbf{J}_{\text{phase}} = 0$, and the equations reduce to classical ones.

Fundamental Consequences

- The magnetic field is always dipole—a monopole is fundamentally impossible as it violates projection geometry.
- Magnetic effects depend on the phase state of space and the mask $M(x)$.
- Under conditions of strong fields or time crystals, anomalous magnetic projections are possible.
- The law of conservation of homothety ensures conservation of magnetic flux.

Experimental Predictions

- **The absence of magnetic monopoles** is explained as a geometric necessity.
- **Phase magnetism:** An anomalous magnetic field distribution is predicted in superconductors with broken phase symmetry.
- **Time crystals:** In systems with time-modulated phase, magnetic field oscillations are expected without an external source.
- **High-energy limit:** At energies above $E_{\text{crit}} \sim m_e c^2/M(x)$, violations of standard magnetic relations are possible.
- **Flux quantization:** In TCS, the quantization of $\Phi_0 = h/2e$ follows from the condition of unique phase in 4D space.

Verification The TCS predictions for magnetism can be verified:

- In experiments with topological insulators and superconductors.
- In studying the magnetic properties of materials under extreme pressure.
- In precision measurements of the magnetic moment of particles.
- In analyzing cosmic magnetic fields in the early Universe.

E The Basic Solution of the Curved Space Theory and Spin Branches

Bilateral Symmetry as a Fundamental Principle In the Curved Space Theory (CST), the basic solution is based on the principle of bilateral "round-trip"symmetry, which ensures the self-consistency of the theory and maintains the mean speed of light equal to c.

Operator formulation of the TCS-covariant factorization We define the TCS-covariant momentum as a two-sided symmetrization:

$$\hat{\Pi}_{\text{TCS}} = \left(M^{-1/2}\hat{\Pi}M^{+1/2}\right)_{\rightarrow} \oplus \left(M^{+1/2}\hat{\Pi}M^{-1/2}\right)_{\leftarrow} \tag{74}$$

The energy equation in operator form:

$$\hat{H}^2 = c^2 \cdot \tfrac{1}{2}\left(\hat{\Pi}_{-}\hat{\Pi}_{+} + \hat{\Pi}_{+}\hat{\Pi}_{-}\right) + m^2c^4M^2[\mathcal{H}] \tag{75}$$

where $M[\mathcal{H}]$ is a TCS scalar of the homothetic invariant $\mathcal{H}$.

Geometric mean of branches: physical justification The solution is constructed as the geometric mean of the outward and outward branches, which ensures:

$$\Psi(r,t) = A\exp\left[\frac{i}{\hbar}\left(\sqrt{p_{-}p_{+}}r - \sqrt{E_{-}E_{+}}t\right)\right] \tag{76}$$

where $p_{\pm}$ and $E_{\pm}$ are the eigenvalues of the operators for specific modes. The geometric mean is chosen as the only way:

- Maintain symmetry between the outward/backward branches
- Ensure decay of the solution when one of the branches disappears
- Satisfy the homothety conservation law
- Avoid choosing a preferred reference frame

Generalization to spin degrees of freedom When taking spin (up/down, out/back) into account, the four branches enter multiplicatively:

$$\Psi_{\text{spin}}(r,t) = \Psi_{\uparrow\rightarrow} \otimes \Psi_{\uparrow\leftarrow} \otimes \Psi_{\downarrow\rightarrow} \otimes \Psi_{\downarrow\leftarrow} \tag{77}$$

The final form is preserved, and $p_{\pm}, E_{\pm}$ become effective quantities incorporating the spin indices. Geometric averaging suppresses unstable channels and ensures the stability of the solution.

Relation to the Homothety Conservation Law The use of the geometric mean follows directly from the homothety conservation law:

$$\partial_\mu J^\mu_{\text{hom}} = 0 \quad \Rightarrow \quad \frac{d}{dt}(a^3 \rho_{\text{hom}}) = 0 \tag{78}$$

This ensures the conservation of the phase volume and the internal consistency of the theory.

Mathematical Correctness and Limiting Cases

- In the classical limit, the solution transforms into the standard equations of motion
- As $M(x) \to 1$, ordinary quantum mechanics is restored
- In the relativistic limit, consistency with special relativity is ensured
- The geometric mean guarantees the reality of the phase factor for any $p_\pm, E_\pm$

Experimental Consequences

- Prediction of Interference Effects in Strong Fields
- Modification of the Tunneling Effect in Anisotropic Media
- Specific Features of Particle Decay in the TCS Formalism
- Possibility of Experimental Verification through Precision Phase Measurements

This approach provides a unified description of quantum processes within the TCS formalism. maintaining the calibrationinvariance and conformity with known physical laws within appropriate limits.

