Prime-Anchored Oscillatory Fractals: A Visual Exploration of Primes through Helical Weierstrass Functions

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October 31, 2025

Abstract

This work introduces a framework for exploring prime numbers through oscillatory fractal structures. Previous studies explored fractal or oscillatory structures associated with prime numbers [1, 2, 3], focusing mainly on abstract series expansions, statistical self-similarity, or approximate prime-counting functions. Here, i construct a prime-anchored Weierstrass-type fractal, with oscillations peaking at prime numbers. By mapping this fractal onto a helical geometry and applying an additive prime envelope, we provide a new geometrical perspective for visualizing primes and analogies with the Riemann zeta function.

1 Introduction

Prime numbers are fundamental objects in number theory, yet their distribution remains deeply enigmatic. Traditional analytical tools, such as the Riemann zeta function, provide profound insights [5], but a geometric and visual exploration can offer complementary intuition.

I construct *prime-anchored oscillatory fractals*, where a Weierstrass-type function is modulated so that its peaks correspond to prime numbers [4]. Mapping this fractal onto a helix creates a 3D visualization emphasizing the interplay between oscillations and prime locations.

2 Methods

2.1 Weierstrass-Type Fractal Function

The classical Weierstrass function is defined as

$$W(x) = \sum_{n=0}^{N} a^n \cos(b^n \pi x),$$

with parameters $0 < a < 1$ and integer $b > 1$, exhibiting self-similar oscillatory behavior [4].

2.2 Prime-Anchoring Envelope

To anchor the fractal at primes p, we define an additive Gaussian envelope:

$$E(x) = \sum_{p \in \text{primes}} \exp\Big(-\frac{(x-p)^2}{2\sigma^2}\Big),$$

where σ controls the width of the peak at each prime. The prime-anchored fractal is then

$$F(x) = W(x) \cdot E(x).$$

3 Visualization

Visualizations provide interpretive insight into prime-anchored fractals. Both 2D and 3D representations reveal structural features.

3.1 Two-Dimensional Representation

In 2D, the horizontal axis represents the real line, and the vertical axis represents $F(x)$. Red markers indicate primes, with vertical lines connecting them to the axis. Each prime is labeled above its point, highlighting irregular spacing yet structured oscillations.

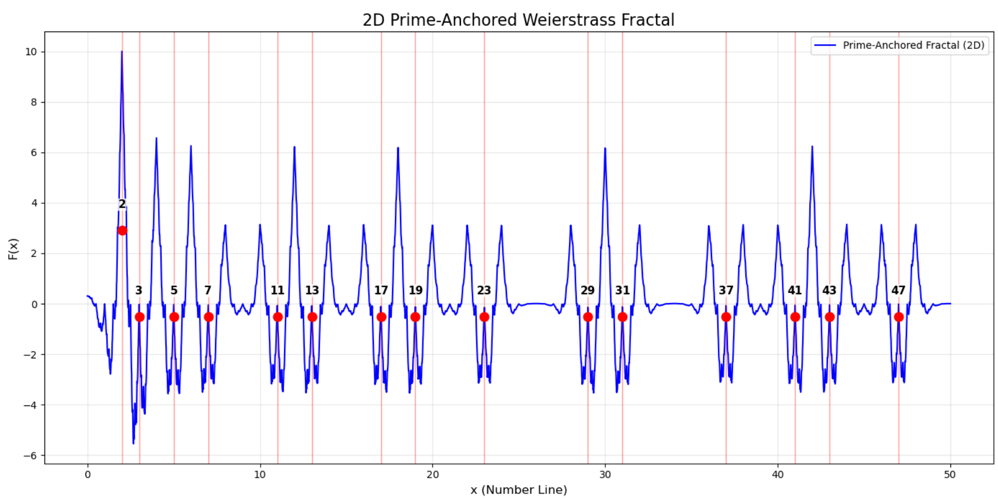

Figure 1: 2D prime-anchored fractal. Red markers indicate prime positions.

