

Resolving the Problem of Time in Quantum Mechanics: The General Covariant Dirac Adjoint in the Qg Framework.

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Abstract

The integration of quantum mechanics with gravitation requires a reformulation of time itself. In standard quantum theory, time appears as an external evolution parameter, while in general relativity it is an intrinsic and dynamical part of spacetime geometry. This paper resolves this long-standing inconsistency within the biquaternionic (BQ) rotor formalism by extending the Dirac adjoint to a fully covariant operator in which the gravitational rotor field Qg generates the local tetrad and the observer's four-velocity defines the temporal direction. The scalar product constructed from this adjoint is invariant under both local Lorentz and observer transformations, and the Dirac equation becomes a law of self-parallel transport along the gravitational flow. Time is thereby internalised and geometrised: it arises as the direction in spacetime determined by the adjoint itself. The resulting framework unifies quantum and gravitational dynamics into a single first-order structure, completing the development of the Qg theory that introduced the gravitational rotor, the Bernoulli–Noether closure, and the exact linearisation of Einstein's equations. This approach provides an observer-independent formulation of quantum mechanics in curved spacetime, eliminating the need for an external time parameter and establishing a coherent first-order synthesis of matter, geometry, and temporal flow.

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1 Introduction

The present work continues a long-term programme that seeks to unify quantum mechanics and gravitation inside a single first-order algebraic framework. Earlier studies demonstrated that the Dirac formalism itself can be constructed purely from the biquaternionic (BQ) algebra and its internal rotor symmetries [1], revealing that Lorentz and spinor transformations share a common geometric origin. The subsequent introduction of the gravitational rotor field $Q_g(x)$ extended this structure to curved spacetime, identifying the local gravitational lapse and shift within the rotated Dirac adjoint [2]. This led to the Bernoulli–Noether closure (BNC)—a first-order flow law that replaces the Einstein equations as the generative principle of spacetime geometry [3]. The same approach was shown to yield an exact linearisation of the Einstein equations in terms of Q_g , establishing gravity as a linear, self-parallel transport process in the Dirac algebra itself [4].

The central focus of the present paper is the *problem of time*: in conventional quantum theory, evolution depends on an external parameter t , while in general relativity the notion of time is intrinsic and dynamical. This dichotomy obstructs any consistent quantum–gravitational framework. Here, the Dirac adjoint is generalised to $\bar{\Psi} = \Psi^\dagger Q_g \beta_\mu Q_g^{-1} u^\mu$, embedding the observer’s four-velocity u^μ directly into the geometry of the rotor field. The scalar product $\bar{\Psi}\Psi$ thereby becomes covariant for all observers, and the time evolution of the spinor proceeds through the geometric transport equation $u^\mu D_\mu \Psi = 0$. Time is no longer external but emerges from the same gravitational structure that generates the metric itself.

This development completes the conceptual synthesis begun in the earlier papers: the Dirac field and the gravitational rotor now share a common first-order dynamics. The resulting framework yields a self-consistent integration of quantum evolution, observer covariance, and gravitational flow. It connects to previous algebraic formulations of spacetime by Dirac [5], Hestenes [6], and the ADM decomposition of general relativity [7, 8], while addressing the relational and geometric conceptions of time discussed in the works of Rovelli [9] and Isham [10]. Within the present formulation, the “problem of time” is not resolved by eliminating time but by embedding it inside the gravitational algebra as a local, observer-dependent direction of self-parallel transport.

2 Extent of the Dirac Adjoint in Curved Space: β_0 versus β_μ

In flat spacetime, the Dirac adjoint $\bar{\Psi} = \Psi^\dagger \beta_0$ anchors the scalar product to the global time direction. When extending the formalism to curved spacetime through the gravitational rotor

field Q_g , the question arises whether one should replace β_0 by $Q_g\beta_0Q_g^{-1}$ or by the full set $Q_g\beta_\mu Q_g^{-1}$.

2.1 1. Flat–space meaning of the adjoint

The operator β_0 defines the Lorentz–invariant inner product $\bar{\Psi}\Psi = \Psi^\dagger\beta_0\Psi$. It singles out the timelike direction and ensures that $J^0 = \Psi^\dagger\Psi$ is positive definite. The spatial matrices β_i form the spatial part of the basis but do not participate in the adjoint.

2.2 2. Curved–space extension

In curved spacetime, the local time direction is generated by

$$/G = Q_g\beta_0Q_g^{-1}.$$

This defines the gravitational lapse and shift: the curved–space generalisation of the time axis. Using $/G$ in the adjoint,

$$\bar{\Psi} = \Psi^\dagger Q_g\beta_0Q_g^{-1},$$

preserves local Lorentz covariance because $/G$ transforms covariantly under $Q_g \rightarrow Q_g R$ with $R \in \text{Spin}(1, 3)$.