E.1 Entangled pair e^+e^- as a single phase system in the TCS

Phase-consistent state of the pair In the TCS, the entangled electron-positron pair is described as a single phase-consistent state:

$$\Psi_{e^+e^-}(r,t) = \mathcal{N} \exp\left[\frac{i}{\hbar}\left(\sqrt{p_-p_+}r - \sqrt{E_-E_+}t\right)\right] \cdot M_{e^-}(\phi) M_{e^+}(\phi) \tag{79}$$

where the masks for the electron and positron are:

$$M_{e^-}(\phi) = \sqrt{1 - 2q_0\phi}, \quad M_{e^+}(\phi) = \sqrt{1 + 2q_0\phi} \tag{80}$$

Charge Compensation and Homothetic Consistency The product of the masks compensates for the linear responses:

$$M_{e^-}(\phi) M_{e^+}(\phi) = \sqrt{1 - (2q_0\phi)^2} \approx 1 - 2(q_0\phi)^2 + O((q_0\phi)^4) \tag{81}$$

This eliminates the charge sign at the pair level, preserving homothetic consistency ($\det Q = 1$) and ensuring state stability during measurements.

The Annihilation Process in the TCS Formalism In the annihilation mode, the transition occurs:

$$M_{e^-}M_{e^+} \to 1, \quad \Psi_{e^+e^-} \to \mathcal{N} \exp\left[\frac{i}{\hbar}\left(\sqrt{p_-p_+}r - \sqrt{E_-E_+}t\right)\right] \tag{82}$$

with subsequent energy transfer into field (photon) channels without invoking the concept of "negative energies"or time reversal.

Quantization and Entanglement

- Entanglement arises naturally from phase coherence
- The state describes the correlated pair as a single entity
- Spatial separation does not violate phase coherence
- Measuring a single particle instantly determines the state of the system

Differences from the standard approach

- Does not require introducing negative-frequency solutions
- Naturally describes the creation and annihilation of pairs
- Preserves locality in TCS space
- Consistent with charge and energy conservation

This description provides a self-consistent picture of entangled states within the TCS formalism.

E.2 Operational Examples: Tunneling and Pair Production in TCS

Tunneling Effect in the TCS Formalism We consider tunneling through a barrier of thickness d within the TCS approach. Below the barrier, the effective momenta of the branches become imaginary:

$$p_\pm = i\kappa, \quad \sqrt{p_-p_+} = i\kappa \tag{83}$$

The wave function below the barrier:

$$\Psi(r) \propto \exp\left(-\frac{\kappa}{\hbar}r\right), \quad T \sim \exp\left(-\frac{2\kappa d}{\hbar}\right) \tag{84}$$

For the pair e^+e^-, the linear field distortions are suppressed by the factor $M_{e^-}M_{e^+}$, leaving a quadratic sensitivity of $\sim (q_0\phi)^2$.

Geometric Interpretation In standard quantum mechanics, the tunneling effect is described by the exponential decay of the wave function under a barrier.

In the TCS, this phenomenon receives a natural geometric interpretation: under a barrier, the particle does not "break through"the obstacle, but

enters the fourth dimension, where the barrier is absent as it is in a three-dimensional slice.

Formalism Under the barrier, the effective momenta of the branches become imaginary:

$$p_{\pm} = i\kappa, \qquad \sqrt{p_{-}p_{+}} = i\kappa,$$

and the wave function in a three-dimensional slice takes the form:

$$\Psi(r) \propto \exp\Big(-\frac{\kappa}{\hbar} r\Big).$$

However, in 4D geometry, this corresponds to a continuous trajectory that bypasses the barrier along an additional coordinate.

Physical Meaning

- In a 3D slice, exponential decay is observed.
- In 4D space, the motion remains continuous and causal.
- The probability of tunneling is determined by the geometry of the bypass in the fourth dimension.

Consequences

- The tunneling effect in the TCS does not violate causality: in 4D, motion is continuous.
- Exponential decay in 3D is a projection of the 4D trajectory.
- This explains the universality of the tunneling effect without resorting to "classically forbidden regions."

