

# The Quantum as Minimal Difference: A Causal–Information Foundation

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## Abstract

We derive the existence of a universal minimal action scale from a causal–information principle: the difference between two causally equivalent infinite systems must be finite and observable. Starting from this principle and a holographic bound on boundary information, we prove that any system with finite boundary information capacity must possess a minimal phase-space cell  $\Delta\Omega_{\min} > 0$ . Under symmetry assumptions this implies a universal constant with dimensions of action, identified with the Planck constant  $\hbar$ . The framework unifies vacuum fluctuations, horizon quantization, and holography at the kinematic level without postulating quantum mechanics. We present a rigorous axiomatic derivation and testable predictions.

## 1 Causal Principle and Information Finiteness

### 1.1 The Observable Difference Principle

Consider two physical systems  $A$  and  $B$  that are *causally equivalent*, meaning they share identical causal structure and dynamics. In any operational comparison, we only measure their *difference* through finite boundaries over finite times.

**Principle of Causal Observable Difference (PCOD).** If two causally equivalent infinite systems  $A$  and  $B$  are compared, the result of any physical measurement of their difference must be:

1. *Finite:* The outcome cannot be infinite, as this would require infinite information transfer in finite time, violating causality.
2. *Observable:* The difference must be resolvable within the finite information capacity of the measurement apparatus.

This principle states that  $\infty - \infty$  cannot be undefined in nature; it must yield a finite result. We take PCOD as a foundational physical principle, logically prior to quantum mechanics.

### 1.2 Axiom: Holographic Information Capacity

Motivated by PCOD, we assume that any finite boundary has finite information capacity.

**Axiom 1 (Holographic Bound).** *For any closed boundary  $\Sigma$  of area  $A$ , the total number of bits  $N_{bits}$  that can ever be reliably encoded on it is bounded by*

$$N_{bits} \leq \eta A, \tag{1}$$

where  $\eta$  is a universal information density (bits per unit area). We define the maximal achievable value

$$N_{bits,max} \equiv \eta A, \quad (2)$$

and say that the bound is saturated for maximally informative systems.

Equivalently, the maximum number of distinguishable microstates for a maximally informative boundary is

$$N_{states,max} = 2^{N_{bits,max}} = 2^{\eta A}. \quad (3)$$

## 2 Axioms of Phase-Space Resolution

### 2.1 Symplectic Structure and Accessibility

**Axiom 2** (Symplectic Phase Space). *The space of all kinematically possible configurations is a  $2n$ -dimensional symplectic manifold  $(\mathcal{M}, \omega)$ . For any compact region  $R \subset \mathcal{M}$ , its symplectic volume is*

$$\Omega(R) = \int_R \frac{\omega^n}{n!}. \quad (4)$$

**Axiom 3** (Finite Accessible Region). *The physically accessible region  $\mathcal{M}_{acc} \subseteq \mathcal{M}$  has finite symplectic volume*

$$\Omega_{total} \equiv \Omega(\mathcal{M}_{acc}) < \infty. \quad (5)$$

### 2.2 Finite Distinguishability and Granularity

**Axiom 4** (Finite Distinguishability). *There exists a surjective map  $\Phi : \mathcal{M}_{acc} \rightarrow \Sigma$ , where  $\Sigma$  is a finite set of  $N_{states}$  physically distinguishable outcomes, with*

$$N_{states} \leq N_{states,max}. \quad (6)$$

The pre-images

$$C_\sigma = \Phi^{-1}(\{\sigma\}), \quad \sigma \in \Sigma, \quad (7)$$

are called cells. Each cell  $C_\sigma$  contains at least one open set in  $\mathcal{M}_{acc}$ .

