First–Order Gravitation in the Dirac Algebra: The Bernoulli–Noether Closure and the Self–Organising Spacetime Field

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Abstract

This study develops a first-order formulation of gravitation inside the Dirac-biquaternionic algebra, continuing a research programme that embeds the geometry of general relativity directly within the Dirac formalism. The gravitational field is represented by a rotor, denoted Qg, whose adjoint defines the local lapse and shift and thereby replaces the metric tensor of general relativity. The corresponding covariant derivative remains first order in derivatives, so that curvature arises as an internal commutator, making gravity an intrinsic property of the Dirac medium rather than an external geometry. From this structure emerges the Bernoulli–Noether Closure, a first-order constitutive law expressing the self-organisation of spacetime as an inviscid perfect fluid with constant total metric energy along each streamline. The stationary solutions of this law reproduce the Schwarzschild, Kerr, and Schwarzschild–de Sitter metrics as equilibrium configurations, while also admitting a new class of axisymmetric, irrotational "constant-Lagrangian spiral" solutions that account for the observed flat galactic rotation curves without invoking dark matter. In this framework, Einstein's second-order field equations appear as the steady-state limit of the first-order rotor dynamics, and the conserved rapidity norm of the flow provides a direct thermodynamic interpretation of gravitational energy, entropy, and equilibrium. The resulting theory unifies quantum spinor dynamics, gravitational curvature, and large-scale galactic structure within a single algebraic principle, establishing the Qg–BNC formalism as a first-order, kinetic completion of general relativity.

Contents

1 Introduction

The present study continues a programme that seeks to reformulate the interaction between quantum theory and gravitation *inside the Dirac algebra itself* [1, 2]. The first paper in this series reconstructed the Lorentz and Dirac operators within the biquaternionic (BQ) formalism, showing that boosts and rotations can be represented as internal rotor operations acting on slashed quantities $\not{P} = \beta^\mu P_\mu$ and that the Dirac algebra is naturally expressed in the real basis of the Pauli–Dirac doubling of the quaternion units. The second paper extended this structure by identifying a *gravitational rotor field* $Q_g(x)$, which acts on the local time basis rather than on spinor components. Its adjoint, $\not{G} = Q_g \beta_0 Q_g^{-1}$, defines both the gravitational lapse and shift and introduces a distinct *gravitational boost* that operates on the Dirac environment itself, rather than in an external metric.

In the formulation developed here, the Dirac Lagrangian is augmented by the field connection $(\not{\partial} Q_g) Q_g^{-1}$, yielding a covariant first–order operator that replaces the metric and spin connection of conventional general relativity. Gravitational curvature appears as an internal commutator, $F_{\mu\nu} = [\not{D}_\mu, \not{D}_\nu]$, so that gravity is no longer an external geometry but a *dynamical property of the Dirac medium*. The resulting framework provides a direct algebraic route from the Dirac equation to gravitational dynamics without introducing a second derivative level.

The work proceeds by analysing the algebraic difference between kinematic and gravitational boosts (Sections 2–4), tracing the origin of the gravitational rotor from the dual freedom of basis and component transformations (Section 6), and showing why gravity, in this formalism, cannot be interpreted as a gauge symmetry (Section 7). Subsequent sections reconstruct the Schwarzschild, Kerr, and Schwarzschild–de Sitter geometries from the same rotor field, derive the first–order Bernoulli–Noether Closure (BNC) as the constitutive law of stationary gravity, and demonstrate that the *constant–Lagrangian spiral* disk emerges as a stationary, self–organising solution that reproduces galactic rotation curves without dark matter.

This approach belongs to the long tradition of algebraic reformulations of relativity and quantum mechanics initiated by Dirac [3], extended by Hestenes in the geometric algebra of spinor relativity [4], and cast into the geometric framework of general relativity by Misner, Thorne, and Wheeler [5]. The connection with the Arnowitt–Deser–Misner (ADM) formalism [6] is explicit: the lapse and shift functions of ADM arise directly as components of the adjoint $\not{G} = Q_g \beta_0 Q_g^{-1}$, providing a purely algebraic realisation of the ADM decomposition within the Dirac–BQ language. Within this representation, the gravitational field equations become first–

order in derivatives and fully covariant under local Lorentz rotors, while remaining consistent with the Einstein geometry in its stationary limit.

The resulting theory thus provides a continuous bridge from quantum spinor dynamics to classical curvature, and introduces a new dynamical regime in which spacetime behaves as a self–organising medium governed by the first–order Bernoulli–Noether law. This first–order, thermodynamic closure explains both the stationary metrics of general relativity and the large–scale structure of galaxies as manifestations of the same internal rotor dynamics of the Dirac field.

2 Reconstructing GR Solutions in the Qg Formalism: The Schwarzschild Case

2.1 General reconstruction recipe

Any solution of general relativity can be represented in the Q_g formalism as a *rotor field* $Q_g(x) \in \mathrm{Spin}(1,3)_{\mathbb{C}}$ whose adjoint and connection reproduce the same local geometry. The correspondence proceeds as follows:

Step 1: Specify the spacetime symmetry. Identify Killing vectors and invariant subspaces. For example, stationarity and spherical symmetry imply that Q_g depends only on the radius r and generates boosts in the (β_0, β_r) plane.

Step 2: Construct the rotor ansatz. Write

$$Q_g = \exp\Big(\frac{\psi(r)}{2}\,\beta_r\beta_0\Big), Q_g quad /\!\!\!G = Q_g\,\beta_0\,Q_g^{-1} = \cosh\psi\,\beta_0 + \sinh\psi\,\beta_r.$$

The local rapidity $\psi(r)$ (or the equivalent inflow speed $v(r) = c\tanh\psi$) encodes the lapse and shift of the spacetime foliation.

Step 3: Compute the first-order connection. The covariant derivative in the Dirac–Q_g algebra is

$$\not{D} = \not{\partial} + (\not{\partial} Q_g)Q_g^{-1}.$$

Its curvature, $F_{\mu\nu} = [\not{D}_\mu, \not{D}_\nu]$, represents the gravitational field strength.

Step 4: Impose the field conditions. In vacuum, require that the Ricci contraction of $F_{\mu\nu}$ vanish, or equivalently that the invariants built from $F_{\mu\nu}$ match those of the desired GR solution.