Pair production mechanism in strong fields When the critical field strength is reached, $|\phi| > \phi_{\text{crit}}$ the neutral phase channel ($\alpha = 0$) becomes unstable and branches:

$$\alpha : 0 \longrightarrow \{+\tfrac{\pi}{2}, -\tfrac{\pi}{2}\}, \quad q(\alpha) = q_0 \sin\alpha \tag{85}$$

This process corresponds to the production of an e^+e^- pair without invoking the concept of negative energies:

- Phase channel switching occurs when the field threshold is reached.
- Masks M_1, M_2 ensure a smooth transition between channels.
- The process maintains homothetic invariance.
- Energy is drawn from the external field, not from the "Dirac sea."

Quantitative Description of the birth threshold The critical field is determined from the condition of instability of the neutral channel:

$$\phi_{\text{crit}} = \frac{mc^2}{2q_0} \cdot \frac{1}{\sqrt{M[\mathcal{H}]}} \tag{86}$$

where $M[\mathcal{H}]$ is a TCS scalar mask that takes into account homothetic corrections.

Comparison with QFT Predictions

- In weak fields ($\phi \ll \phi_{\text{crit}}$), the TCS predicts the same probabilities as QFT
- In strong fields, corrections due to homothety appear
- The divergence disappears at $\phi \to \infty$
- Unitarity is preserved at all field scales

Experimental Verification

- Modification of the Pair Production Threshold in Strong Laser Fields
- Corrections to the Tunneling Probability in Nanostructures
- Specific Correlations in Pair Production
- Possibility of Verification at Facilities Like ELI or XFEL

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F Idea 3D Projections and Round-Trip Symmetrization

Let the observed 3D line be measured as

$$ds^2_{\text{3D}} = c^2\,dt^2 - \left(\tfrac{s_1+s_2}{2}\right)^2,$$

where s_1 and s_2 are the effective "one-way"(round-trip) lengths along the projection, and $\frac{s_1+s_2}{2}$ is the symmetrized "two-way"(round-trip) length. The metric reciprocity condition

$$s_1 = s_2 \equiv s$$

ensures that the projection does not introduce anisotropy into the distance measurement. If $s_1 \neq s_2$ initially, then to restore metric symmetry, we can introduce a scaling of the time component, or equivalently, the "effective speed of light":

$$ds^2_{\text{3D}} \;\longrightarrow\; c^2\,\beta^2\,dt^2 - s^2.$$

Here β is a dimensionless factor compensating for the asymmetry of unidirectional lengths when reducing $s_1 = s_2$.

F.1 Equivalence with Special Relativity: Necessary and Sufficient Conditions

The Minkowski metric in Special Relativity (in units with a Euclidean 3D part) has the form

$$ds^2_{\text{SR}} = c^2\,d\tau^2 - s^2.$$

To interpret $ds^2_{\text{3D}} = c^2\,\beta^2\,dt^2 - s^2$ as a STR metric, it is sufficient to introduce a new "proper"or "operational"time interval

$$d\tau \equiv \beta\,dt,$$

after which

$$ds_{3D}^2 = c^2\, d\tau^2 - s^2 \equiv ds_{SR}^2.$$

Therefore, "changing time"(scaling dt) or "changing c"(replacing $c \to c\beta$) are equivalent redefinitions of units with a constant β:

$$\text{if} \quad \beta = \text{const}, \quad \text{then} \quad c^2\, \beta^2\, dt^2 = (c\beta)^2\, dt^2 = c^2\, d\tau^2.$$

This gives an exact correspondence to the Minkowski metric and allows the standard Lorentz group in the new units.

F.2 When the correspondence is violated

If β depends on coordinates or time,

$$\beta = \beta(t, \mathbf{x}),$$

then

$$ds_{3D}^2 = c^2\, \beta^2(t, \mathbf{x})\, dt^2 - s^2$$

is a *conformal-Minkowskian* metric (a scalar field scaling the time component). In this case:

- **Non-global STR:** global Lorentz invariance is violated because the light cone and norms change with coordinates.
- **Local SR approximation:** with a slow variation of β, one can assume $\beta \simeq$ const in a small neighborhood and recover the local Minkowskian form.
- **Physical interpretation:** $\beta(t, \mathbf{x})$ is equivalent to a scalar "refraction"field (the refractive index of time), which encodes the projection anisotropy (the remainder of $s_1 \neq s_2$). This is not special relativity, but a metric theory with an effective scalar modulus.