2.3 3. Why not include all β_μ ?

The adjoint’s role is to define the *scalar product* by projecting onto the local time direction. The full tetrad $Q_g\beta_\mu Q_g^{-1}$ already appears in the covariant derivative term of the Dirac equation:

$$i\hbar\gamma^\mu D_\mu\Psi = mc\Psi, \quad \gamma^\mu = Q_g\beta^\mu Q_g^{-1}.$$

Including all β_μ in the adjoint would duplicate the metric contribution already represented in the kinetic term.

2.4 4. Geometric interpretation

Object	Definition	Physical Role
$Q_g\beta_\mu Q_g^{-1}$	Full tetrad	Generates the curved metric $g_{\mu\nu}$ (lapse + shift).
$Q_g\beta_0 Q_g^{-1}$	Adjoint operator	Defines invariant scalar product and local time direction.
$\bar{\Psi} = \Psi^\dagger Q_g\beta_0 Q_g^{-1}$	Dirac adjoint	Anchors the spinor to the metric time axis.

2.5 5. General observer–based form

A more general adjoint can include an arbitrary observer’s four–velocity u^μ :

$$\bar{\Psi} = \Psi^\dagger Q_g\beta_\mu Q_g^{-1}u^\mu,$$

which reduces to $\Psi^\dagger Q_g\beta_0 Q_g^{-1}$ in the local rest frame $u^\mu = (1, 0, 0, 0)$. This form remains covariant because u^μ transforms as a four–vector under local Lorentz transformations.

2.6 6. Summary

- The minimal covariant adjoint $Q_g\beta_0Q_g^{-1}$ correctly generalises the flat–space operator.
- The full $Q_g\beta_\mu Q_g^{-1}$ describes the tetrad and already enters the Dirac kinetic term.
- A generalised form with u^μ provides complete observer covariance.
- The adjoint projects onto the local time direction; it need not reconstruct the full metric.

3 Covariance of the Scalar Product with the General Covariant Adjoint

The Dirac adjoint guarantees covariance of spinor bilinears under Lorentz (and, in curved spacetime, tetrad) transformations. Here we extend the adjoint from the fixed time axis β_0 to the observer–covariant form $Q_g\beta_\mu Q_g^{-1}u^\mu$ and show that the scalar $\bar{\Psi}\Psi$ remains invariant.

3.1 1. Flat spacetime: why the adjoint guarantees covariance

With basis $\{\beta_\mu\}$ and spinor transformation $\Psi' = R\Psi$ for $R \in \text{Spin}(1, 3)$, the defining relation

$$R^\dagger\beta_0R = \beta_0$$

implies

$$\bar{\Psi}'\Psi' = \Psi^\dagger R^\dagger\beta_0R\Psi = \Psi^\dagger\beta_0\Psi = \bar{\Psi}\Psi.$$

Thus the scalar bilinear $\bar{\Psi}\Psi$ is invariant: the inner product is Lorentz covariant.

3.2 2. Curved spacetime: the minimal covariant adjoint

Let $Q_g(x)$ be the local gravitational rotor (tetrad). The minimal curved–space adjoint is

$$\bar{\Psi} = \Psi^\dagger Q_g\beta_0Q_g^{-1},$$

so that

$$Q_g^\dagger(Q_g\beta_0Q_g^{-1})Q_g = Q_g\beta_0Q_g^{-1},$$

and therefore

$$\bar{\Psi}\Psi = \Psi^\dagger Q_g\beta_0Q_g^{-1}\Psi$$

is invariant under local tetrad rotations $Q_g \mapsto Q_gR$ with $R \in \text{Spin}(1, 3)$.

3.3 3. The generalized adjoint

Introduce an arbitrary future–directed unit timelike u^μ (the observer’s 4–velocity) and define

$$\bar{\Psi} = \Psi^\dagger Q_g\beta_\mu Q_g^{-1}u^\mu.$$

Equivalently, with the tetrad $\beta'_\mu := Q_g\beta_\mu Q_g^{-1}$, the operator $\beta'(u) := \beta'_\mu u^\mu$ is the observer–aligned time direction.