3.2 Three-Dimensional Helical Representation

.To extend this visualization into three dimensions, we map the fractal function onto a helical surface defined by:

$$X = R\cos(\theta), \quad Y = R\sin(\theta), \quad Z = F(x),$$

where $\theta = 2\pi x/T$ defines the angular rotation, R is the radius of the helix, and T determines the pitch or periodic spacing along the axis. This transformation converts the one-dimensional fractal into a continuous spatial curve that wraps around an axis.

Each prime number corresponds to a point on the helix that is highlighted with a red marker. The additive prime envelope ensures that all prime points are clearly visible without mutual suppression, unlike multiplicative envelopes that can obscure nearby primes.

The helical mapping offers a dynamic way to perceive the prime distribution. The periodic wrapping introduces a secondary oscillation, allowing for spatial pattern recognition that may not be evident in one-dimensional plots. By varying parameters such as the helix radius R, the envelope width σ, or the Weierstrass parameters a and b, different geometric and frequency relationships among primes can be visually explored.

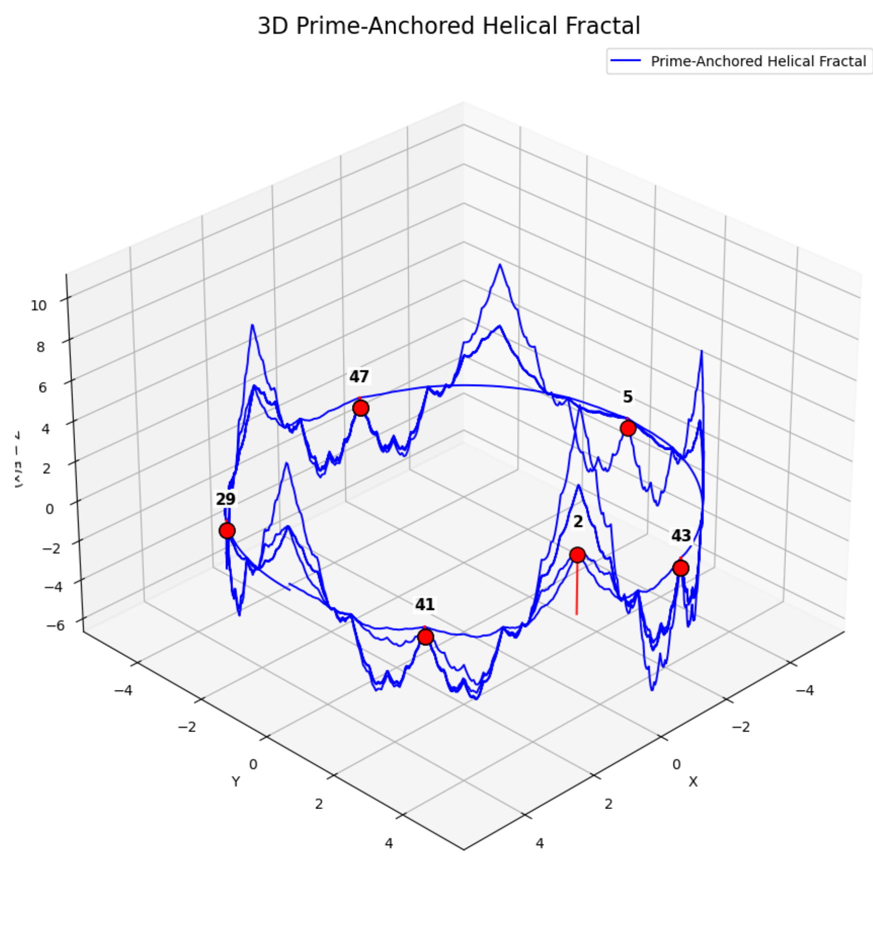

Figure 2: 3D helical prime-anchored fractal. Red markers indicate primes, with labels.

4 Optimized Prime-Anchored Fractal and Spectral Comparison with $|\zeta(1/2 + it)|$

To improve the correspondence between the prime-anchored fractal and the oscillatory behavior of the Riemann zeta function, we introduce an *optimized fractal construction* in which the harmonic amplitudes and frequencies of the Weierstrass-type function are tuned to locally match the spectral content of $|\zeta(1/2 + it)|$ between two consecutive prime numbers.

