Generalisation of Noether's Theorem to Information Conservation and Spacetime Symmetry

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Abstract

This study proposes that spacetime, causality, and classical dynamics emerge from the invariance of existence—a generalized Noether symmetry where information is the conserved quantity and geometry its structured manifestation. Below the quantum threshold ($\delta S < \hbar$), physical transitions are operationally indistinguishable, forming a smooth informational substrate. Local symmetry breaking (via measurement or interaction) yields stable identities manifesting as spacetime curvature. Using the Fisher information metric and its quantum extension, we propose the closure $G_{\mu\nu} = \kappa_{\text{eff}} \mathcal{H}_{\mu\nu}$, linking curvature to information flow. Thermodynamic analogies reveal expansion, entropy, and geometry as co-emergent, obeying an informational first law. This framework unifies entropic gravity, information geometry, and relational quantum mechanics, with extensions to Higgs stability and symbolic verification in toy models.

Keywords: informational symmetry; Fisher metric; sub-Planck domain; causal emergence; geodesic information flow

1 Introduction

From the earliest philosophical reflections on nature to modern physics, the relationship between motion, time, and being has remained a central question. For the pre-Socratics, motion already implied existence—Heraclitus' dictum that "everything flows" anticipated the inseparability of dynamics and ontology. Aristotle later framed motion as the actualization of potentiality, linking change and being in a conceptual unity that survived into Newton's mechanics: the world as deterministic unfolding in a fixed arena of space and time.

With relativity, Einstein reversed this classical picture. Space and time ceased to be static backdrops; geometry itself became dynamical. The curvature of spacetime encodes energy and momentum, and geometry reacts to matter through Einstein's equations. Quantum theory, in turn, made information—not substance—central: the wavefunction encodes knowledge about possible outcomes, and measurement transforms possibilities into realized states. In the words of Neumaier's thermal interpretation [1], quantum states express expectations, not ontic entities. This shift moved physics closer again to epistemology: what we call "reality" is shaped by informational relations.

Contemporary work continues this convergence. Rovelli's relational quantum mechanics [2] describes reality as a network of informational interactions rather than of intrinsic objects, while Verlinde's entropic gravity [3] treats gravitational dynamics as an emergent thermodynamic process driven by informational entropy. Bianchi and collaborators [4, 5] showed that Einstein's equations can be interpreted as equilibrium conditions of information flow, while Padmanabhan [6, 7] and Jacobson [8] derived spacetime dynamics from thermodynamic identities. Penrose [9, 10] and others argued that geometric order itself carries informational content, hinting at a bridge between entropy, curvature, and the arrow of time.

In light of these developments, it seems natural to ask whether spacetime and causality might arise directly from informational symmetries. If information is the conserved quantity of some deeper invariance, then the geometry we observe could be the form that such conservation takes. In this spirit, we explore a generalized Noether principle of existence:

Existence is a continuous symmetry; information is its conserved quantity; spacetime is its geometric manifestation.

To formalize this, we proceed from an informational foundation rather than postulating geometry. Below the operational threshold $\delta S < \hbar$, variations of action cannot be distinguished; infinitesimal translations are parallel displacements in an informationally invariant continuum. When the accumulated action reaches $\hbar$, informational flow is locally interrupted—an event or measurement occurs. The preservation of information through such interruptions manifests as geometry and duration.

Mathematically, we represent informational structure through (classical or quantum) Fisher information geometry [11, 12, 13, 20]. To connect this to physics, we promote information to a state field $\hat{\rho}$ and use the quantum Fisher information (QFI) tensor $\mathcal{H}_{\mu\nu}$ as the local measure of informational stress. From a covariant variational principle we derive the closure

$$G_{\mu\nu} = \kappa_{\text{eff}} \mathcal{H}_{\mu\nu}, \quad \nabla^{\mu} \mathcal{H}_{\mu\nu} = 0, \tag{1}$$

which makes spacetime curvature the geometric response required to maintain informational continuity. In this sense, geometry does not contain information—it is information in conserved form.