Step 5: Extract the metric. From the adjoint $/\!\!\!G = Q_g\beta_0 Q_g^{-1}$ identify the lapse and shift:

$$N = \cosh\psi, Q_g quad N^r = c\sinh\psi.$$

These reproduce the metric coefficients in the 3+1 ADM decomposition.

Step 6: Optional: apply a re-slicing rotor $\mathcal{W}(x)$. Multiplying Q_g by a spatially dependent rotor $\mathcal{W}$ performs a local change of slicing (e.g. Painlevé–Gullstrand $\leftrightarrow$ diagonal Schwarzschild), without altering the underlying curvature $F_{\mu\nu}$.

2.2 Static spherical vacuum: the Schwarzschild field

For a stationary, spherically symmetric vacuum, choose the purely radial boost field

$$Q_g(r) = \exp\Big(\frac{\psi(r)}{2}\,\beta_r\beta_0\Big), Q_g quad v(r) = c\tanh\psi(r).$$

The adjoint $\not{G} = Q_g\beta_0 Q_g^{-1} = \cosh\psi\,\beta_0 + \sinh\psi\,\beta_r$ defines a local time axis tilted by rapidity $\psi(r)$. The induced line element from this adjoint is

$$ds^2 = -(c^2 - v^2(r))dt^2 - 2\,v(r)\,dr\,dt + dr^2 + r^2 d\Omega^2. \tag{1}$$

Equation (1) is the Painlevé–Gullstrand (PG) form of the Schwarzschild metric.

Imposing the vacuum condition on the curvature $F_{\mu\nu} = [\not{D}_\mu, \not{D}_\nu]$ fixes the rapidity profile to the familiar Bernoulli–inflow law

$$v^2(r) = \frac{2GM}{r}, Q_g quad\psi(r) = \operatorname{arctanh}\Big(-\sqrt{\tfrac{2GM}{rc^2}}\Big).$$

The corresponding curvature invariants of $F_{\mu\nu}$ match those of the Schwarzschild solution outside the source. Thus, the classical metric solution emerges as the squared (second-order) image of a first-order rotor field in the Dirac algebra.

2.3 The re-slicing rotor $\mathcal{W}(r)$

The PG representation has lapse $N{=}1$ and shift $N^r{=}v(r)$. To obtain the diagonal Schwarzschild form ($N^r{=}0$, $N = \sqrt{1 - \frac{2GM}{rc^2}}$), introduce a second, purely temporal rotor $\mathcal{W}(r)$ that performs a radial re-slicing:

$$\widetilde{Q_g}(r) = \mathcal{W}(r)Q_g(r), Q_g quad\widetilde{\not{G}} = \widetilde{Q_g}\beta_0\widetilde{Q_g}^{-1} = N(r)\beta_0.$$

This operation corresponds to a local hyperbolic rotation in the (β_0, β_r) plane that exactly cancels the radial component of $\not{G}$. In coordinate language it is equivalent to the time transformation

$$t_{\text{Schw}} = t_{\text{PG}} - \int \frac{v(r)}{c^2 - v^2(r)}\,dr = t_{\text{PG}} - \int \frac{\sqrt{2GM/r}}{c^2 - 2GM/r}\,dr,$$

which redefines the simultaneity slices. Algebraically, $\mathcal{W}(r)$ acts as a *gauge rotor* that changes the local time basis while leaving the curvature $F_{\mu\nu}$ invariant. Different choices of $\mathcal{W}$ thus correspond to the familiar freedom of foliation in general relativity: the physics—the curvature encoded in $F_{\mu\nu}$—remains unchanged, while the coordinate appearance of the metric (lapse and shift) is a matter of gauge.

3 Recovering the Schwarzschild metric from the Adjoint $\not{G}$

3.1 1. Metric from the rotor algebra

In the Dirac–BQ basis the elements β_μ satisfy

$$\beta_\mu\beta_\nu + \beta_\nu\beta_\mu = 2\eta_{\mu\nu}, Q_g quad\eta_{\mu\nu} = \operatorname{diag}(-1, 1, 1, 1),$$

so that the scalar part of their product gives the Minkowski metric: $\langle\beta_\mu\beta_\nu\rangle_0 = \eta_{\mu\nu}$. Define the biquaternionic position vector

$$\not{R} = R_\mu\beta^\mu = r_0\beta_0 + r_i\beta_i, Q_g quad d\not{R} = dR_\mu\beta^\mu.$$

In flat space the invariant quadratic is

$$d\not{R}\,d\not{R} = \langle d\not{R}\,d\not{R}\rangle_0 = \eta_{\mu\nu}\,dR^\mu dR^\nu = ds^2\not{1}. \tag{2}$$

3.2 2. Local rotation of the differential

In a gravitational field the local basis is rotated by the gravitational rotor $Q_g(x)$:

$$d\!\!\not{R}' = Q_g\, d\!\!\not{R}\, Q_g^{-1}, Q_g quad \beta'_\mu = Q_g\, \beta_\mu\, Q_g^{-1}.$$

The adjoint $\not{G} = Q_g\, \beta_0\, Q_g^{-1}$ defines the local time direction of the field. Because Q_g is an element of $\mathrm{Spin}(1,3)_{\mathbb{C}}$, the scalar quadratic is invariant:

$$\langle d\!\!\not{R}'\, d\!\!\not{R}' \rangle_0 = \langle d\!\!\not{R}\, d\!\!\not{R} \rangle_0 = ds^2 \mathbb{1}.$$

Hence the geometry is encoded in the rotated differential $d\!\!\not{R}'$, while the scalar line element ds^2 remains invariant.

3.3 3. Metric tensor from the rotor field

The metric tensor arises as the scalar part of the product of rotated basis elements,

$$g_{\mu\nu}(x) = \left\langle Q_g\, \beta_\mu\, Q_g^{-1}\, Q_g\, \beta_\nu\, Q_g^{-1} \right\rangle_0 = \langle \beta'_\mu \beta'_\nu \rangle_0, \tag{3}$$

and the spacetime interval follows as

$$ds^2 = \left\langle (Q_g\, d\!\!\not{R}\, Q_g^{-1})^2 \right\rangle_0 = g_{\mu\nu}\, dR^\mu dR^\nu. \tag{4}$$

Equation (4) provides the compact rotor-invariant definition of the line element in the Q_g formalism.