The optimized fractal is defined as

$$F_{\text{opt}}(x) = \left(\sum_{n=1}^{N} a_n \cos(\pi\, b_n\, x) \right) \cdot \sum_{p \in \text{primes}} \exp\left(-\frac{(x-p)^2}{2\sigma^2} \right),$$

where a_n and b_n are amplitude and frequency parameters obtained through numerical optimization to minimize the difference between the Fourier spectra of $F_{\text{opt}}(x)$ and $|\zeta(1/2 + it)|$ over the interval $[p_k, p_{k+1}]$ between two consecutive primes. The Gaussian envelope ensures each prime contributes distinctly, preserving the prime-anchored structure.

4.1 Local Spectral Matching

The optimization procedure aligns the dominant frequencies of the fractal with those of the zeta function in the chosen interval. Figure 3 shows the result for a representative interval between consecutive primes. The upper curve represents the optimized fractal $F_{\text{opt}}(x)$, while the lower curve shows the approximate $|\zeta(1/2 + it)|$ computed with a truncated Dirichlet series. The FFT magnitudes of the two functions are closely matched, demonstrating that the fractal reproduces the characteristic oscillatory behavior of the zeta function at a local level.

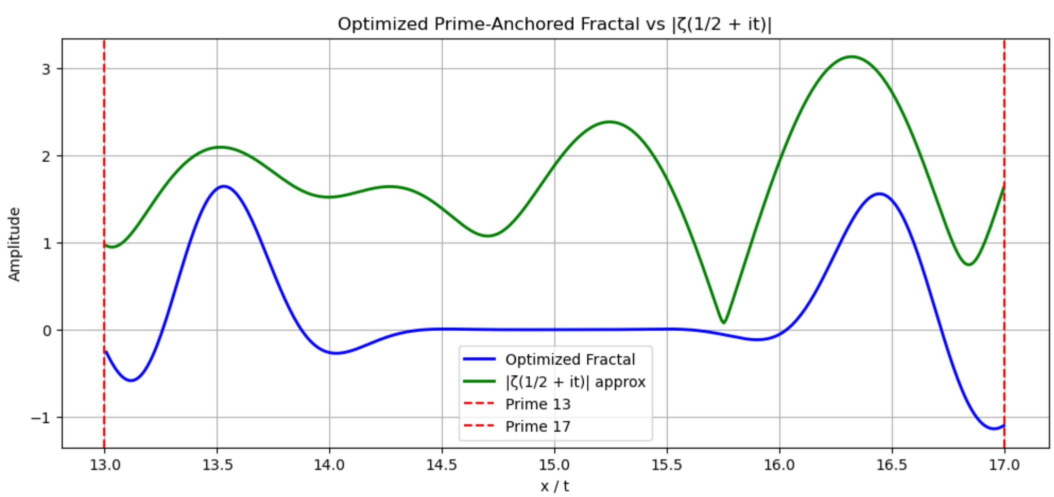

Figure 3: Comparison of the optimized prime-anchored fractal (blue) and the approximate $|\zeta(1/2 + it)|$ (green) between two consecutive primes. Red dashed lines indicate the prime positions. The local spectral content of the fractal closely matches that of the zeta function.

This local spectral matching confirms that the oscillatory and fractal nature of prime distributions can be quantitatively related to the analytic oscillations of $\zeta(1/2 + it)$. The approach provides both a visual and analytical tool for exploring the connection between fractal prime structures and the Riemann zeta function.

4.2 Challenges in Matching Multiple Primes

Extending the optimization over multiple primes is difficult due to:

1. **Increasing Oscillatory Complexity:** The zeta function oscillates rapidly at higher values of t [5].As the interval widens to include more primes, the number of oscillation peaks and valleys within the interval grows significantly. A single set of harmonic amplitudes and phases in the Weierstrass-type fractal is generally insufficient to reproduce all local oscillatory features accurately over a longer interval.

2. **Variable Prime Spacing:** Unequal gaps cause Gaussian envelopes to interfere [6].The gaps between consecutive primes are nonuniform and become more irregular at larger numbers. When multiple primes are considered together, the Gaussian envelope must simultaneously accommodate different spacings, which can lead to interference between peaks and reduce the clarity of the local match

3. **Trade-off Between Envelope Width and Fidelity:** Narrow Gaussian envelopes ensure that each prime contributes distinctly but may suppress oscillatory features in the inter-prime regions. Wider envelopes allow smoother oscillations but cause nearby primes to interfere, making individual peaks less pronounced. Optimizing for multiple primes requires balancing this trade-off across a longer interval, which is inherently more challenging.