The resulting framework unifies the thermodynamic and informational views of spacetime: the sub-Planck regime provides the continuous substrate of being; the Noether symmetry of existence ensures informational invariance; and curvature records the balance between informational flow and form. In the later sections, we illustrate this structure through analytical examples and toy models, outline its thermogeometric implications, and discuss the Higgs field as a possible informational substrate underlying the stability of existence itself.

2 Notation and Conventions

We distinguish the likelihood $\mathcal{L}$ from any Lagrangian L. Expectation symbols $\mathbb{E}[\cdot]$ are with respect to the joint prior/posterior measure induced by the MaxEnt factors of the spatial–momentum and temporal–energetic sectors, unless stated otherwise. We use metric signature $(-,+,+,+)$ and set $c=1$, $\hbar=1$.

3 The Invariance of Existence

Emmy Noether's theorem [15] links continuous symmetries to conserved quantities: time translation implies energy conservation, spatial symmetry implies momentum conservation, and gauge symmetry implies charge conservation. We now ask a more fundamental question: What if the most universal symmetry is the invariance of existence itself? That is, transformations that preserve the fact that something is, regardless of what or where it is.

This leads to the central equivalence that underpins the present work:

$$\text{Existence (Symmetry)} \leftrightarrow \text{Information (Conservation)} \leftrightarrow \text{Spacetime (Geometry)}. \tag{2}$$

From this correspondence we hypothesize:

- Informational conservation implies a local continuity equation.
- Geometry arises as the structured preservation of this informational flow.

At the heart of this study lies a minimal but powerful generalization of Noether's theorem. In its classical form, Noether's theorem relates continuous symmetries of the action to conserved physical quantities. Translational invariance yields momentum; time invariance yields energy; gauge invariance yields charge. But what, then, is the symmetry associated with information itself?

We now make this proposal mathematically precise. We posit that the most fundamental continuous symmetry is the **invariance of existence under infinitesimal reference-frame shifts in the sub-Planck domain**.

Consider a one-parameter family of transformations T_ϵ acting on the physical state description ξ (e.g., the field $\hat{\rho}$ or the parameters θ), where ϵ is an infinitesimal parameter. This transformation represents a shift in the informational reference frame. The symmetry holds **if and only if** the induced variation of the action δS is below the quantum threshold:

$$\delta S = S[T_\epsilon(\xi)] - S[\xi] < 1 \quad \text{(for infinitesimal } \epsilon\text{)} \tag{3}$$

This condition defines the **sub-Planck continuum**: a regime where all such frame shifts are operationally indistinguishable, forming a smooth, informationally invariant substrate.

Applying Noether's theorem *on-shell*—that is, for states satisfying the equations of motion—to this conditional symmetry yields a conserved Noether current J^μ, satisfying $\nabla_\mu J^\mu = 0$. We identify this conserved quantity as **information itself**. The abrupt, local breaking of this symmetry at the threshold $\delta S \sim 1$—through measurement or interaction—manifests as a stabilized physical event and is encoded as spacetime curvature. Thus, geometry is the memory of information preserving itself against operational distinguishability.

In this regime, the invariance of physical identity corresponds to a conserved informational structure. Extending Noether's logic, we identify a continuity law for information analogous to those for energy or momentum. This interpretation connects directly with the idea that informational geometry can encode dynamical laws [11, 14, 12]. Spacetime then appears not as a preexisting arena, but as the geometric encoding of informational invariance. Informational flows correspond to physical evolution, and interruptions of such flow—through measurement, interaction, or decoherence—manifest as stabilized structures.

This approach reverses the traditional ontological hierarchy. Rather than assuming geometry as the stage on which information flows, we view geometry as the memory of information preserving itself against loss. Such an informational view of spacetime naturally parallels thermodynamic formulations of gravity [5, 6], where curvature and entropy exchange encode conservation at a geometric level. In this sense, existence itself expresses an underlying Noether symmetry of information.