3.4 4. Example: static spherical symmetry

For the spherically symmetric rotor $Q_g(r) = \exp(\frac{\psi(r)}{2} \beta_r \beta_0)$, the adjoint and rotated spatial basis are

$$\beta'_0 = \cosh\psi\, \beta_0 + \sinh\psi\, \beta_r, Q_g quad \beta'_r = \cosh\psi\, \beta_r + \sinh\psi\, \beta_0.$$

Substituting these into Eq. (4) gives

$$\begin{aligned} ds^2 &= \left\langle (\beta'_0\, dr_0 + \beta'_r\, dr_r + r\beta_\theta d\theta + r \sin\theta\, \beta_\phi d\phi)^2 \right\rangle_0 \\ &= -(c^2 - v^2(r)) dt^2 - 2v(r)\, dr\, dt + dr^2 + r^2 d\Omega^2, \end{aligned}$$

with $v(r) = c \tanh\psi(r)$, the Painlevé–Gullstrand form of the Schwarzschild metric.

3.5 5. Re-slicing by a secondary rotor $\mathcal{W}(r)$

A secondary local rotor $\mathcal{W}(r) = \exp(-\frac{\chi(r)}{2} \beta_r \beta_0)$ can reorient the local time direction:

$$\widetilde{d\!\!\not{R}} = \mathcal{W}(r)\, d\!\!\not{R}\, \mathcal{W}^{-1}(r), Q_g quad dt' = \cosh\chi\, dt - \frac{\sinh\chi}{c}\, dr.$$

Choosing $\chi(r)$ so that the mixed term in ds^2 vanishes yields the diagonal Schwarzschild form. The rotor $\mathcal{W}(r)$ therefore acts as a *gauge re-slicing* of the local time–space plane: it alters the lapse and shift but leaves the curvature tensor $F_{\mu\nu} = [\not{D}_\mu, \not{D}_\nu]$ and the scalar line element $\langle d\!\!\not{R}\, d\!\!\not{R} \rangle_0$ invariant.

Summary

$$\boxed{ds^2 = \left\langle (Q_g\, d\!\!\not{R}\, Q_g^{-1})^2 \right\rangle_0, Q_g quad d\!\!\not{R} = dR_\mu \beta^\mu, Q_g quad g_{\mu\nu} = \left\langle Q_g\, \beta_\mu\, Q_g^{-1} Q_g\, \beta_\nu\, Q_g^{-1} \right\rangle_0.} \tag{5}$$

The scalar part of the squared, rotor-rotated differential replaces the tensor construction of the metric: geometry emerges directly from the local rotor field $Q_g(x)$, and coordinate freedom corresponds to additional local re-rotations such as $\mathcal{W}(r)$.

4 Kerr as a Composite Q_g Rotor: Five Steps and Summary

4.1 Step 1: Symmetry data (stationary, axisymmetric)

Kerr spacetime is stationary and axisymmetric about the z–axis. In the Dirac–BQ algebra with Pauli content $K_i \in \{I, J, K\}$, choose a spherical spatial frame

$$\beta_r,\ \beta_\theta,\ \beta_\phi \quad \text{with} \quad \{\beta_a, \beta_b\} = 2\delta_{ab}\,\mathbb{1}, \qquad \beta_0^2 = -\mathbb{1},\ \beta_a^2 = +\mathbb{1},$$

and $\beta_0\beta_a + \beta_a\beta_0 = 0$. Axisymmetry singles out β_ϕ as the azimuthal basis element.

4.2 Step 2: Composite gravitational rotor (radial boost + azimuthal rotation)

Generalise the Schwarzschild rotor by adding an azimuthal piece:

$$\boxed{Q_g(r,\theta) \;=\; \exp\!\Big[\tfrac{1}{2}\,\psi(r)\,\beta_r\beta_0 + \tfrac{1}{2}\,\omega(r,\theta)\,\beta_\phi\beta_0\Big]\,.} \tag{6}$$

Here $\psi(r)$ encodes the radial inflow rapidity and $\omega(r,\theta)$ the stationary frame–dragging angular rapidity. Define physical velocities

$$v_r(r) = c\,\tanh\psi(r), \qquad v_\phi(r,\theta) = r\sin\theta\,\omega(r,\theta).$$

4.3 Step 3: Rotated differential and line element

Write the differential in the spherical Dirac basis,

$$d\not{R} = \beta_0\,c\,dt + \beta_r\,dr + \beta_\theta\,r\,d\theta + \beta_\phi\,r\sin\theta\,d\phi.$$

Rotating by (6) gives

$$\big(Q_g\,d\not{R}\,Q_g^{-1}\big)^2 = ds^2\,\mathbb{1}, \qquad ds^2 = \Big\langle \big(Q_g\,d\not{R}\,Q_g^{-1}\big)^2 \Big\rangle_0. \tag{7}$$

Evaluating the scalar part with the Clifford anticommutation relations yields the Painlevé–Gullstrand / Doran–type stationary form

$$\boxed{ds^2 = -(c^2 - v_r^2 - v_\phi^2)\,dt^2 - 2\,v_r\,dr\,dt - 2\,v_\phi\,r\sin\theta\,d\phi\,dt + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\,d\phi^2\,.} \tag{8}$$

This may also be expressed as

$$ds^2 = -c^2dt^2 + \big(dr - v_r dt\big)^2 + r^2d\theta^2 + \big(r\sin\theta\,d\phi - v_\phi\,dt\big)^2, \tag{9}$$

which displays the inflow (v_r) and frame–dragging (v_ϕ) structure explicitly.

4.4 Step 4: Field profiles from the first–order curvature

The gravitational connection is first order,

$$\not{D} = \not{\partial} + (\not{\partial}Q_g)Q_g^{-1}, \qquad F_{\mu\nu} = [\not{D}_\mu, \not{D}_\nu].$$

The vacuum condition (vanishing Ricci contraction of $F_{\mu\nu}$) determines the field profiles. In the GR (Kerr) limit this reproduces the known stationary laws:

$$v_r^2(r,\theta) \;\longrightarrow\; \frac{2GM\,r}{\rho^2}, \qquad \rho^2 = r^2 + a^2\cos^2\theta, \tag{10}$$

$$\omega(r,\theta) \;\longrightarrow\; \Omega_{\text{drag}}(r,\theta) \quad \text{(Lense–Thirring angular velocity; weak field } \Omega\!\sim\!2GJ/(c^2r^3)\text{)}. \tag{11}$$

With $v_\phi = r\sin\theta\,\omega$, inserting (10)–(11) into (8) yields the Doran/PG–type Kerr line element; a further time/azimuth re–slicing (see Step 5) maps it to Boyer–Lindquist form.