3.4 4. Covariance of the scalar product

Under a local Lorentz rotation $R(x)$,

$$\Psi' = R\Psi, \quad Q'_g = Q_gR, \quad \beta'_\mu = Q'_g\beta_\mu Q_g^{-1} = Q_gR\beta_\mu R^{-1}Q_g^{-1}.$$

Using $R\beta_\mu R^{-1} = \Lambda_\mu{}^\nu\beta_\nu$ and letting u^μ transform as $u'^\mu = \Lambda^\mu{}_\nu u^\nu$, we obtain

$$\bar{\Psi}'\Psi' = \Psi^\dagger R^\dagger Q'_g\beta_\mu Q_g^{-1}u^\mu R\Psi = \Psi^\dagger Q_g(\beta_\nu u'^\nu)Q_g^{-1}\Psi = \Psi^\dagger Q_g\beta_\mu Q_g^{-1}u^\mu\Psi = \bar{\Psi}\Psi.$$

Hence the generalized adjoint preserves full covariance of $\bar{\Psi}\Psi$ provided u^μ transforms as a 4–vector.

3.5 5. Physical meaning

- $Q_g\beta_0Q_g^{-1}$: adjoint anchored to a specific local rest frame (metric time axis); ensures covariance under local tetrad rotations.
- $Q_g\beta_\mu Q_g^{-1}u^\mu$: adjoint anchored to an arbitrary observer's 4-velocity; ensures *observer-covariant* inner products and currents $J^\mu = \bar{\Psi}\gamma^\mu\Psi$.

Thus the generalized adjoint is the more complete construction: it is covariant under both local Spin(1, 3) rotations and observer changes.

3.6 6. Limiting case check

In the local rest frame $u^\mu = (1, 0, 0, 0)$ one has

$$\beta'(u) = Q_g\beta_0Q_g^{-1},$$

so the generalized adjoint reduces to the minimal curved-space adjoint. It is therefore a covariant generalization, not a different notion.

Summary

- The flat-space adjoint with β_0 guarantees Lorentz invariance of $\bar{\Psi}\Psi$.
- In curved spacetime, $\bar{\Psi} = \Psi^\dagger Q_g\beta_0Q_g^{-1}$ is the minimal covariant choice.
- The generalized adjoint $\bar{\Psi} = \Psi^\dagger Q_g\beta_\mu Q_g^{-1}u^\mu$ preserves *full* covariance if u^μ transforms as a 4-vector.
- Physically, it anchors the inner product to the observer's time direction; the minimal case is recovered in the local rest frame.

4 Resolution of the Problem of Time via the General Covariant Adjoint

The standard formulation of quantum mechanics assumes an external, absolute time parameter t that drives the Schrödinger evolution,

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi,$$

while in general relativity time is a dynamical coordinate inseparable from spacetime geometry. Reconciling these two views of time is one of the long-standing conceptual tensions in physics, known as the *problem of time*. The Q_g -based formalism introduces a *general covariant adjoint* that effectively resolves this issue.

4.1 1. The problem of time in conventional quantum theory

In flat-space quantum mechanics, time is an external classical variable; the wavefunction $\Psi(t, \mathbf{x})$ evolves in a fixed background. The adjoint $\bar{\Psi} = \Psi^\dagger\beta_0$ defines a positive-definite inner product $\bar{\Psi}\Psi = \Psi^\dagger\Psi$, but this relies on a globally defined time direction β_0 . Once spacetime curvature or observer-dependence is introduced, this structure fails: there is no global β_0 .

4.2 2. The role of the adjoint in flat spacetime

The flat-space Dirac adjoint $\bar{\Psi} = \Psi^\dagger \beta_0$ ensures Lorentz covariance of the scalar bilinear $\bar{\Psi}\Psi$ because

$$R^\dagger \beta_0 R = \beta_0 \quad \Rightarrow \quad \bar{\Psi}' \Psi' = \bar{\Psi} \Psi.$$

Here β_0 fixes the unique time direction in Minkowski space and defines the probability density $J^0 = \Psi^\dagger \Psi$.

4.3 3. Breakdown in curved spacetime

In curved space, there is no globally defined β_0 . The minimal covariant generalisation replaces it by the gravitationally rotated time basis $/G = Q_g \beta_0 Q_g^{-1}$, but this still presumes a chosen slicing of spacetime. Time evolution remains tied to a specific local observer, not the geometry itself.

4.4 4. The general covariant adjoint

The extended definition,

$$\bar{\Psi} = \Psi^\dagger Q_g \beta_\mu Q_g^{-1} u^\mu,$$

introduces the local observer's four-velocity u^μ as a geometric component. The operator $Q_g \beta_\mu Q_g^{-1} u^\mu$ is the *observer-aligned time direction* in the curved tetrad basis. The scalar product

$$\bar{\Psi} \Psi = \Psi^\dagger Q_g \beta_\mu Q_g^{-1} u^\mu \Psi$$

is invariant under local Lorentz transformations provided u^μ transforms as a four-vector. Thus the adjoint and the scalar product remain fully covariant for arbitrary observers.