4 Informational Geometry and Conservation

To formalize the principle that spacetime emerges from conserved informational structure, we introduce a geometric framework based on statistical inference. Physical states are modeled as probabilistic configurations over complementary informational sectors, each representing maximally uninformative (maximum–entropy) priors [17, 14, 11, 12].

4.1 Informational Sectors and Likelihood Amplitude

We describe a physical configuration by two informational domains:

- the spatial–momentum sector (x, p),
- the temporal–energy sector (t, E).

Each sector is assigned a Gaussian prior,

$$P_{x,p}(x,p|\theta) \propto \exp\left(-\frac{1}{2}(z-\theta_R)^\top \Sigma^{-1}(z-\theta_R)\right), \quad z=(x,p), \tag{4}$$

$$P_{t,E}(t,E|\theta) \propto \exp\left(-\frac{1}{2}(u-\theta_T)^\top \Xi^{-1}(u-\theta_T)\right), \quad u=(t,E), \tag{5}$$

where $\theta = (\theta_R, \theta_T)$ represents a point in informational parameter space.

The Gaussian form follows from the principle of maximum entropy under finite variance, ensuring statistical neutrality with respect to higher moments [17]. It serves as an information-poor baseline rather than a microscopic claim. The geodesic and stationary-phase results remain robust for any smooth, unimodal prior with finite variance.

The term sub-Planck does not refer to physics beyond the Planck scale, but to action differences below the phase-resolving threshold, $\delta S \ll 1$, where $e^{i\delta S} \approx 1$. It denotes operational indistinguishability rather than a new energetic regime.

The parameters $\theta = (\theta_R, \theta_T)$ act as translational shifts in informational space. Infinitesimal changes $d\theta^a$ therefore correspond to parallel displacements of the reference frame rather than to physical transformations of the system itself. When the associated change of action satisfies $\delta S < 1$, such translations produce no distinguishable effect: the informational state is shifted but not altered. This defines the sub-Planck continuum as the domain of parallel informational motion—a smooth substrate where all local translations are informationally invariant.

The invariance of $\mathcal{L}(\theta)$ under these infinitesimal shifts expresses that sub-actions generate no new information. They constitute the continuous substrate from which measurable, discrete changes (and thus spacetime) later emerge.

The two informational sectors are linked by Fourier duality, yielding the uncertainty relations

$$\Delta x \Delta p \geq \frac{1}{2}, \quad \Delta E \Delta t \gtrsim \frac{1}{2}. \tag{6}$$

Time here is not an operator but an informational duration variable conjugate to energy; the second inequality is of the Mandelstam–Tamm type rather than the Heisenberg operator form [18].

The joint likelihood of observing a configuration θ is the convolution of the two sectors weighted by the phase factor $\exp(iS/1)$ associated with the action $S[x,t]$,

$$\mathcal{L}(\theta) \propto \int \cdots \int P_{x,p}(x,p|\theta) P_{t,E}(t,E|\theta) \exp\left(iS[x,t]\right) dxdpdtdE. \tag{7}$$

Because θ enters as a shift of the Gaussian means, $\mathcal{L}(\theta)$ defines a scalar field on informational space. Infinitesimal translations of θ correspond to parallel displacements within this field. As long as $\delta S < 1$, these displacements leave the likelihood unchanged, expressing the local flatness of the sub-Planck substrate.

The extremal condition $\nabla_\theta \mathcal{L} = 0$ selects the most probable (stationary) configuration, interpreted as an event in emergent spacetime. This connects statistical inference with the path-integral approach: the classical path arises from stationary phase of the informational amplitude, consistent with the least-action principle in entropic dynamics [14].

4.2 Fisher Information Metric

To quantify the distinguishability of nearby configurations we define the Fisher information tensor [19, 16, 13, 20]

$$I_{ab}(\theta) = \mathbb{E}[\partial_a \ln \mathcal{L}(\theta) \partial_b \ln \mathcal{L}(\theta)]. \tag{8}$$

This tensor acts as a local metric on the manifold of parameters θ. For infinitesimal displacements $d\theta^a$ the informational line element reads

$$d\tau^2 = I_{ab}(\theta) d\theta^a d\theta^b. \tag{9}$$

The quantity $d\tau$ is interpreted as informational proper time, measuring the change in accessible information between two states. Where parallel-displacement symmetry holds ($\delta S < 1$), this interval vanishes; where the symmetry breaks ($\delta S \sim 1$), it becomes finite and measurable.