4.5 Step 5: Slicing freedom via a secondary rotor

As with Schwarzschild, a secondary local rotor in the (β_0, β_r) and (β_0, β_ϕ) planes,

$$\mathcal{W}(r,\theta) = \exp\Big[-\tfrac{1}{2}\,\chi_r(r,\theta)\,\beta_r\beta_0 - \tfrac{1}{2}\,\chi_\phi(r,\theta)\,\beta_\phi\beta_0\Big],$$

performs a re–slicing, $\widetilde{Q}_g = \mathcal{W}\,Q_g$, that can remove cross–terms (g_{tr}, modify $g_{t\phi}$) and yield the preferred coordinates (Doran or Boyer–Lindquist). Curvature invariants from $F_{\mu\nu}$ remain unchanged.

Summary

$$\boxed{\begin{aligned}
&\textbf{Rotor:}\quad Q_{g,\mathrm{Kerr}}(r,\theta) = \exp\Big[\tfrac{1}{2}\,\psi(r)\,\beta_r\beta_0 + \tfrac{1}{2}\,\omega(r,\theta)\,\beta_\phi\beta_0\Big],\\
&\textbf{Velocities:}\quad v_r = c\,\tanh\psi, \qquad v_\phi = r\sin\theta\,\omega,\\
&\textbf{Line element from algebra:}\quad (Q_{g,\mathrm{Kerr}}\,d\!\!\not{R}\,Q_{g,\mathrm{Kerr}}^{-1})^2 = ds^2\,\not{1},\\
&\textbf{Scalar form:}\quad ds^2 = -(c^2 - v_r^2 - v_\phi^2)\,dt^2 - 2\,v_r\,dr\,dt - 2\,v_\phi\,r\sin\theta\,d\phi\,dt + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\,d\phi^2,\\
&\textbf{Profiles (Kerr limit):}\quad v_r^2 \to 2GM\,r/\rho^2, \quad \omega \to \Omega_{\mathrm{drag}}(r,\theta), \quad \rho^2 = r^2 + a^2\cos^2\theta.
\end{aligned}} \tag{12}$$

In words: the Kerr metric arises from a *composite first–order rotor*—a radial boost plus an azimuthal rotation—acting on the differential $d\!\!\not{R}$ in the Pauli–embedded Dirac basis. The scalar of the squared rotated differential yields the PG/Doran–type Kerr metric, and a secondary rotor implements the coordinate slicings without changing the curvature.

5 Origin of the Conserved Rapidity Norm in the Q_g Field

5.1 Meaning of the constant rapidity norm

The Bernoulli–Lagrangian invariant of the metric flow can be written as

$$\tfrac{1}{2}\,v^2 = \tfrac{1}{2}\Big(v_{r,\mathrm{eff}}^2 + v_{\mathrm{orb}}^2\Big) = \mathrm{const}(R), \qquad v_{r,\mathrm{eff}} = \sqrt{\tfrac{2GM}{r}} - H_z r,$$

where R labels a streamline of the metric inflow. In the Q_g formulation this becomes a statement of constant *rapidity amplitude*

$$\tanh^2\psi(r;R) = \frac{v^2(r;R)}{c^2} = \mathrm{const}(R),$$

so the local Q_g rotor

$$Q_g = \exp\Big[\tfrac{1}{2}\,\psi_0(R)\big(\cos\alpha(r;R)\,\beta_r + \sin\alpha(r;R)\,\beta_\phi\big)\beta_0\Big]$$

has constant magnitude $\psi_0(R)$ and only its direction $\alpha(r;R)$ varies along the flow. The constant rapidity norm is therefore the algebraic equivalent of a Bernoulli invariant.

5.2 Self–organisation as the physical cause

The metric flow behaves as a stationary, inviscid perfect fluid obeying

$$\nabla_\mu(\rho v^\mu) = 0, \qquad v^\nu\nabla_\nu v_\mu = -\nabla_\mu\Phi_{\mathrm{eff}},$$

with an effective potential

$$\Phi_{\mathrm{eff}}(r) = -\frac{GM}{r} + \tfrac{1}{2}H_z^2 r^2.$$

Integration of the Euler equation gives the Bernoulli invariant $\frac{1}{2}v^2 + \Phi_{\rm eff} = \text{const}$. Because the potential is already encoded in $v_{r,\rm eff}$, this is equivalent to $\frac{1}{2}(v_{r,\rm eff}^2 + v_{\rm orb}^2) = \text{const}$, the defining condition of a self–organised steady flow. Hence the constancy of the rapidity norm arises from the self–organisation of spacetime as a perfect fluid that minimises internal energy at fixed total energy per streamline.

5.3 Role of the Hubble boundary condition

Locally, the self–organised flow is closed and energy conserving; globally, the Hubble term H_z sets the *outer boundary* where inflow and expansion balance:

$$v_{r,\rm eff} \to 0 \quad \Rightarrow \quad r_c = (2GM/H_z^2)^{1/3}.$$

Beyond r_c the stationary Bernoulli equilibrium cannot persist and the spacetime fluid merges into the cosmological background. Thus H_z determines the boundary of the self–organised region but not the conservation itself.

5.4 Thermodynamic interpretation

In the thermodynamic view of spacetime as a perfect fluid, the rapidity amplitude ψ acts as an *enthalpy potential*. The Bernoulli constant corresponds to constant specific enthalpy $h = N \cosh\psi$ along streamlines. Constancy of $\psi_0(R)$ therefore represents an adiabatic, isentropic configuration: the local “temperature” (time dilation) and “pressure” (curvature) adjust to maintain equilibrium. The conserved rapidity norm is thus the macroscopic signature of a steady–state, isentropic spacetime fluid.

5.5 Dynamical form in the Q_g equations

In compact rotor form,

$$v^\mu \not{D}_\mu Q_g = 0 \qquad \Rightarrow \qquad \partial_\mu(\tanh^2\psi)\, v^\mu = 0,$$

so that the rapidity magnitude ψ is conserved along the flow lines. This is the Q_g analogue of the Euler–Bernoulli law.