**Theorem 1** (Existence of Minimal Phase-Space Cell). *Under Axioms 2–4, there exists a strictly positive minimal symplectic volume per distinguishable state:*

$$\Delta\Omega_{min} = \min_{\sigma \in \Sigma} \Omega(C_\sigma) > 0. \quad (8)$$

*Proof.* By Axiom 4, the accessible region  $\mathcal{M}_{acc}$  is partitioned into a finite number  $N_{states}$  of cells  $C_\sigma$ . Each cell has symplectic volume  $\Omega(C_\sigma) \geq 0$ . Since every  $C_\sigma$  contains an open set, and the Liouville measure induced by  $\omega$  assigns strictly positive volume to any nonempty open subset, we have  $\Omega(C_\sigma) > 0$  for all  $\sigma$ . The set  $\{\Omega(C_\sigma)\}_{\sigma \in \Sigma}$  is finite and consists of strictly positive numbers, therefore its minimum exists and is strictly positive. Define  $\Delta\Omega_{min}$  as this minimum. ■

**Corollary 1** (Upper Bound on Distinguishable States). *The total number of distinguishable states satisfies*

$$N_{states} \leq \frac{\Omega_{total}}{\Delta\Omega_{min}}. \quad (9)$$

*Proof.* We have

$$\Omega_{total} = \sum_{\sigma \in \Sigma} \Omega(C_\sigma) \geq \sum_{\sigma \in \Sigma} \Delta\Omega_{min} = N_{states} \Delta\Omega_{min}, \quad (10)$$

which implies the stated inequality. ■

**Axiom 5** (Homogeneous Resolution). *All cells  $C_\sigma$  have equal symplectic volume:*

$$\Omega(C_\sigma) = \Delta\Omega_{\min} \quad \text{for all } \sigma \in \Sigma. \quad (11)$$

Physically, this corresponds to a maximally informative, homogeneous discretization of phase space, in which both the holographic bound (Axiom 1) and the volume bound of the corollary are saturated.

**Theorem 2** (Universal Action Constant). *Under Axioms 2–5, there exists a universal constant  $a_*$  with dimensions of action such that*

$$\Delta\Omega_{\min} = (2\pi a_*)^n. \quad (12)$$

*Proof.* By Axiom 5, all cells have equal volume  $\Delta\Omega_{\min}$ . For any integrable subsystem, we can introduce action–angle variables  $(J_i, \theta_i)$ ,  $i = 1, \dots, n$ , with  $\theta_i \sim \theta_i + 2\pi$  and

$$\omega = \sum_{i=1}^n dJ_i \wedge d\theta_i. \quad (13)$$

A fundamental cell can be chosen as the product of intervals  $\Delta\theta_i = 2\pi$  and  $\Delta J_i = a_*$ , giving symplectic volume

$$\Delta\Omega_{\text{cell}} = \prod_{i=1}^n \int_0^{2\pi} d\theta_i \int_0^{a_*} dJ_i = (2\pi a_*)^n. \quad (14)$$

Homogeneous resolution implies  $\Delta\Omega_{\text{cell}} = \Delta\Omega_{\min}$ , hence  $\Delta\Omega_{\min} = (2\pi a_*)^n$ . The dimension of  $\omega$  in classical mechanics is [action], so  $[\Delta\Omega_{\min}] = [\text{action}]^n$  and  $[a_*] = [\text{action}]$ . ■

We now identify this constant with the experimentally measured Planck constant:

$$\hbar \equiv a_*. \quad (15)$$

## 3 Physical Interpretations

### 3.1 Vacuum Fluctuations as Minimal Action Events

In standard quantum theory, the time–energy uncertainty relation

$$\Delta E \cdot \Delta t \gtrsim \frac{\hbar}{2} \quad (16)$$

can be interpreted as a statement that any physically resolvable process requires a minimal action budget of order  $\hbar$ .

Within our framework, this emerges as follows: any transition between distinguishable states must traverse at least one minimal phase-space cell of volume  $(2\pi\hbar)^n$ , so the integrated action along the trajectory cannot be smaller than  $\mathcal{O}(\hbar)$ . Vacuum fluctuations correspond to the system exploring neighboring cells in  $\mathcal{M}$  consistent with this minimal action cost. The usual ultraviolet divergence of zero-point energy is suggestively tamed kinematically by the finite number of cells  $N_{\text{states}}$  available within any bounded region of phase space.