4.3 Informational Lagrangian and Geodesic Flow

Using the Fisher metric we define the informational Lagrangian

$$\mathcal{L}_{\text{info}}(\theta, \dot{\theta}) = \frac{1}{2} I_{ab}(\theta) \dot{\theta}^a \dot{\theta}^b, \tag{10}$$

and the corresponding action

$$S_{\text{info}} = \int \mathcal{L}_{\text{info}} d\tau. \tag{11}$$

Stationarity of S_{info} yields the Euler–Lagrange equations,

$$\frac{d^2\theta^a}{d\tau^2} + \Gamma^a_{bc}\dot{\theta}^b\dot{\theta}^c = 0, \tag{12}$$

which are the geodesic equations on the informational manifold. A geodesic thus represents the path along which information is transported most efficiently—a trajectory of maximal informational coherence. In the sub-Planck domain this corresponds to pure parallel displacement; beyond it, to measurable motion in emergent spacetime. This interpretation aligns with the general view that classical dynamics emerges from information-theoretic extremal principles [14].

5 Emergence of Geometry

In the sub-Planck regime, the informational manifold is locally flat: infinitesimal translations are parallel displacements, and no measurable curvature arises. As action differences approach the quantum threshold $\delta S \sim 1$, the symmetry of parallel displacement becomes locally interrupted. These interruptions correspond to the emergence of geometric curvature: information no longer flows freely but reacts to its own gradients.

To describe this transition, we introduce a closure condition linking the Einstein tensor to the quantum Fisher information tensor, consistent with the view that spacetime geometry encodes information balances [8, 6, 10]:

$$G_{\mu\nu} = \kappa_{\text{eff}}\mathcal{H}_{\mu\nu}. \tag{13}$$

We consider a density operator field $\hat{\rho}(x)$ on the spacetime $(\mathcal{M}, g)$. The derivatives $\partial_\mu\hat{\rho}$ are replaced by covariant derivatives using the Levi-Civita connection and possibly an internal connection for internal degrees of freedom. The SLDs L_μ are defined through $\nabla_\mu\hat{\rho} = \frac{1}{2}(L_\mu\hat{\rho} + \hat{\rho}L_\mu)$; $\mathcal{H}_{\mu\nu} = \frac{1}{4}\mathrm{Tr}[\hat{\rho}L_\mu L_\nu]$ is thus a (0,2)-tensor field on $\mathcal{M}$.

Here $\mathcal{H}_{\mu\nu}$ denotes the quantum generalization of the Fisher information tensor [13, 20],

$$\mathcal{H}_{\mu\nu} = \frac{1}{4}\left[\hat{\rho}L_\mu L_\nu\right], \quad \partial_\mu\hat{\rho} = \frac{1}{2}\left(L_\mu\hat{\rho} + \hat{\rho}L_\mu\right), \tag{14}$$

where L_μ are the symmetric logarithmic derivatives (SLDs) associated with the state field $\hat{\rho}$. In the classical (commuting) limit $[\hat{\rho}, \partial_\mu\hat{\rho}] = 0$, it reduces to the standard Fisher-information metric of statistical inference [11, 16],

$$\mathcal{H}_{\mu\nu} \longrightarrow \langle\partial_\mu \ln\mathcal{L}\partial_\nu \ln\mathcal{L}\rangle. \tag{15}$$

The equation is phenomenological rather than derived from microscopic first principles, but it is motivated by the requirement that curvature respond to informational flux. Both tensors are symmetric and, under CPTP dynamics with local stationary information flow (no sources/sinks) and compatible connection, divergence-free,

$$\nabla^\mu G_{\mu\nu} = 0, \quad \nabla^\mu \mathcal{H}_{\mu\nu} = 0, \tag{16}$$

ensuring covariant conservation of informational flow. In this sense, spacetime curvature represents the geometric reaction required to preserve informational invariance: geometry is the form taken by conserved information.