F.3 Relation to the condition $s_1 = s_2$ and operational consequences

The projection asymmetry (the difference $s_1 - s_2$) can be reduced to zero in two first-approximation equivalent ways:

$$\text{(i) path symmetrization: } s_1 \to s,\ s_2 \to s; \qquad \text{(ii) time scaling: } dt \to d\tau = \beta\, dt.$$

Both steps lead to the same effective Minkowski metric *only for constant β*. Then:

1. **Global compatibility with STR:** yes, the metric becomes exactly Minkowskian, and the physical predictions of STR are preserved (Lorentz transformations, light-cone invariance).
2. **Consistency with TCS:** β can be interpreted as a projection modulus depending on the hypercone geometry; the requirement $\beta =$ const means that the projection does not introduce macroscopic inhomogeneity into the measurement of time (redefinition of units).

Minimal conditions for a correct SR interpretation

- **Constant:** $\beta = \text{const}$ in the region of interest. Isotropy: Path symmetry $s_1 = s_2$ (absence of unidirectional anisotropy). Unit calibration: Either redefine $c \to c\beta$ or $dt \to d\tau = \beta\, dt$ (equivalently). Symmetry group: Keep the standard Lorentz group for the variables $(\tau, \mathbf{x})$.

Conclusiona: for $s_1 = s_2$ and constant scale β, the transition

$$ds^2_{\text{3D}} = c^2\beta^2\, dt^2 - s^2 \quad \longleftrightarrow \quad ds^2_{\text{SR}} = c^2\, d\tau^2 - s^2$$

gives a correct interpretation in terms of special relativity after redefining the units of time (or c). If $\beta = \beta(t, \mathbf{x})$, then we obtain not special relativity, but a conformal Minckowskian metric with a physical scalar modulus reflecting the projection nature (residual anisotropy)—this is a different class of models.

G Rigorous Scheme TCS: From Projection to Metric

G.1 1. Projection Operator and Definition of β as a Functional of M

We introduce an explicit projection operator $\mathcal{P}$ from 4+0 to the observed 3+1 space. Let χ be the radial coordinate of the hypercone, and $\mathcal{M}(x, \chi)$ be the 4D mask field. The projected scalar field into 3+1 is:

$$M(x) \equiv \int d\chi\, K(\chi)\, \mathcal{M}(x, \chi),$$

where $K(\chi)$ is the projection kernel (fixed by the class of hypercone geometries).

We define the kinematic modulus of time $\beta(x)$ as a local functional of M:

$$\boxed{\beta(x) \equiv \mathcal{P}[M](x) = b_0 + b_1\, M(x) + b_2\, \ell^2\, \Box M(x) + b_3\, \ell^2\, \partial_\mu M\, \partial^\mu M + \cdots}$$

where ℓ is the projection length (the hypercone radius), and b_i are constants defining the projection class; $\Box$ is the d'Alembert operator in the background Minkowskian metric.

G.2 2. Action: Field sector for M and metric sector for β

We formulate a local action functional that combines the dynamics of M and β:

$$S[M, \beta; g] = \int d^4x \sqrt{-g} \left\{ \underbrace{\frac{1}{2} Z_M\, \partial_\mu M\, \partial^\mu M - U(M)}_{\text{field sector}} + \underbrace{\frac{1}{2} Z_\beta\, \partial_\mu \beta\, \partial^\mu \beta - V(\beta)}_{\text{metric modulus}} + \underbrace{\lambda \Big[\beta - \mathcal{P}[M]\Big]}_{\beta\text{–M coupling}} + \mathcal{L}_{\text{m}}[g, \Psi]$$

Here g is the metric (in a weak field, we take the conformal Minkowskian form), $\lambda(x)$ is the Lagrangian field, ensuring $\beta = \mathcal{P}[M]$ as the coupling equation, $\mathcal{L}_{\text{m}}$ is the matter Lagrangian.

The kinetic normalizations Z_M, Z_β and potentials U, V are the parameters of the phenomenology of the TCS.

Varying with respect to λ yields the strict condition:

$$\boxed{\beta(x) - \mathcal{P}[M](x) = 0 \quad \Rightarrow \quad \beta(x) = \mathcal{P}[M](x).}$$

G.3 3. Equations of Motion and Explicit Dynamics of β

Varying the action with respect to M and β:

(a) For M:

$$Z_M \Box M + U'(M) \; - \; \lambda \frac{\delta \mathcal{P}[M]}{\delta M} = \mathcal{J}(x),$$

where $\mathcal{J}$ is the effective source (related to projection and matter; included via $\mathcal{L}_{\mathrm{m}}$).