Summary

The conserved rapidity norm in the Q_g field arises from the self–organisation of spacetime as an inviscid, barotropic perfect fluid. Along each stationary streamline the metric's kinetic energy per unit mass (the Bernoulli invariant) remains constant because the spacetime medium continually adjusts its local rapidity and direction to maintain equilibrium. The cosmic expansion rate H_z merely fixes the outer boundary where this self–organised state connects to the cosmological background.

6 Status of the Rapidity Field within General Relativity

6.1 Constraints within standard GR

In conventional general relativity the geometry of spacetime is defined by a metric $g_{\mu\nu}(x)$ satisfying Einstein's equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

All gravitational information—lapse, shift, and curvature—is contained in the metric itself; there is no independent dynamical object corresponding to a rapidity or rotor field.

- **Static case (Schwarzschild):** the line element can be written in Painlevé–Gullstrand form, $ds^2 = -(c^2 - v_r^2)\,dt^2 - 2v_r\,dr\,dt + dr^2 + r^2 d\Omega^2$ with $v_r = \sqrt{2GM/r}$. This inflow velocity is purely a coordinate representation of the static geometry; it carries no separate dynamics.
- **Stationary case (Kerr):** the additional degree of freedom is a shift $N^\phi(r,\theta)$ representing frame dragging. Again, N^ϕ is an algebraic part of the metric, not an independent flow field.

Einstein's equations are second-order in the metric components and do not impose or permit a separate *first-order* condition such as a conserved rapidity magnitude.

6.2 The new structure introduced by the Q_g field

In the rotor formulation the metric is defined as

$$ds^2 = \langle (Q_g\, d\!\!\not{R}\, Q_g^{-1})^2 \rangle_0,$$

and the quantity $(\not{\partial} Q_g)Q_g^{-1}$ functions as the gravitational connection. The *rapidity amplitude*

$$\tanh\psi(r;R) = \frac{v(r;R)}{c} = \frac{1}{c}\sqrt{v_{r,\mathrm{eff}}^2 + v_{\mathrm{orb}}^2}$$

becomes an independent field variable rather than a coordinate artefact. The constancy of ψ along a metric streamline is a first–order balance law,

$$\tfrac{1}{2}v^2 + \Phi_{\mathrm{eff}} = \mathrm{const},$$

which expresses the self–organisation of spacetime as a steady Bernoulli flow. Thus the Q_g formalism extends GR by introducing a dynamical relation between lapse and shift: the lapse N and the shift N^i become orthogonal projections of a single rotor field.

6.3 Relation to Schwarzschild and Kerr geometries

The Q_g field thus contains the Schwarzschild and Kerr geometries as special stationary configurations:

- When $v_\phi = 0$ and $H_z = 0$ the rapidity field reproduces the Schwarzschild inflow exactly.
- Adding a fixed azimuthal orientation (constant α) yields a stationary helical flow that resembles Kerr in symmetry but remains irrotational (no frame–dragging vorticity).
- Allowing $\alpha(r;R)$ to vary according to the constant–Lagrangian condition gives a self–organised field that has no analogue in standard GR.

6.4 Compatibility with GR geometry

Any Q_g rotor defines a metric by $g_{\mu\nu} = \langle Q_g\beta_\mu Q_g^{-1}\, Q_g\beta_\nu Q_g^{-1}\rangle_0$. If that metric satisfies Einstein's equations for some stress–energy $T_{\mu\nu}$, it is admissible within GR. However, the *first–order conservation law*

$$v^\mu \not{D}_\mu Q_g = 0 \quad \text{or} \quad \frac{d\psi}{ds} = 0$$

has no counterpart in GR. It constitutes an additional, constitutive relation for the spacetime medium: a dynamical rule governing the internal organisation of the metric flow. Hence the Q_g formalism is compatible with GR but goes beyond it by endowing spacetime with fluid–like constitutive behaviour.

Aspect	Schwarzschild	Kerr	Q_g Bernoulli Field
Fundamental variable	Metric $g_{\mu\nu}$	Metric $g_{\mu\nu}$	Rotor $Q_g \in \mathrm{Spin}(1,3)$
Equation type	Second–order (Einstein)	Second–order (Einstein)	First–order (Euler–Bernoulli)
Degrees of freedom	Lapse $N(r)$	Lapse $N(r,\theta)$, shift N^ϕ	Rapidity magnitude $\psi(r;R)$, direction $\alpha(r;R)$
Flow character	Radial, irrotational	Radial + rotational (vorticity)	Helical, irrotational, self–organised
Curvature source	Central mass M	M and spin a	Spatial gradients of Q_g (self–curvature)
Physical interpretation	Static gravitational potential	Rotating mass, frame dragging	Stationary metric inflow of spacetime medium
Limiting behaviour	$v_r = \sqrt{2GM/r}$	v_r, v_ϕ from (M,a)	v_r, v_ϕ from (M,H_z,R) under constant–L balance

Table 1: Comparison of the Schwarzschild, Kerr, and Q_g Bernoulli formulations. The tabularx layout ensures full page width without overflow.

6.5 Empirical and conceptual position

In the weak–field regime the rapidity field reproduces the observational predictions of GR: gravitational redshift, time dilation, and Keplerian motion. In stronger or cosmological regimes it provides a first–principles energy balance for the metric flow through the Bernoulli invariant, which links local galactic morphology and the cosmic boundary condition set by H_z. Conceptually, the Q_g field converts Einstein's static curvature picture into a *first–order geometric dynamics*: spacetime curvature becomes the macroscopic expression of a steady, self–organising rapidity field.

Summary

The Q_g rapidity field is not an alternative metric of GR but a deeper, first–order dynamical structure that reproduces the Schwarzschild and Kerr solutions as stationary states. It supplements the Einstein geometry with a conserved–rapidity (Bernoulli) law that describes spacetime as an inviscid, barotropic perfect fluid. This self–organising behaviour is absent from—yet fully compatible with— the traditional geometric formulation of general relativity.

7 Thermodynamic Interpretation of the Self–Organising Q_g Field

7.1 First law: energy conservation in the Q_g flow

In the Q_g dynamics, the local energy density of spacetime per unit rest mass is

$$\mathcal{E} = \tfrac{1}{2}v^2 + \Phi_{\text{eff}}, \qquad \Phi_{\text{eff}} = -\frac{GM}{r} + \tfrac{1}{2}H_z^2 r^2.$$

The Bernoulli condition

$$\frac{d\mathcal{E}}{ds} = 0$$

is the *first law of thermodynamics* for the spacetime medium: no net work is done on a fluid element following the stationary metric flow. In Q_g notation this reads

$$v^\mu \not{D}_\mu Q_g = 0, \qquad \Rightarrow \qquad \partial_\mu\left(\tfrac{1}{2}v^2 + \Phi_{\rm eff}\right) v^\mu = 0,$$

showing that the total (kinetic + potential) energy of the metric flow is conserved. The Bernoulli invariant thus represents the thermodynamic first law expressed in geometric form.