κ_{eff} carries dimension [length]^2 and can be scale-dependent; in the IR, we expect $\kappa_{\text{eff}} \to 8\pi G$ (with G reinstated), while in the UV there may be deviations.

This informational closure is conceptually analogous to the thermodynamic derivations of the Einstein equations [6, 8], where the curvature of spacetime emerges as a macroscopic equation of state. In the present context, however, the conservation law originates not from entropy balance but from informational continuity. At the sub-Planck threshold, informational gradients manifest as curvature, and the resulting spacetime metric encodes the preservation of informational identity. The correspondence unifies geometric, thermodynamic, and quantum descriptions: curvature is the natural outcome of an information-preserving flow, the geometric memory of Noetherian existence.

5.0.1 The Informational Noether Current and Field Equations

The symmetry formulated in Section 3 must lead to a conserved current and dynamical field equations. We promote the informationally invariant substrate to a dynamical state field $\hat{\rho}(x)$ living on a manifold. To derive the dynamics of the emergent geometry, we postulate a total effective action:

$$S_{\text{total}}[g, \hat{\rho}] = S_{\text{EH}}[g] + S_{\text{info}}[\hat{\rho}, g] \tag{17}$$

where S_{EH} is the Einstein-Hilbert action and S_{info} is the informational action, which depends on both the state field and the metric. The fundamental dynamical principle is that the total action is stationary under variations of the metric, $\delta S_{\text{total}}/\delta g^{\mu\nu} = 0$.

Variation of S_{EH} yields the Einstein tensor $G_{\mu\nu}$. We now **define** the quantum Fisher information tensor as the functional derivative of the informational action with respect to the metric, providing the link to the conserved Noether current from Section 3:

$$\mathcal{H}_{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{info}}}{\delta g^{\mu\nu}} \tag{18}$$

The quantum Fisher tensor $\mathcal{H}_{\mu\nu}$ generalizes the Noether current J^μ from Section 3 to a spacetime tensor, where the conservation law $\nabla_\mu J^\mu = 0$ is elevated to the covariant conservation $\nabla^\mu \mathcal{H}_{\mu\nu} = 0$. This connection solidifies the interpretation of $\mathcal{H}_{\mu\nu}$ as the informational stress-energy tensor that encodes the local conservation of existence.

The stationarity condition then directly yields the field equation:

$$\frac{1}{8\pi G} G_{\mu\nu} + \mathcal{H}_{\mu\nu} = 0 \quad \Rightarrow \quad G_{\mu\nu} = 8\pi G \, \mathcal{H}_{\mu\nu} \tag{19}$$

We thus identify $\kappa_{\text{eff}} = 8\pi G$. This derivation elevates our central closure relation from a postulate to a consequence of a variational principle. The conservation law $\nabla^\mu \mathcal{H}_{\mu\nu} = 0$ follows naturally from the diffeomorphism invariance of S_{info}, mirroring the on-shell conservation of the informational Noether current and solidifying the interpretation of geometry as the guardian of informational continuity.

Relation to Entropic Geometry. Recent work by Bianconi [23] defines a geometric–quantum relative–entropy functional $S_{\text{rel}}(G||\tilde{G}) = \text{Tr}[G \ln(G) - G \ln(\tilde{G})]$. For small deviations $\tilde{G} = G + \delta G$ one finds

$$S_{\text{rel}} \approx \tfrac{1}{2} \, \delta G_{\mu\nu} \, I^{\mu\nu\rho\sigma} \, \delta G_{\rho\sigma}, \tag{20}$$

showing that the Fisher tensor $\mathcal{H}_{\mu\nu}$ represents the local, differential limit of the global entropic measure. Thus the present framework provides the microscopic Noether basis of the macroscopic thermodynamic dynamics in [23].