(b) For β:

$$Z_\beta \Box \beta + V'(\beta) + \lambda = \mathcal{S}(x),$$

where $\mathcal{S}$ is a possible direct source of β (usually zero if β is a purely projection modulus).

We eliminate λ using the constraint equation $\beta = \mathcal{P}[M]$. In the linear approximation $\mathcal{P}[M] \approx b_0 + b_1 M$ and for $\mathcal{S} = 0$, we obtain a closed equation for β:

$$\boxed{Z_\beta \Box \beta + V'(\beta) \; = \; b_1 \left[\lambda \right]}$$

and for λ from the equation for M:

$$\lambda \approx \frac{1}{b_1} \big(Z_M \Box M + U'(M) - \mathcal{J} \big).$$

Substituting $M \approx (\beta - b_0)/b_1$ to first order, we obtain an explicit effective equation for β:

$$Z_\beta \Box \beta + V'(\beta) \; \simeq \; Z_M \Box \Big(\tfrac{\beta - b_0}{b_1} \Big) + U' \Big(\tfrac{\beta - b_0}{b_1} \Big) - \mathcal{J}(x).$$

This relates the dynamics of β directly to the sources and potential of the field M. In the simplest linear regime $U(M) \approx \frac{1}{2} m_M^2 M^2$, $V(\beta) \approx \frac{1}{2} m_\beta^2 \beta^2$, we obtain

$$\Big(Z_\beta - \tfrac{Z_M}{b_1} \Big) \Box \beta + \Big(m_\beta^2 - \tfrac{m_M^2}{b_1} \Big) \beta \; \simeq \; - \mathcal{J}(x) + \mathrm{const}.$$

Thus, β behaves like a scalar potential emitted by $\mathcal{J}$, with effective parameters expressed in terms of $(Z_M, Z_\beta, m_M, m_\beta, b_1)$.

G.4 4. Weak Field: β as a Newtonian Potential

In a weak field, we take the metric

$$ds^2 = c^2 \, \beta^2(x) \, dt^2 - (1 + \sigma) \, \delta_{ij} dx^i dx^j, \quad |\beta - 1| \ll 1, \; |\sigma| \ll 1,$$

and set $\sigma = -\epsilon$ to restore the standard linear gauge. The gravitational potential in GR is $\epsilon(x) = 2\Phi(x)/c^2$. For compatibility, we require

$$\beta(x) \approx 1 + \epsilon(x) \approx 1 + \frac{2\Phi(x)}{c^2}.$$

Then the equation for β must reduce to the Poisson equation:

$$\boxed{\nabla^2 \beta(x) \simeq \frac{8\pi G}{c^2} \rho(x), \quad \beta \approx 1 + \frac{2\Phi}{c^2}.}$$

Compared with the previous equation for β, this imposes a gauge condition on the parameters:

$$Z_\beta - \frac{Z_M}{b_1} \; \to \; 1, \qquad m_\beta^2 - \frac{m_M^2}{b_1} \; \to \; 0 \quad \text{(Newtonian limit)},$$

and fits $\mathcal{J}(x)$ into $\mathcal{J}(x) \equiv -\frac{8\pi G}{c^2}\rho(x)$ (up to constants and signs fixed by the normalization of the action).

G.5 5. Weak Field Equivalence GR: Verification Procedure

To demonstrate the correspondence to GR at the level of linear gravity:

1. Choose $\mathcal{P}$ in linear form: $\beta = b_0 + b_1 M$.
2. Fix $Z_\beta, Z_M, m_\beta, m_M, b_1$ so that the equation for β reduces to $\nabla^2\beta = 8\pi G\rho/c^2$.
3. Associate β with ϵ: $\beta - 1 = \epsilon = 2\Phi/c^2$.
4. Check the standard effects: gravitational redshift ($\Delta\nu/\nu = \Phi/c^2$), lensing, and Shapiro delay—all should follow from β.

If all three steps are satisfied, the TCS metric with β reproduces the Newtonian limit and the linear predictions of GR.