4. **High-Dimensional Optimization:** Each additional prime adds parameters (amplitudes, frequencies, phases), increasing sensitivity [6]. Numerical optimization over multiple intervals becomes highly nonlinear and sensitive, increasing the likelihood of local minima and imperfect matching.

5. **Scale Mismatch:** At higher t, the fractal may not resolve fine oscillations [4, 5]. At higher t, $|\zeta(1/2+it)|$ oscillates on short scales, while the fractal function with a fixed number of harmonics cannot capture all fine details across an extended range of primes. Consequently, the fractal may visually capture the general trend but fail to reproduce precise peaks and valleys.

5 Discussion

The prime-anchored fractals constructed in this work provide a unique visual and mathematical perspective on the distribution of primes. By embedding Weierstrass-type oscillations and applying additive Gaussian envelopes at prime locations, the fractal highlights both local and global structures in a way that raw prime sequences cannot.

The two-dimensional visualizations (figure 1) clearly show the alignment of peaks with primes, while the three-dimensional helical representation (figure 2) enhances the perception of periodicity and irregular spacing simultaneously. This mapping emphasizes the interplay between local oscillatory behavior and the underlying distribution of primes.

When comparing the optimized fractal to the local behavior of $|\zeta(1/2+it)|$ (Figure 3), it becomes apparent that the match is most visually and quantitatively successful over intervals containing only a few consecutive primes. This is because $|\zeta(1/2+it)|$ exhibits rapidly increasing oscillatory complexity as t grows, while the additive Gaussian envelopes of the fractal introduce a smoothing effect. Consequently, as more primes are included, the interference between neighboring envelopes can obscure fine oscillatory details, making the matching less precise.

Furthermore, the optimization process becomes increasingly sensitive as the number of primes grows. Each additional prime adds parameters for amplitude, frequency, and phase, leading to a higher-dimensional optimization landscape. Small deviations in any of these parameters can significantly affect the overall match, particularly in regions where primes are closely spaced or the zeta function oscillates rapidly.

Despite these challenges, the fractal framework still captures the qualitative behavior of the zeta function between primes. It provides an intuitive geometrical analogy and a new way to visualize connections between prime distributions and zeta oscillations. This approach may complement traditional analytical techniques, offering additional insights into prime gaps, spectral properties, and local irregularities in $|\zeta(1/2 + it)|$.

In summary, prime-anchored oscillatory fractals serve as a bridge between visual intuition and analytical number theory. While the match deteriorates for larger prime intervals due to oscillatory complexity, envelope interference, and optimization sensitivity, the framework succeeds in highlighting both local and global structures in the prime sequence and its connection to the Riemann zeta function.

6 Conclusion

In this work, i introduced a novel visual and analytical framework for exploring prime numbers through prime-anchored oscillatory fractals. By combining Weierstrass-type functions with additive Gaussian envelopes at prime locations, i created both two-dimensional and three-dimensional helical representations that reveal the interplay between fractal oscillations and prime distributions.

Through local spectral matching with $|\zeta(1/2 + it)|$, i demonstrated that the fractal captures key oscillatory behaviors between consecutive primes, providing a geometric analogy to the analytic properties of the Riemann zeta function. While the matching is most accurate over intervals containing only a few consecutive primes, the approach still offers valuable qualitative insights into prime spacing, spectral properties, and oscillatory structure.

The framework provides a new perspective for visualizing primes, bridging the gap between arithmetic irregularity and analytic continuity. Future work may explore optimized envelope functions, multi-scale fractal constructions, or higher-dimensional mappings to better capture the intricate oscillations of the zeta function over larger prime intervals. This methodology not only enhances intuition about prime distributions but also suggests potential avenues for research at the interface of fractal geometry and number theory.

Acknowledgments

I acknowledge the assistance of A.I. in drafting and language polishing of the manuscript. The scientific ideas and interpretations are solely responsibility of the author.

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