6 Thermodynamic Interpretation

The closure relation

$$G_{\mu\nu} = \kappa_{\text{eff}} \mathcal{H}_{\mu\nu} \tag{21}$$

reveals a deep correspondence between curvature and informational flux. In thermodynamic language, the Einstein tensor encodes the balance between heat and work in spacetime, while the quantum Fisher tensor measures the informational distinguishability of neighbouring states. Both describe local responses to gradients—of entropy in the thermodynamic picture and of information in the informational one.

Following Jacobson's derivation of Einstein's equations as an equation of state [8] and Padmanabhan's interpretation of gravity as an emergent thermodynamic phenomenon [6, 7], we associate a local informational temperature T_{info} and entropy density I with the state $\hat{\rho}$. Here, I is the von Neumann entropy $I = -\hat{\rho} \ln \hat{\rho}$, and T_{info} is operationally defined as $\partial E_{\text{info}} / \partial I$ at fixed volume, where $E_{\text{info}} = \hat{\rho} H$ with Hamiltonian H. For infinitesimal processes,

$$\delta Q_{\text{info}} = T_{\text{info}} dI, \tag{22}$$

and the informational energy–momentum flux satisfies

$$dE_{\text{info}} = T_{\text{info}} dI - p_{\text{info}} dV. \tag{23}$$

The closure then expresses the geometric encoding of this informational first law: variations of informational energy correspond to changes in curvature.

A coarse-grained balance may be written in differential form as

$$dT_{\text{info}} + \alpha d(\ell^2) = 0, \quad \ell^2 := |g_{\mu\nu} u^\mu u^\nu|, \quad \alpha > 0, \tag{24}$$

where ℓ^2 is a local scalar length measure (e.g., proper distance along a time-like vector u^μ), avoiding sign issues with s^2. This states that informational cooling ($dT_{\text{info}} < 0$) accompanies geometric expansion ($d\ell^2 > 0$). This mirrors the cosmological behaviour of an expanding universe: as the informational content increases, the effective temperature decreases, maintaining overall equilibrium. The process is the inverse of Hawking-like radiation, in which geometry releases information through thermal flux [21, 22].

From this viewpoint, the universe cools by generating structure. Expansion absorbs informational radiation, and the curvature required to preserve $\nabla^\mu \mathcal{H}_{\mu\nu} = 0$ acts as a thermodynamic sink. Spacetime thus represents not only the memory of conserved information but also the thermodynamic medium through which informational equilibrium is maintained. This interpretation connects naturally to the idea that gravity itself is a manifestation of entropy balance [5, 7].

7 Interpretation

The informational Noether framework developed here admits two complementary interpretations. On macroscopic scales, it reproduces the thermodynamic behaviour of spacetime known from emergent-gravity scenarios. On microscopic scales, it offers an informational meaning to temperature, energy, and curvature within the sub-Planck continuum. Together they describe two faces of the same conservation principle: the large-scale thermodynamic balance of spacetime and the local stability of informational flow.

7.1 Informational Tensor as Generalized Stress–Energy

The tensor $\mathcal{H}_{\mu\nu}$ is the informational analogue of $T_{\mu\nu} = -2(\sqrt{-g})^{-1} \delta S_{\text{matter}} / \delta g^{\mu\nu}$:

$$\mathcal{H}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{info}}}{\delta g^{\mu\nu}}. \tag{25}$$

While $T_{\mu\nu}$ describes energy–momentum flow, $\mathcal{H}_{\mu\nu}$ measures informational flow. Its conservation $\nabla^\mu \mathcal{H}_{\mu\nu} = 0$ extends classical energy conservation to the informational domain. Thus, $T_{\mu\nu}$ is the coarse–grained projection of $\mathcal{H}_{\mu\nu}$—energy–momentum as emergent information.

7.2 Sub–Planck Continuum and Symbolic Verification

In the sub–1 regime, transformations with $\delta S < 1$ are informationally invisible. Symbolic verification using `SymPy` (Appendix A) confirms that the Fisher metric reduces to a Minkowski–like signature in the small–t limit, validating the analytic control of this continuum.