G.6 6. Gradient Corrections and Deviations from GR

With the terms $b_2\ell^2\Box M$ and $b_3\ell^2(\partial M)^2$ in $\mathcal{P}$, the equation for β acquires nonlocal/nonlinear corrections:

$$\beta \simeq b_0 + b_1 M + b_2\ell^2\Box M + b_3\ell^2(\partial M)^2 \quad \Rightarrow \quad \Box\beta \simeq b_1\Box M + b_2\ell^2\Box^2 M + \cdots$$

This leads to modified Newtonian predictions (screening, potential dispersion).

They can be constrained by laboratory experiments (optical clocks, atomic interferometers) and astrophysical observations (pulsars, lensing).

G.7 7. Laboratory Observability Assessment

In the laboratory, ρ is very small in units of G/c^2, so $|\beta - 1| \sim 2\Phi/c^2$ is at the level of $\lesssim 10^{-16}$–10^{-18} for typical altitudes/gradients—observable only with precision clocks.

Strong fields (neutron stars, BH) yield $|\beta - 1| \sim 0.1$–1, which is verified astrophysically.

G.8 8. Brief Summary (Rigorous)

- **The β–M relation is defined by the projection operator:** $\beta = \mathcal{P}[M]$ as a local functional with gradient corrections.
- **The dynamics of β are derived from the action:** via the Lagrangian coupling and the equations for M, we obtain a closed evolution equation for β.
- **Weak field:** calibration of the parameters leads to $\nabla^2\beta = 8\pi G\rho/c^2$, $\beta = 1 + 2\Phi/c^2$, consistent with GR.
- **Deviations:** gradient/nonlocal terms in $\mathcal{P}$ generate testable modifications that yield predictions for laboratory and astrophysical tests.

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H Compton Scattering in TCS: Geometric Derivation

Prerequisites and Invariants

- Absolute time (4+0): $t' = t$, the photon's light-like phase is conserved: $K^\mu K_\mu = 0$.
- Electron before collision: $P^\mu P_\mu = (m_e c)^2$; after: $P'^\mu P'_\mu = (m_e c)^2$.
- Projection operator Q (local homothety): $\det Q = 1$, consistent with the photon phase on the null subspace (lightlikeness is not violated).

H.1 4D Kinematics and 3D Projection

Let $K^\mu = (\omega/c, \boldsymbol{k})$, $K'^\mu = (\omega'/c, \boldsymbol{k}')$ be the 4-vectors of the photon before and after. For light: $\omega = c|\boldsymbol{k}|$, $\omega' = c|\boldsymbol{k}'|$. The scattering angle θ is defined as the angle between the 3D vectors $\boldsymbol{k}$ and $\boldsymbol{k}'$:

$$\cos\theta \equiv \frac{\boldsymbol{k}\cdot\boldsymbol{k}'}{|\boldsymbol{k}||\boldsymbol{k}'|}.$$

Conditions on the projection Q

- **Null constraint:** for any K with $K^\mu K_\mu = 0$, $(QK)^\mu (QK)_\mu = 0$ (the phase is light-like).
- **Volume conservation:** $\det Q = 1$ (the homothety law).
- **Momentum exchange axis:** one proper direction Q coincides with the intersection line of the 3D slices (the longitudinal axis is without scale), the transverse axis is subject to an anisotropic scale consistent with the two conditions above.

These requirements fix the *angular* dependence of the result and leave the numerical coefficient tied to the electron invariant $(m_e c)^2$.

H.2 Rigorous derivation of the Compton shift

Preservation of 4D invariants and the rotation geometry define a unique scalar combination of a pair of wave 4-vectors:

$$K\cdot K' \;=\; \frac{\omega\omega'}{c^2} - \boldsymbol{k}\cdot\boldsymbol{k}' \;=\; \frac{\omega\omega'}{c^2} - |\boldsymbol{k}||\boldsymbol{k}'|\cos\theta \;=\; \frac{\omega\omega'}{c^2}(1-\cos\theta),$$

since $|\boldsymbol{k}| = \omega/c$, $|\boldsymbol{k}'| = \omega'/c$. To exchange with an electron:

$$P'^\mu - P^\mu = K^\mu - K'^\mu, \qquad P'^2 = P^2 = (m_e c)^2.$$

Expanding $(P + K - K')^2 = (m_e c)^2$ and using $P^2 = (m_e c)^2$, $K^2 = K'^2 = 0$, we obtain (purely kinematically):

$$2P\cdot(K - K') = 0 \;\Rightarrow\; P\cdot K = P\cdot K'.$$

In the electron's rest frame before the interaction, $P^\mu = (m_e c, \mathbf{0})$, and therefore

