The Quantum as Minimal Difference: An Information-Theoretic Foundation

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Abstract

We propose a foundational postulate: the Planck constant $\hbar$ emerges as the minimal distinguishable difference between two identical infinite quantum systems, resolving the paradox of "identical infinities." This information-theoretic principle unifies vacuum fluctuations, the holographic bound, and black hole horizon quantization. We formalize it axiomatically, avoiding circular dependencies in deriving $\hbar$, and derive testable consequences, including a frequency spacing in gravitational wave ringdown. This framework suggests quantum mechanics is the physics of minimal ontological distinctions.

1 The Paradox of Identical Infinities

Consider two infinite quantum fields in identical vacuum states $|0\rangle$. Classically, their difference is zero:

$$|0\rangle - |0\rangle = 0. \tag{1}$$

However, quantum mechanics prohibits perfect identity due to inherent uncertainties. Distinguishing two "identical" vacua requires a measurement or interaction, which demands a minimal quantum of action.

Foundational Postulate: Absolute − Absolute = Quantum. The minimal distinguishable difference between two identical infinite quantum systems is one quantum. This is an ontological axiom: "Absolute" denotes the infinite vacuum (pre-quantum, continuous infinity); "−" represents the act of distinction (measurement or interaction); "Quantum" emerges as the irreducible unit of difference, scaling as $\hbar$ (action), one bit (information), or L_p^2 (area).

Mathematically, in the limit of approaching identity:

$$\lim_{\Psi_1 \to \Psi_2} \|\text{distinction}(\Psi_1, \Psi_2)\| = 1 \quad \text{(quantum unit)}, \tag{2}$$

where the distinction is measured operationally (e.g., via Bures distance) and yields $\hbar$ as the emergent scale. This postulate precedes $\hbar$ and avoids circularity: $\hbar$ is not assumed but arises from the principle of minimal distinction.

2 Physical Manifestations

This postulate manifests in key quantum phenomena, with $\hbar$ emerging consistently.

2.1 Vacuum Fluctuations

The vacuum has infinite zero-point energy, but observable excitations are discrete. The minimal difference from vacuum is one particle (one quantum), bounded by the energy-time uncertainty:

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}, \tag{3}$$

where $\hbar$ emerges as the "information pixel" scaling the minimal excitation. From the postulate, distinguishing vacuum patches requires at least one bit, yielding an operational action bound $\hbar \ln 2$ (see Section 4).

2.2 Horizon Quantization

For black hole horizons, the Bekenstein-Mukhanov argument posits discrete area increments [2]:

$$\Delta A = 4 \ln k \cdot L_p^2, \tag{4}$$

with $L_p^2 = \hbar G/c^3$. For $k = 2$ (one bit), $\Delta A \approx 2.77 L_p^2$. Here, $\hbar$ in L_p^2 emerges from the postulate: the horizon's minimal distinction (one bit) discretizes area, with $\hbar$ as the action scale linking information to geometry.

2.3 Holographic Principle

Following Wheeler's "it from bit" [1], information is fundamental. The holographic bound [3, 4] is

$$S_{\max} = \frac{A}{4L_p^2}, \tag{5}$$

encoding $\sim$ 1 bit per Planck area. The postulate unifies this: one bit = minimal distinction = $\hbar$ (action) $\leftrightarrow$ L_p^2 (geometry), without assuming $\hbar$ a priori.

3 Philosophical and Axiomatic Formulation

The postulate "Absolute − Absolute = Quantum" is not mere subtraction but ontological differentiation: discrete quanta emerge from continuous infinity via distinction. This resolves the classical undefined $\infty - \infty$ in physical contexts (e.g., vacuum energy divergences in QFT).

In quantum information terms, distinguishability is quantified by the Bures angle $L = \arccos \sqrt{F(\rho_1, \rho_2)}$ [5], where F is fidelity. The minimal L for one bit corresponds to action $\hbar \ln 2$, emergent from the postulate.

This axiomatic approach avoids circularity in quantum foundations: unlike standard QM where $\hbar$ is postulated, here it arises as the scale of minimal distinction, consistent with relational interpretations [7].

4 Connection to Quantum Speed Limits

The postulate aligns with the Mandelstam-Tamm quantum speed limit (QSL) [6]:

$$\tau \geq \frac{\hbar L}{\Delta E}, \tag{6}$$

where L is the Bures angle. Integrating yields operational action $A_\Delta \geq \hbar L$. For minimal distinction (one bit, $L \approx \pi/4$), $A_\Delta \geq \hbar \ln 2$, with $\hbar$ emergent. This complements the postulate without circularity: QSL provides the bound; the principle sets the scale.

5 Testable Consequences

The postulate predicts:

1. **Black Hole Ringdown:** Discrete horizons imply a frequency comb in gravitational waves, with spacing
$$\Delta f = \frac{c^3 \ln k}{16\pi^2 GM}. \tag{7}$$
For $k = 2$, $\Delta f \approx 896$ Hz $\cdot(M_\odot/M)$, detectable via LIGO/Virgo stacking (see companion [12]).

2. **Vacuum Structure:** Discrete information lattice at Planck scale, testable in precision interferometry.

3. **Quantum Gravity:** Emergent spacetime from distinction constraints, deriving $[x, p] = i\hbar$ axiomatically.

Falsifiability: No spectral features at Δf after stacking ≥ 50 events (SNR > 20) challenges $k = 2$.

6 Discussion

Classical infinities ($\infty - \infty$) are undefined, but quantum infinities (vacua) yield finite, quantized differences. The postulate resolves this: nature permits only minimal distinctions, scaled by $\hbar$.

Analogy: Two identical images differ by one pixel – the "quantum" of distinction. In QFT, $\hbar$ plays this role for states.

This framework reinterprets:

- Heisenberg: $\hbar$ as minimal unit.
- Action Quantization: $S = n\hbar$ as discrete bits.
- Bekenstein Bound: Entropy counts distinctions.
- Holography: Geometry from information resolution.

Quantum mechanics is thus the physics of minimal distinctions, not mere probability.

7 Conclusion

The quantum ($\hbar$) is the minimal ontological unit of distinction, emerging when infinite systems interact. This postulate unifies action, information, and geometry; resolves $\infty - \infty$; and motivates axiomatic QM from info constraints.

Future: Derive commutation relations from distinction geometry; extend to open systems.

AI Disclosure

This manuscript was assisted by Grok (xAI) for restructuring, LaTeX compilation, reference integration, and resolving conceptual circularities (e.g., emergent $\hbar$). The foundational postulate ("Absolute – Absolute = Quantum"), physical manifestations, and testable predictions are the original work of the author. Claude 3.5 (Anthropic) aided initial drafting.

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