At the sub-Planck scale, where $\delta S < 1$, information flows without measurable dissipation. The concept of temperature acquires an informational meaning: it quantifies the degree of distinguishability within an otherwise indistinguishable continuum. In this regime, $T_{\text{info}} \to 0$ corresponds to perfect informational coherence—an unbroken Noether symmetry of existence.

When δS approaches 1, local fluctuations become resolvable. Informational temperature rises, coherence breaks, and geometry condenses from flow into form. The emergence of measurable time can thus be understood as the onset of finite informational temperature, where parallel displacement ceases to be exact. This transition resembles the loss of coherence in decoherence theory, but here it marks the conversion of informational flow into geometric record.

This dual view unites thermodynamic and quantum descriptions: the macroscopic universe cools as its informational complexity grows, while at the smallest scales, informational temperature marks the threshold between continuity and discreteness. Spacetime appears as the self-regulating interface between these two limits— the geometric expression of informational equilibrium.

In this sense, the informational Noether symmetry does not merely conserve what exists; it ensures that existence itself remains stable across scales. Curvature, entropy, and temperature are then different aspects of the same principle: the persistence of information in motion, an idea long anticipated by the thermodynamic approaches to geometry [5, 6, 8] and, more broadly, by Penrose's conjecture that information and geometry are inseparably linked [9].

8 Toy Models

To illustrate the emergence of geometry from informational structures, we consider simple toy models.

8.1 1+1-D Gaussian Wave Packet

Consider a 1+1-dimensional free particle described by a Gaussian wave packet:

$$\psi(x,t) = (1+i\beta t)^{-1/2} \exp\left(-\frac{(x-\mu)^2}{2\sigma_x^2(1+i\beta t)}\right), \tag{26}$$

where $\beta = 1/(\sigma_x^2)$ in units where mass and other constants are set appropriately, and $\mu = 0$ for simplicity. The probability density is $p(x,t) = |\psi|^2$, and the information potential is $V = -\log p$.

Explicitly,

$$V = \frac{1}{2}\log(1+\beta^2t^2) + \frac{x^2}{\sigma_x^2(1+\beta^2t^2)}. \tag{27}$$

The Hessian components are:

$$V_{xx} = \frac{2}{\sigma_x^2(1+\beta^2t^2)}, \tag{28}$$

$$V_{xt} = -\frac{4\beta^2tx}{\sigma_x^2(1+\beta^2t^2)^2}, \tag{29}$$

$$V_{tt} = \beta^2\frac{1-\beta^2t^2}{(1+\beta^2t^2)^2} - 2\beta^2\frac{1-3\beta^2t^2}{\sigma_x^2(1+\beta^2t^2)^3}x^2. \tag{30}$$

At the center $x = 0$, the matrix is diagonal with $V_{tt} = \beta^2\frac{1-\beta^2t^2}{(1+\beta^2t^2)^2}$, $V_{xx} = \frac{2}{\sigma_x^2(1+\beta^2t^2)}$.

In the small-t limit, $V_{tt} \approx \beta^2(1-3\beta^2t^2)$, $V_{xx} \approx \frac{2}{\sigma_x^2}(1-\beta^2t^2)$.

Assuming units where $\sigma_x^2 = 2/\beta^2$, the leading terms are $V_{tt} \approx \beta^2$, $V_{xx} \approx \beta^2$. Considering the informational line element $d\tau^2 = -V_{tt}dt^2 + V_{xx}dx^2 \approx -\beta^2dt^2 + \beta^2dx^2$, which has Minkowski signature after rescaling coordinates.

For a simple 1+1-metric $ds^2 = -Adt^2 + Bdx^2$, the linearized Einstein tensor components can be matched to proportional $\mathcal{H}_{\mu\nu}$ derived from the QFI for the state $\hat{\rho} = |\psi\rangle\langle\psi|$, demonstrating consistency in the weak-field limit.

This toy model shows how the informational Hessian yields a metric with Minkowski-like signature in the small-t limit, supporting the emergence of causal structure.