### 3.2 Horizon Quantization

For a black-hole horizon (boundary  $S^2$ ) the number of physically distinct states is finite and proportional to the area. Bekenstein and Mukhanov argued that the horizon area should be quantized in discrete steps [1, 2],

$$\Delta A = 4 \ln k \cdot \frac{\hbar G}{c^3}, \quad (17)$$

where  $k$  is an integer counting the degeneracy of each area level.

Within our framework, this quantization can be seen as a consequence of:

1. finite boundary information capacity (Axiom 1),
2. a minimal action quantum  $\hbar$  arising from  $\Delta\Omega_{\min} = (2\pi\hbar)^n$ ,
3. and the gravitational coupling  $G$  relating area and energy.

The appearance of the same  $\hbar$  in both phase-space cell volumes and horizon area increments reflects a common origin in finite-resolution kinematics, rather than a separate “quantum gravity” postulate.

### 3.3 Holographic Principle and Information Density

The Bekenstein–Hawking entropy bound for a black hole of horizon area  $A$  [8] reads

$$S_{\max} = \frac{k_B A}{4L_p^2} = \frac{k_B A c^3}{4\hbar G}. \quad (18)$$

If we identify  $S_{\max}$  with the maximum Shannon entropy on the boundary,

$$S_{\max} = k_B \ln N_{\text{states,max}} = k_B N_{\text{bits,max}} \ln 2 = k_B \eta A \ln 2, \quad (19)$$

then comparison yields

$$\eta = \frac{c^3}{4\hbar G \ln 2}. \quad (20)$$

In other words, the Bekenstein–Hawking relation fixes the information density  $\eta$  in terms of the fundamental constants  $c$ ,  $G$  and  $\hbar$ .

Logically, our framework reverses the usual interpretation: in Axiom 1,  $\eta$  is introduced as a primitive information-density parameter; the existence of a minimal action scale  $a_*$  follows from Axioms 2–5; and empirical black-hole thermodynamics then fixes the numerical value of  $\eta$  and confirms that  $a_*$  coincides with the observed Planck constant  $\hbar$ .

## 4 Connection to Quantum Information Geometry

In quantum information theory, distinguishability between states  $\rho_1$  and  $\rho_2$  is quantified by the Bures angle

$$L(\rho_1, \rho_2) = \arccos \sqrt{F(\rho_1, \rho_2)}, \quad (21)$$

where  $F$  is the Uhlmann fidelity [3]. For a pair of pure states carrying one classical bit of distinguishable information (orthogonal states),  $F = 0$  and  $L = \pi/2$ .

If we associate to a trajectory in Hilbert space an “action length”  $A = \hbar L$ , then evolving between orthogonal states costs an action of order  $\sim (\pi/2)\hbar$ . This suggests an interpretation of  $\hbar$  as a natural conversion factor between geometric distinguishability (Bures/Fubini–Study distance) and physical action. The comparison with Landauer’s bound [7] then indicates that erasing or creating classical information generally involves action scales of order  $\hbar$  per bit, consistent with the finite-resolution picture.

## 5 Quantum Speed Limits as Kinematic Constraints

The Mandelstam–Tamm quantum speed limit (QSL) [4] states that the minimal time  $\tau$  required for a system with energy uncertainty  $\Delta E$  to evolve between two pure states with Bures angle  $L$  obeys

$$\tau \geq \frac{\hbar L}{\Delta E}. \quad (22)$$

For orthogonal states ( $L = \pi/2$ ) this gives  $\tau \geq \pi\hbar/(2\Delta E)$ .

Within our framework, this bound can be reinterpreted kinematically: any evolution that takes the system from one distinguishable state to another must traverse at least one minimal phase-space cell of volume  $(2\pi\hbar)^n$ . The QSL then expresses the fact that the “clock speed” of state transitions is limited by the available energy budget per unit action quantum. The presence of  $\hbar$  in QSLs is thus not a mysterious dynamical assumption, but a manifestation of finite ontological resolution.

