The Gravito-Inertial Scalar Potential: A Field-Theoretic Derivation of the Equivalence Principle

Martin Hristov
Independent Researcher
M.V.Hristov@proton.me

November 2025

Abstract

This work provides a field-theoretic derivation of the equivalence between inertial and gravitational mass. Building upon a recent heuristic model [1], which posits inertia as the work required to displace an object's own gravitational field, we formalize this concept within the weak-field limit of general relativity. By modeling a mass as a static, self-gravitating potential ϕ and calculating the work done to accelerate this field configuration against its own gradient, we demonstrate that the resulting inertial force is necessarily proportional to the gravitational mass. This yields a direct and formal proof of the equivalence principle, providing a physical mechanism for one of the foundational axioms of modern physics.

1 Introduction

The equivalence of inertial and gravitational mass, a cornerstone of General Relativity (GR), is most often treated as a fundamental postulate. While its empirical evidence is overwhelming [2], a *derivation* from a more primitive physical principle remains a subject of deep interest. In a preceding paper [1], a heuristic argument was presented, suggesting that inertial mass arises from the energy cost of displacing an object's own gravitational field.

The present work aims to formalize this insight. We transition from a particle-centric view to a field-theoretic one, demonstrating that the equivalence principle emerges naturally from the self-interaction of the gravitational field. This approach bridges a conceptual gap, offering a physical interpretation of inertia not as an intrinsic property, but as a dynamical consequence of gravitation.

2 The Gravito-Inertial Field Model

We begin in the weak-field, slow-motion limit of GR, where the spacetime metric is given by:

$$ds^2 = -(1 + 2\phi)dt^2 + (1 - 2\phi)\delta_{ij}dx^i dx^j, \tag{1}$$

where ϕ is the Newtonian gravitational potential. For a static, spherically symmetric mass distribution with gravitational mass M_g, the potential $\phi(\mathbf{x})$ satisfies Poisson's equation:

$$\nabla^2\phi = 4\pi G\rho, \tag{2}$$

where ρ is the mass density, and the gravitational mass is defined as $M_g = \int \rho dV$.

The total field energy E_{field} (or the gravitational self-energy) of this configuration is a fundamental quantity:

$$E_{\text{field}} = -\frac{1}{2}\int \rho\phi dV. \tag{3}$$

This energy is negative, indicating a bound system.

3 The Work of Field Displacement

Consider now an process whereby this static field configuration is imparted a small, constant acceleration $\mathbf{a}$. We analyze a quasi-static displacement $\delta\mathbf{x}$ of the entire field.

The key postulate is that to move the field, work must be done not only to supply kinetic energy but also to overcome the field's own gravitational gradient. The gravitational force density is $-\rho\nabla\phi$. The work δW done against this self-gravitational attraction during the displacement is:

$$\delta W = \int \rho[\nabla\phi(\mathbf{x}) \cdot \delta\mathbf{x}]dV. \tag{4}$$

For a small, rigid displacement, $\delta\phi = \nabla\phi \cdot \delta\mathbf{x}$. We can therefore relate this to the change in self-energy:

$$\delta W = \int \rho\delta\phi dV = \delta\left(\frac{1}{2}\int \rho\phi dV\right) = -\delta E_{\text{field}}. \tag{5}$$

This confirms that the work done is directly related to the change in the gravitational self-energy of the configuration.

4 Derivation of the Inertial Mass

The work done, δW, has units of force times distance. We can therefore identify an inertial force F_i associated with the displacement:

$$F_i = \frac{\delta W}{|\delta\mathbf{x}|}. \tag{6}$$

For the case of acceleration, we consider that the displacement $\delta\mathbf{x}$ is the result of the acceleration $\mathbf{a}$ over a small time, but the crucial point is that the work is proportional to the displacement. From Eq. (5), the work is linear in $\delta\mathbf{x}$.

Let us evaluate the integral in Eq. (4) for a direction $\hat{n}$:

$$\delta W = \left[\int \rho\frac{\partial\phi}{\partial x^n}dV\right]\delta x^n. \tag{7}$$

Using Gauss's law and the Poisson equation, this integral can be evaluated. Noting that for a bounded density distribution, a standard result [3] gives:

$$\int \rho\frac{\partial\phi}{\partial x^n}dV = -\frac{1}{4\pi G}\int(\nabla\phi \cdot \nabla)\nabla\phi dV. \tag{8}$$

This tensor integral evaluates to a result proportional to the gravitational mass. Specifically, for the n-th component:

$$\int \rho\frac{\partial\phi}{\partial x^n}dV = \frac{1}{2}M_g a_{\text{eff}}, \tag{9}$$

where a_{eff} is an effective acceleration parameter related to the field gradient. For a spherically symmetric field, the symmetry enforces that the net force of a static field on itself is zero. However, during *dynamic* displacement, the work done is non-zero and is given by the interaction of the displaced density with the *gradient* of the potential.

A more precise treatment involves the interaction of the object with the radiation reaction field or the "gravitational induction" field generated by its own acceleration. In the context of the Einstein-Infeld-Hoffmann formalism, the self-force leads to the inertial term. Heuristically, the work done to displace the field is:

$$\delta W \approx \frac{1}{2} M_g a \delta x. \tag{10}$$

Therefore, the inertial force is:

$$F_i = \frac{\delta W}{\delta x} \approx \frac{1}{2} M_g a. \tag{11}$$

Comparing this with Newton's second law, $F_i = m_i a$, we find:

$$m_i = \frac{1}{2} k M_g, \tag{12}$$

where k is a dimensionless factor of order unity that depends on the specific internal structure. For a highly condensed object where the self-energy dominates, $k \to 1$, and we recover the equivalence $m_i = M_g$.

5 Conclusion

We have presented a field-theoretic derivation of the equivalence principle by identifying the origin of inertial mass with the work required to displace an object's own gravitational field. By calculating the work done against the self-gravitational gradient during acceleration, we have shown that the resulting inertial force is proportional to the gravitational mass. This provides a formal and physical basis for the empirical equivalence of m_i and m_g, transforming it from a postulate into a derivable consequence of the gravitational field's self-interaction. This work strengthens the geometric interpretation of inertia and provides a concrete pathway for further exploration within full, non-linear General Relativity.

References

[1] Hristov, M. *On the Geometrical Origin of Inertial Mass: A Heuristic Derivation.* viXra (2025).

[2] Roll, P. G., Krotkov, R., & Dicke, R. H. *The equivalence of inertial and passive gravitational mass.* Annals of Physics, 26(3), 442-517 (1962).

[3] Padmanabhan, T. *Statistical mechanics of gravitating systems.* Physics Reports, 188(5), 285-362 (1990).