7.2 Second law: entropy and self–organisation

Within the Q_g ontology, spacetime acts as an inviscid, barotropic perfect fluid. For a stationary flow,

$$v^\mu \nabla_\mu s = 0,$$

so the entropy per streamline is constant. Over cosmological times, however, the field self–organises into stationary rapidity norms that minimise curvature dissipation. This corresponds to an entropy maximum or a least–action principle for curvature:

$$\delta \int R_{\rm eff}\, dV = 0, \qquad R_{\rm eff} \propto |\not{D} Q_g|^2.$$

Hence the stationary Q_g configuration represents a maximum–entropy, minimum–curvature–dissipation state—the geometric realisation of the second law of thermodynamics.

7.3 Third law: asymptotic self–stabilisation of the cosmic rapidity

As the universe expands and H_z tends to a constant, the rapidity field $\psi_g(t)$ relaxes toward a uniform value:

$$\dot{\psi}_g \to 0, \qquad v_{r,\rm eff} \to 0.$$

In this regime the global Q_g field becomes isothermal in rapidity space:

$$T_g \sim \frac{c^2}{\cosh \psi_g} = \text{const},$$

indicating that the spacetime fluid reaches a uniform "temperature" or time–dilation scale. This stationary equilibrium, where inflow and cosmic expansion exactly balance, is the geometric analogue of the *third law* of thermodynamics: the universe approaches a state of zero curvature change, or zero "temperature" in rapidity space.

7.4 Thermodynamic variables in Q_g–BQ form

The biquaternion algebra renders the thermodynamic quantities of the spacetime medium as geometric invariants:

Thermodynamic quantity	**Q_g–BQ analogue**	**Geometric meaning**
Internal energy U	$\frac{1}{2}v^2 = \frac{1}{2}c^2 \tanh^2 \psi$	Local kinetic energy of spacetime flow
Enthalpy h	$N \cosh \psi$	Time–dilation factor along Killing field
Temperature T_g	$c^2/\cosh \psi$	Curvature response to rapidity
Pressure p_g	$\rho c^2(\cosh^2 \psi - 1)$	Stress–energy feedback of curvature
Entropy s_g	$\int dQ_g/Q_g$	Logarithmic measure of geometric microstates
Work term $p\,dV$	$\langle(\not{D} Q_g) Q_g^{-1}\, d\not{R}\rangle_0$	Geometric volume strain

Because rotations and boosts are conjugate variables in the biquaternion algebra, their invariants play roles analogous to conserved thermodynamic quantities such as energy, enthalpy, and entropy.

7.5 Self–organisation as thermodynamic equilibrium

The self–organised, stationary Q_g state—constant rapidity norm, irrotational flow, and Bernoulli invariance—constitutes the thermodynamic *equilibrium* of the spacetime medium. Deviations in ψ produce curvature stresses that restore uniform rapidity:

$$\frac{\delta \mathcal{L}_{Q_g}}{\delta \psi} = 0 \quad \Rightarrow \quad \not{D}_\mu(\tanh^2 \psi \, v^\mu) = 0,$$

the same condition underlying the Bernoulli law. In this sense, the laws of thermodynamics emerge directly from the geometric dynamics of the Q_g rotor field.

Summary

The Q_g–BQ formalism obeys thermodynamic laws in geometric form: the Bernoulli invariant embodies the *first law* (energy conservation); the steady–state self–organisation of the rapidity field realises the *second law* (maximum entropy or minimal curvature); and the asymptotic uniform rapidity $\psi_g \to \text{const}$ constitutes a geometric analogue of the *third law*. Spacetime thus appears as a self–organising thermodynamic medium whose macroscopic order arises from its microscopic Q_g rotor dynamics.

8 Relation Between the Q_g Formulation and General Relativity

8.1 What remains identical to GR

The Q_g formalism reproduces the empirical and geometrical content of general relativity whenever the rotor–defined metric

$$g_{\mu\nu} = \langle Q_g \beta_\mu Q_g^{-1} \, Q_g \beta_\nu Q_g^{-1} \rangle_0$$

satisfies Einstein's field equations. All observable phenomena that depend only on this metric therefore coincide with GR:

- The geodesic motion of test bodies follows the same covariant form $\nabla_u u^\mu = 0$;
- Gravitational redshift, time dilation, and deflection of light are identical when the same $g_{\mu\nu}$ is inserted;
- The weak–field, Newtonian limit yields $v_r = \sqrt{2GM/r}$, identical to the Painlevé–Gullstrand representation of the Schwarzschild geometry;
- The Kerr metric and its frame–dragging terms appear as specific stationary rotor configurations when $\omega(r, \theta) \neq 0$.

Thus, at the level of curvature and motion, the Q_g approach is fully compatible with Einstein geometry.

8.2 Reinterpretation of existing GR structures

While the empirical geometry is the same, the ontology differs. Quantities that in GR are treated as purely geometric acquire dynamical meaning in Q_g:

- The **lapse function** N and **shift vector** N^i are no longer independent metric coefficients but orthogonal projections of a single rapidity rotor,

  $$N = \cosh \psi, \qquad N^i = \sinh \psi \, \hat{n}^i,$$

  representing the local "tilt" of the spacetime medium through time.

- The **connection coefficients** are interpreted as derivatives of the rotor field: $\Gamma_\mu = (\not{\partial}_\mu Q_g)Q_g^{-1}$, so curvature arises from the non–commutativity of local boosts and rotations.

- The **metric inflow velocity** v_r is not a coordinate effect but the real velocity of the spacetime medium relative to the cosmic frame.

These reinterpretations turn Einstein's static geometric picture into a dynamic description of spacetime as a structured medium.

8.3 New elements absent in GR

Beyond reinterpretation, the Q_g formulation introduces genuinely new first–order dynamics:

- A **Bernoulli–type conservation law**

$$\frac{1}{2}v^2 + \Phi_{\text{eff}} = \text{const}, \qquad v^\mu \not{D}_\mu Q_g = 0,$$

  which governs the self–organisation of the metric flow. This is a first–order law that GR does not contain; Einstein's equations are second order in $g_{\mu\nu}$.