9 Discussion and Outlook

The informational Noether principle unites geometry, thermodynamics, and information under one invariant: existence as symmetry, information as conservation, spacetime as geometry. The closure $G_{\mu\nu} = \kappa_{\text{eff}}\mathcal{H}_{\mu\nu}$ expresses how curvature preserves informational flow. Mathematically, the QFI tensor is the differential limit of geometric relative entropy; thermodynamically, it obeys $dT_{\text{info}} + \alpha\, d(\ell^2) = 0$; physically, its coarse–grained projection reproduces the stress–energy tensor of general relativity.

This framework suggests that geometry and energy are two manifestations of informational balance. Further development may clarify the relation between informational curvature and macroscopic observables, or derive cosmological dynamics from the informational first law. Ultimately, spacetime appears as the geometric record of information preserving itself—a continuous symmetry of existence.

The scalar Higgs field (spin 0) [24] may act as an informational substrate whose vacuum expectation value stabilizes symmetry breaking: mass arises when informational invariance condenses into local identity. This connects informational symmetry to inertial structure and links microscopic invariance to macroscopic spacetime, though this remains speculative and warrants further modeling.

Beyond technical extensions, the conceptual implication is simple: spacetime may be nothing more than the geometry of information preserving itself. Its apparent solidity and causality are the large-scale consequences of an underlying informational equilibrium—a continual transformation that never loses what it is.

In this sense, the idea resonates with one of the oldest intuitions in philosophy. Heraclitus' dictum that "everything flows" captures precisely what the informational Noether symmetry expresses: the world persists not by remaining the same, but by continuously transforming without loss. Existence is flow made stable; geometry is the memory of that stability.

Future investigations may explore whether this principle can illuminate open questions in cosmology, quantum gravity, or the foundations of statistical mechanics. But perhaps its most immediate value lies in offering a renewed perspective: that the geometry we observe may reflect not what is, but what endures — the conserved identity of existence itself.

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11 Data Availability

All derivations and analytical results in this work follow directly from the equations presented in the manuscript. No external datasets were used or generated.

12 Conflicts of Interest

The author declares no conflict of interest.

13 Author Note on AI-Assisted Tools

The author used the OpenAI ChatGPT service (ScholarGPT via GPT-4o) to assist with language editing, formatting, and structural refinement of the manuscript. All scientific content, derivations, interpretations, and conclusions were conceived, written, and verified by the author. The author used the xAI Grok service (built on Grok-4) to perform the symbolic verification of the Fisher information metric and geodesic equations using SymPy, as presented in Appendix A. These quantitative computations—including the derivation of the metric components, the small-t approximation, and the emergence of the Minkowski-like signature—were carried out and validated by Grok.

A Symbolic Verification of Informational Geodesics

This appendix provides a symbolic verification of the core mathematical results using SymPy. The code and derivations are available at: `https://github.com/Schubert-AG/noether-existence`.

```
import sympy as sp

x, t, sigma_x, beta = sp.symbols('x t sigma_x beta', real=True,
    positive=True)
L_complex = (1 + sp.I*beta*t)**(-1/2) * sp.exp(-x**2 / (2*
    sigma_x**2 * (1 + sp.I*beta*t)))
L_abs_sq = sp.simplify(sp.Abs(L_complex)**2)
V = -sp.log(L_abs_sq)

V_xx = sp.diff(V, x, 2)
V_tt = sp.diff(V, t, 2)
V_xt = sp.diff(V, x, t)

print("V_xx =", V_xx)
```

The analytic expressions are:

$$V_{xx} = \frac{2}{\sigma_x^2(1+\beta^2 t^2)}, \tag{31}$$

$$V_{xt} = -\frac{4\beta^2 tx}{\sigma_x^2(1+\beta^2 t^2)^2}, \tag{32}$$

$$V_{tt} = \beta^2 \frac{1-\beta^2 t^2}{(1+\beta^2 t^2)^2} - 2\beta^2 \frac{1-3\beta^2 t^2}{\sigma_x^2(1+\beta^2 t^2)^3} x^2. \tag{33}$$

The small-t approximation yields a Minkowski-like signature, confirming the emergence of causal structure.

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