$$P\cdot K = m_e\omega, \qquad P\cdot K' = m_e\omega'.$$

Therefore, $m_e\omega = m_e\omega'$, whichin its pure form contradicts the observed shift. Resolution: *after* the exchange, the electron is no longer at rest (not the same P^μ), and the condition $P'^2 = (m_e c)^2$ yields:

$$(P + K - K')^2 = (m_e c)^2 \ \Rightarrow\ 2P \cdot (K - K') = 2K \cdot K'.$$

That is,

$$m_e(\omega - \omega') = \frac{\omega\omega'}{c^2}(1 - \cos\theta).$$

Let's move on to the wavelengths: $\omega = 2\pi c/\lambda$, $\omega' = 2\pi c/\lambda'$:

$$m_e\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right) = \frac{c^2}{\lambda\lambda'}(1 - \cos\theta).$$

Multiply by $\frac{\lambda\lambda'}{c}$:

$$m_e(\lambda' - \lambda) = \frac{h}{c}\,(1 - \cos\theta).$$

And finally:

$$\boxed{\Delta\lambda \equiv \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) = \lambda_C(1 - \cos\theta)}$$

where the quantum energy-frequency coupling relation $E = h\nu = \hbar\omega$ (for the photon) and the standard definition of the Compton length $\lambda_C = h/(m_e c)$ are used. The entire derivation relies on *4D invariants* and the geometry of the angle between projections, without introducing masks or adjustable parameters.

Independence on the shape of Q

The result uses only:

1. lightlikeness $K^2 = K'^2 = 0$ (phase invariance on the null subspace),
2. electron rest invariant $P^2 = P'^2 = (m_e c)^2$,
3. 3D projection rotation geometry (angle θ) compatible with $\det Q = 1$.

Any local homothety of Q satisfying these three conditions leads to the same angular dependence $(1 - \cos\theta)$ and the same coefficient fixed by m_e.

H.3 Dynamic Interpretation in TCS

Energy and momentum transfer is understood as phase matching of a 4D oscillation (of a photon) with the local proper geometry of an electron in projection. The phase-matching condition and invariants are:

$$\Delta P^\mu = K^\mu - K'^\mu, \quad \Delta P^2 = 2K \cdot K', \quad P'^2 = (m_e c)^2,$$

does not require "particle collisions"and is formulated entirely in terms of 4D phase geometry and its 3D projection.

Cross-Section Perspective (Klein–Nishina Analog)

Kinematics yields an angular law. For the probabilities (cross sections), it is necessary to determine the projection mode density and the transition functional:

$$\frac{d\sigma_{\text{TCS}}}{d\Omega} \propto \left|\langle QK|K'\rangle\right|^2 \cdot \rho(\boldsymbol{k} \to \boldsymbol{k}')$$

with controlled anisotropy in ϕ_{4D}. In the limit of projection angle isotropy, the Klein–Nishina form is restored; experimental tests of the TCS determine the deviations.

Experimental Implications

- **One-sided anisotropy:** testing the dependence on ϕ_{4D} in the differential cross section.
- **Two-way invariance:** Round-trip averaging eliminates orientational anisotropy (consistency with Michelson-Morley).
- **Strong fields:** modification of the projection density of modes $\Rightarrow$, corrections to the angular distribution, and small corrections to $\Delta\lambda$.

I Geometric derivation of the Heisenberg uncertainty relation in TIS

I.1 Projection geometry and natural length

In the Theory of Curved Space (TIS), the observed 3D picture is obtained as a projection of a 4D oscillation onto a 3D slice. The light-like internal dynamics generate a finite amplitude along the unobservable coordinate, defining the natural projection length:

A = mc.

This length is the physical "thickness"of the intersection of the 4D soliton with the 3D slice: at the peak of the 4D oscillation, the local projection metric is stretched; at the decline, it is compressed. The finite amplitude and the light-like nature of the metric restoration generate nonzero geometric position spreads and a finite spectral width.

I.2 Operator Lower Bound as a Universal Framework

For a one-dimensional projection with normalized wave function $\psi(x)$, we define

$$\sigma_x^2 = \langle x^2\rangle - \langle x\rangle^2, \quad \sigma_p^2 = \langle p^2\rangle - \langle p\rangle^2, \quad \hat{p} = -i\hbar\,\frac{d}{dx}.$$

The commutator $[\hat{x}, \hat{p}] = i\hbar$ and Robertson's inequality yield a universal lower bound

$$\sigma_x\,\sigma_p \geq \frac{\hbar}{2},$$

which must be satisfied for any physical projection. The geometric derivation in TCS shows where the finite σ_x and σ_p come from and why their product cannot be smaller than $\hbar/2$.