- A **conserved rapidity norm** $\tanh^2\psi = \text{const}(R)$ along streamlines, expressing energy equilibrium of the spacetime fluid.

- A **thermodynamic interpretation**: curvature and energy density correspond to kinetic and enthalpy variables of an inviscid perfect fluid. GR provides no constitutive relation of this type.

- A natural **cosmic boundary condition** via H_z, linking local geometry to cosmological expansion through the effective inflow $v_{r,\text{eff}} = \sqrt{2GM/r} - H_z r$.

Together these additions convert Einstein geometry into a *self–organising dynamical system.*

8.4 Shared mathematical structure

Both GR and the Q_g approach are built on the Lorentz group SO(1, 3) and its spin representation. The difference lies in the choice of fundamental variable:

Framework	**Primary variable**	**Derived quantity**
General Relativity	Metric $g_{\mu\nu}$	Connection $\Gamma^\lambda_{\mu\nu}$ (2nd order)
Q_g–Biquaternion	Rotor Q_g	Metric $g_{\mu\nu} = \langle Q_g\beta_\mu Q_g^{-1} Q_g\beta_\nu Q_g^{-1}\rangle_0$ (1st order)

The Q_g field therefore lowers the differential order of the theory: geometry emerges as the integral of a first–order rotor flow.

8.5 Summary of equivalence and extension

- **Same as GR:** spacetime curvature, test–body geodesics, redshift, time dilation, gravitational lensing, and all metric–based predictions in the appropriate limits.

- **Different from GR:** the interpretation of curvature as the self–organised flow of a spacetime medium with a conserved rapidity norm; the presence of a first–order Bernoulli equation replacing the purely second–order Einstein tensor; and the embedding of local gravity in cosmological boundary conditions.

Summary

The Q_g framework coincides with general relativity whenever the rotor–generated metric satisfies Einstein's equations, but it extends the theory by supplying a first–order, thermodynamic and fluid–dynamic interpretation of those geometries. Einstein's static curvature becomes a manifestation of a deeper, conserved–rapidity flow field whose self–organisation produces the same metrics and phenomenology while offering new, physically transparent dynamics for the structure of spacetime itself.

9 A First–Order Q_g Closure

9.1 Statement: Bernoulli–Noether Closure (BNC)

In a stationary spacetime with a timelike Killing field, let the gravitational state be encoded by a rotor

$$Q_g(x) \in \mathrm{Spin}(1,3), \qquad \not{D}_\mu = \not{\partial}_\mu + (\not{\partial}_\mu Q_g)Q_g^{-1}.$$

Define the *metric flow* 4–velocity via the local rapidity decomposition of the rotated differential:

$$(\, Q_g\, d\not{R}\, Q_g^{-1}\,)^2 = ds^2\, \not{1}, \qquad v^2 = c^2 \tanh^2\psi.$$

BNC postulates the *first–order constitutive law*

$$\boxed{v^\mu \not{D}_\mu Q_g \;=\; 0} \tag{13}$$

as the governing equation of stationary vacuum gravity (with boundary data set by M and, if present, H_z and/or asymptotic rotation).

Immediate consequences. (i) *Conserved rapidity norm along streamlines:*

$$v^\mu \partial_\mu(\, \tanh^2\psi) = 0 \iff \tfrac{1}{2}v^2 = \text{const along a streamline.}$$

(ii) *Bernoulli invariant (metric form):*

$$\tfrac{1}{2}(v_{r,\text{eff}}^2 + v_{\text{orb}}^2) = \text{const}(R), \qquad v_{r,\text{eff}} = \sqrt{\tfrac{2GM}{r}} - H_z r,$$

with R the Lagrangian streamline label. (iii) *Metric generation (PG/Doran form):*

$$ds^2 = -(c^2 - v_r^2 - v_\phi^2)dt^2 - 2\,(v_r\,dr + v_\phi\, r\sin\theta\, d\phi)\,dt + dr^2 + r^2 d\Omega^2,$$

where $v_r = c\,\tanh\psi\cos\alpha$ and $v_\phi = c\,\tanh\psi\sin\alpha$.

Why this is new. Einstein's equations are second order in $g_{\mu\nu}$ and contain *no* first–order constitutive closure like (13). BNC says: *stationary vacuum gravity is a self–organising flow* whose internal degree of freedom Q_g is parallel transported along its own metric streamlines. Yet BNC *reproduces* Schwarzschild (PG), Kottler (PG with H_z), and the Doran form of Kerr (when a genuine rotational mode is present) by choosing the appropriate boundary data.

9.2 Corollary A: Rapidity–Curvature Correspondence (RCC)

For irrotational stationary flows (no frame–dragging mode in the generator), BNC implies a scalar closure between curvature and the rapidity field:

$$\boxed{\mathcal{R}_{\text{stat}} \;\propto\; \nabla\cdot(N\,\nabla\tanh^2\psi)} \tag{14}$$

with N the lapse along the Killing field. In vacuum Schwarzschild/Kottler exteriors, $\mathcal{R}_{\text{stat}} = 0$ and (14) reduces to the radial harmonicity of $\tanh^2\psi$ outside sources (all curvature localised to the mass term)—a *first–order* potential picture of the familiar Ricci–flatness.

9.3 Corollary B: Kelvin–Bernoulli Theorem for Spacetime

Define the (dimensionless) circulation of the metric shift one–form weighted by the lapse,

$$\Gamma_C \;=\; \oint_C N^{-1}\, N_i\, dx^i, \quad N_i \equiv g_{ti}/c.$$

Under BNC, the circulation is conserved along stationary evolution in vacuum:

$$\boxed{\frac{d\Gamma_C}{ds} = 0} \tag{15}$$

(i) For irrotational Schwarzschild/Kottler flows, $\Gamma_C = 0$. (ii) For Kerr–type rotational states, $\Gamma_C \neq 0$ and is fixed by the asymptotic spin (frame–dragging) data—directly paralleling Kelvin's theorem for perfect fluids.

9.4 Falsifiable predictions and checks

(a) **PG/Kottler match:** With $H_z^2 = \Lambda/3$, BNC yields $v_{r,\text{eff}} = \sqrt{2GM/r} - H_z r$ and reproduces the PG form of Schwarzschild–de Sitter without invoking the second–order EEQs.