## 6 Testable Consequences

The information-theoretic framework, with  $\hbar$  interpreted as a minimal action quantum arising from finite phase-space resolution, leads to several potentially testable predictions and reinterpretations.

### 1. Black Hole Ringdown: Discrete Frequency Comb.

If black-hole horizons have discretized areas with spacing

$$\Delta A = 4 \ln k \cdot \frac{\hbar G}{c^3} \quad (23)$$

(Bekenstein–Mukhanov quantization), then the asymptotic quasi-normal mode (QNM) spectrum acquires a discrete frequency spacing. For a Schwarzschild black hole of mass  $M$ , one expects

$$\Delta f = \frac{c^3 \ln k}{16\pi^2 GM}, \quad (24)$$

with  $k = 2$  giving

$$\Delta f \approx 896 \text{ Hz} \cdot \left( \frac{M_\odot}{M} \right). \quad (25)$$

This is a direct probe of  $\Delta\Omega_{\min}$  via horizon area quantization. A null detection of such a comb structure after stacking sufficiently many high-SNR ringdown events would constrain or falsify simple horizon-quantization models.

### 2. Vacuum Structure and Planck-Scale Correlations.

A finite lattice of phase-space cells implies deviations from exact continuum field theory at scales  $\ell \sim L_p$ , leading to modified two-point correlation functions and dispersion relations. High-precision interferometry and astrophysical observations may eventually be sensitive to such Planck-suppressed effects.

### 3. Canonical Commutation Relations from Symplectic Area Quantization.

From  $\Delta\Omega_{\min} = (2\pi\hbar)^n$  one recovers the standard Bohr–Sommerfeld quantization condition for integrable systems,

$$\oint p dq = 2\pi\hbar n, \quad (26)$$

where  $n \in \mathbb{Z}$  counts distinct loop states (cells) in phase space. This suggests that canonical commutation relations can be viewed as an operator-level encoding of minimal symplectic area, rather than as an independent postulate.

## 7 Discussion

In the classical limit one implicitly assumes  $\hbar = 0$ , corresponding to infinite information density on any boundary and vanishing minimal phase-space cell. This leads naturally to divergences of the form  $\infty - \infty$  and ill-defined integrals. Our framework, by contrast, starts from the finiteness of boundary information and derives the existence of a nonzero minimal action quantum.

Several standard features of quantum theory then acquire a unified interpretation:

- Heisenberg uncertainty expresses the fact that one cannot resolve more than one phase-space cell  $(2\pi\hbar)^n$  with finite information capacity.
- Action quantization  $S = n\hbar$  for cyclic processes counts how many minimal-action steps the trajectory executes in phase space.
- Entropy counts distinguishable microstates, which are themselves limited by the number of available cells  $N_{\text{states}}$ .
- The holographic scaling of entropy with area emerges from the interplay between boundary information density  $\eta$  and bulk phase-space resolution.

Quantum mechanics, on this view, can be regarded as the effective theory of systems with finite ontological resolution—a theory of finite-state machines embedded in continuous symplectic geometry. The probabilistic structure of quantum mechanics then reflects incomplete information about which of the finitely many cells the system occupies, rather than a fundamental indeterminism at the level of the symplectic continuum [5, 6].

## 8 Conclusion

From a small set of assumptions—causal finiteness (PCOD), a holographic bound on boundary information, symplectic structure with finite accessible volume, finite distinguishability, and homogeneous resolution—we have argued for the existence of a minimal phase-space cell

$$\Delta\Omega_{\text{min}} = (2\pi\hbar)^n. \quad (27)$$

This defines a universal constant  $a_*$  with the dimensions of action, which we identify with the experimentally measured Planck constant  $\hbar$ .

The information-theoretic foundation presented here unifies action, information, and geometry at a kinematic level; provides a natural regularization of classical infinities; and offers an axiomatic basis for the emergence of quantum mechanics without postulating  $\hbar$  from the outset. Future work should extend the formalism to open systems (information exchange across boundaries), field theories on networks of boundaries, and dynamical derivations of the Schrödinger and path-integral formalisms from the finite-resolution axioms.

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