I.3 Geometric Mechanism of Finite Width: Light-Like Recovery Scale

The light-like nature of the 4D oscillation determines the characteristic full-amplitude crossing time:

$$T_{\text{cross}} \sim \frac{2A}{c}.$$

The finiteness of T_{cross} generates a minimum frequency width $\Delta\omega \gtrsim 1/T_{\text{cross}}$ and, consequently, a finite pulse width

$$\Delta p \sim \hbar \frac{\Delta\omega}{c} \gtrsim \frac{\hbar}{2A}.$$

In parallel, the amplitude of the 4D oscillation itself produces a geometric spread in the order position A. Thus, the purely geometric multiplication "length $\times$ conjugate width"inevitably leads to a scale of $\sigma_x\sigma_p \sim \hbar/2$ and agrees with the operator lower bound.

I.4 Rigorous Calculations for Profile Shapes

To confirm the geometric estimate, we present two typical projection profiles (both normalized), which realize different 4D→3D intersection shapes and yield finite σ_x, σ_p at a scale of $A = \hbar/(mc)$.

I) Gaussian Profile (Robertson Minimum) Let

$$\psi_G(x) = \left(\frac{1}{2\pi\sigma_0^2}\right)^{1/4} \exp\left(-\frac{x^2}{4\sigma_0^2}\right), \quad \sigma_0 = \alpha A.$$

Then the known formulas yield

$$\sigma_x = \sigma_0, \quad \sigma_p = \frac{\hbar}{2\sigma_0}, \quad \Rightarrow \quad \sigma_x\sigma_p = \frac{\hbar}{2}.$$

Geometrically $\sigma_0 \propto A$: a wide 4D peak gives a large rms width σ_x, and a finite T_{cross} gives a finite σ_p, with the Gaussian profile reaching its lower limit.

II) Sech-profile (soliton-like projection) Take

$$\psi_S(x) = \frac{1}{\sqrt{2A}} \operatorname{sech}\left(\frac{x}{A}\right), \quad \int_{-\infty}^{\infty} |\psi_S|^2 dx = 1.$$

An exact calculation yields:

$$\langle x^2 \rangle = \frac{\pi^2}{12}A^2, \quad \sigma_x = \frac{\pi}{2\sqrt{3}}A \approx 0.9069A,$$

$$\langle p^2 \rangle = \frac{\hbar^2}{3A^2}, \quad \sigma_p = \frac{\hbar}{\sqrt{3}A} \approx 0.5774\frac{\hbar}{A}.$$

Therefore, the product

$$\sigma_x\sigma_p = \frac{\pi}{6}\hbar \approx 0.5236\hbar > \frac{\hbar}{2}.$$

This result demonstrates that non-Gaussian (soliton) forms yield a product greater than the minimum, preserving the geometric interpretation of the scale A.

I.5 Geometric-operator consistency summary

- The 4D oscillation amplitude $A = \hbar/(mc)$ defines the physical position scale $\sigma_x \sim A$
- The light recovery scale $T_{\text{cross}} \sim 2A/c$ produces a finite frequency and momentum width $\Delta p \sim \hbar/(2A)$, yielding $\sigma_x \sigma_p \sim \hbar/2$
- Rigorous projection models (Gauss, sech) confirm: Gaussian reaches a minimum $\hbar/2$, sech yields $\sigma_x \sigma_p = \pi\hbar/6 \approx 0.5236\hbar$
- The Robertson operator inequality $\sigma_x \sigma_p \geq \hbar/2$ serves as a universal lower bound, and the geometry of the TCS explains why the widths are finite and comparable to this limit.

Conclusion

In the TCS, the uncertainty relation is not introduced as a postulate: it follows from the projection geometry of the 4D oscillation onto a 3D slice (scale A) and from the finite light time of the metric recovery (scale T_{cross}). The operator framework guarantees a lower bound on $\hbar/2$, and geometric estimates and rigorous calculations for physical profiles show how the finite σ_x and conjugate σ_p arise, and why their product naturally has the order of $\hbar/2$ or slightly larger.

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