4.5 5. Dynamical evolution without external time

The observer's flow u^μ defines the local direction of evolution. Replacing the Schrödinger derivative ∂_t by the covariant transport along u^μ gives

$$u^\mu D_\mu \Psi = 0,$$

where D_μ is the gravitationally covariant derivative. Evolution is now a geometric flow along the observer's worldline rather than with respect to an external parameter. Time becomes an internal, relational direction encoded in the geometry of Q_g .

4.6 6. Comparison with other approaches

Approach	Strategy	Limitation	How Q_g resolves it
Wheeler–DeWitt	Eliminate time ($H\Psi = 0$)	Frozen formalism, no dynamics	$u^\mu D_\mu \Psi = 0$ restores local evolution.
Page–Wootters	Time as one subsystem	Requires subsystem partition	u^μ provides an intrinsic clock vector.
Tomita–Takesaki	Time from state algebra	Abstract, non-geometric	Q_g embeds time directly in geometry.
Q_g adjoint	Embed time in $Q_g \beta_\mu Q_g^{-1} u^\mu$	Requires u^μ field specification	Fully covariant, geometric, observer-based.

4.7 7. Summary

- The general adjoint replaces the absolute time axis β_0 by the observer's geometric direction u^μ within the rotor field Q_g .
- The scalar product $\bar{\Psi}\Psi$ becomes covariant for all observers.
- The Schrödinger derivative is replaced by geometric transport $u^\mu D_\mu$.
- Time evolution and gravitational flow become the same process.
- The “problem of time” is resolved: time is no longer external, but an emergent direction in the spacetime flow generated by Q_g .

In essence: *The general covariant adjoint geometrises time itself, turning quantum evolution into self-parallel transport along the spacetime flow, thereby resolving the problem of time by construction.*

5 Conclusion

The formulation developed in this work extends the Dirac equation to full covariance within the gravitational rotor framework. By generalising the adjoint to

$$\bar{\Psi} = \Psi^\dagger Q_g \beta_\mu Q_g^{-1} u^\mu,$$

time ceases to be an external parameter and becomes a local geometric direction, defined by the observer's four-velocity u^μ and the rotor field $Q_g(x)$ that generates the spacetime metric. The Dirac equation and the Bernoulli–Noether closure thereby merge into a single, first-order law of self-parallel transport,

$$u^\mu D_\mu \Psi = 0, \quad v^\mu D_\mu Q_g = 0,$$

expressing quantum and gravitational evolution as two projections of the same geometric process.

5.1 1. Conceptual synthesis

- The *problem of time* in quantum mechanics is resolved: evolution no longer requires an external time coordinate but proceeds along the observer's flow u^μ embedded in spacetime geometry.
- The scalar product $\bar{\Psi}\Psi = \Psi^\dagger Q_g \beta_\mu Q_g^{-1} u^\mu \Psi$ is invariant under both local Lorentz and observer transformations, providing a globally covariant probability measure.
- The adjoint thus plays a dual role: it defines the local temporal direction and simultaneously guarantees the covariance of the inner product.

5.2 2. Integration of matter and geometry

The same covariant derivative $D_\mu = \partial_\mu + (\not{\partial} Q_g) Q_g^{-1}$ governs both the evolution of the Dirac spinor and the dynamics of the gravitational rotor field. Geometry, matter, and time are not separate sectors but aspects of one self-organising flow. The Einstein tensor $G_{\mu\nu}$ and the Dirac current $J_\mu = \bar{\Psi} \gamma_\mu \Psi$ emerge as complementary projections of a single first-order structure.

5.3 3. Consequences for physics

- The formalism provides a first-order, linearised alternative to Einstein's equations in which the spacetime metric is generated algebraically from Q_g .
- Quantum evolution and gravitational transport are unified; the Dirac equation becomes manifestly covariant and observer-independent in curved spacetime.
- The theory supplies an operationally defined local time direction for quantum systems in strong-field or cosmological environments, offering a consistent basis for quantum information, interferometry, and relativistic metrology.

5.4 4. Outlook

The covariant adjoint introduces a framework in which gravitational dynamics, quantum evolution, and temporal flow are inseparable. Future work will focus on:

- i) the mutual coupling of the (Q_g, Ψ) system, allowing matter back-reaction within the same first-order dynamics;
- ii) the inclusion of massless and gauge fields inside the biquaternion algebra, extending the present formulation toward the Standard Model;
- iii) empirical tests through gravitational phase evolution in clock networks, atom interferometry, and galactic-scale flow phenomena.

In summary, the general covariant adjoint transforms the Dirac equation into a geometrically self-consistent theory of motion in curved spacetime. Time is no longer imposed from outside the quantum system but arises from the same rotor field that builds the metric itself. The resulting synthesis between quantum mechanics and gravitation offers a coherent, linear, and testable foundation for the physics of matter, geometry, and time.

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