(b) **Spiral metric (no vorticity):** With variable pitch $\alpha(r; R)$ but constant $|\psi|$ along R, BNC generates the constant–Lagrangian spiral metric and its redshift–dependent pitch law—an *extra* stationary solution class not singled out by GR alone.

(c) **Kelvin test:** In axisymmetric disks, the line integral of $N^{-1}N_\phi$ around large loops should be zero in non–frame–dragging systems (irrotational BNC) and nonzero with magnitude set by the spin parameter in frame–dragging systems.

Summary

The proposed *Bernoulli–Noether Closure* $v^\mu \not{D}_\mu Q_g = 0$ is a *first–order* law for stationary gravity that (i) reproduces the classic GR geometries when supplied with the usual boundary data, (ii) yields two striking corollaries—a rapidity–curvature link and a Kelvin–type circulation conservation—and (iii) predicts a broader class of stationary, irrotational spiral metrics. Its novelty lies in replacing the purely second–order Einstein dynamics with a constitutive, fluid–like evolution equation for the rotor field while remaining fully metric–compatible.

10 The Bernoulli–Noether Closure and the Constant–Lagrangian Spiral as Its Stationary Solution

10.1 The general closure condition

The proposed universal law governing stationary, self–organising gravitational fields is the *Bernoulli–Noether Closure* (BNC):

$$\boxed{v^\mu \, \not{D}_\mu Q_g = 0} \tag{16}$$

where $Q_g(x) \in \text{Spin}(1,3)$ is the gravitational rotor field and $\not{D}_\mu = \not{\partial}_\mu + (\not{\partial}_\mu Q_g)Q_g^{-1}$ is the corresponding first–order covariant derivative. Equation (16) states that the rotor is parallel–transported along its own flow lines—spacetime is a perfect fluid that carries its internal orientation without distortion along the metric streamlines.

This first–order condition simultaneously encodes:

(i) *Energy conservation:* the Bernoulli invariant $\frac{1}{2}v^2 + \Phi_{\text{eff}} = \text{const}$ along each streamline;

(ii) *Constancy of the rapidity norm:* $v^\mu \partial_\mu(\tanh^2\psi) = 0$, so that $\tanh\psi = c^{-1}\sqrt{v_r^2 + v_\phi^2}$ is constant along streamlines;

(iii) *Metric generation:*

$$(Q_g\, d\!\!\!/R\, Q_g^{-1})^2 = ds^2\, 1\!\!\!/_4, \qquad ds^2 = -(c^2 - v_r^2 - v_\phi^2)dt^2 - 2(v_r\, dr + v_\phi\, r\, d\phi)\, dt + dr^2 + r^2 d\phi^2 + \cdots .$$

Thus BNC serves as the single, first–order closure from which all stationary, irrotational geometries—from Schwarzschild inflow to the constant–Lagrangian spiral—can be generated.

10.2 The spiral solution as a specific rotor configuration

For an axisymmetric, stationary flow the rotor may be written as

$$Q_g(r; R) = \exp\Big[\frac{\psi_0(R)}{2}\,(\cos\alpha(r; R)\,\beta_r + \sin\alpha(r; R)\,\beta_\phi)\,\beta_0\Big],$$

where $\psi_0(R)$ is the constant rapidity amplitude along streamline R and $\alpha(r; R)$ is the local pitch angle of the flow. Substituting this form into (16) yields two scalar relations:

$$v_r\,\partial_r\psi_0(R) = 0 \quad \Rightarrow \quad \psi_0(R) = \text{const along each streamline},$$

and

$$v_r\,\partial_r\alpha(r; R) = -\frac{v_\phi}{r},$$

which couples the pitch evolution $\alpha(r; R)$ to the azimuthal velocity v_ϕ. Together with the Bernoulli invariant $\frac{1}{2}(v_r^2 + v_\phi^2) = \text{const}(R)$, these relations reproduce the *constant–Lagrangian spiral solution* used to describe stationary galactic disks.

10.3 Hierarchy of stationary solutions under BNC

Solution type	**Symmetry**	**Rotor generator B**	**Behaviour of (**
Schwarzschild / PG	Spherical	$\beta_r\beta_0$	$\psi = \psi(r)$, $\alpha = 0$
Kottler (Schwarzschild–de Sitter)	Spherical + cosmic boundary	$\beta_r\beta_0$	same $\psi(r)$ with
Spiral (constant–L disk)	Axisymmetric, irrotational	$(\cos\alpha\,\beta_r + \sin\alpha\,\beta_\phi)\beta_0$	$\psi = \psi_0(R) =$ co
Kerr / Doran	Axisymmetric, vortical	$\beta_r\beta_0 + \omega(r,\theta)\beta_\phi\beta_0$	ψ, α both vary;

All four geometries satisfy $v^\mu D\!\!\!/_\mu Q_g = 0$; they differ only by the non–zero components of the bivector generator and by the dependence of ψ and α on the coordinates. The spiral disk therefore appears as a special, axisymmetric, irrotational, constant–rapidity solution of the same governing law that yields Schwarzschild and Kerr.

10.4 Conceptual implications

Einstein's field equations $G_{\mu\nu} = 0$ are second–order in $g_{\mu\nu}$ and contain no constitutive closure. BNC, by contrast, is first–order and expresses a self–organisation rule:

In stationary vacuum gravity, the rotor field Q_g is parallel–transported along its own flow lines.

From this single statement:

- The *metric inflow* v_r becomes a real velocity of the spacetime medium, not a coordinate artefact;

- Stationary states automatically conserve the Bernoulli invariant;
- The classical GR metrics appear as steady–state configurations of a deeper first–order flow field.

The difference is thus one of dynamical level: Einstein's equations describe the equilibrium geometry, whereas BNC prescribes the *self–organising dynamics* that lead to it.

10.5 Unified form of the stationary flow equations

For all stationary, irrotational solutions generated by (16), the governing relations can be summarised as

$v^\mu \not{D}_\mu Q_g = 0$	(Bernoulli–Noether Closure)
$\frac{1}{2}(v_r^2 + v_\phi^2) = \text{const}(R)$	(Bernoulli invariant)
$\tan\alpha(r;R) = \frac{v_\phi}{\lvert v_r \rvert}$	(Pitch law).

The constant–Lagrangian spiral disk emerges when $\psi = \psi_0(R)$ is uniform and $\alpha(r;R)$ varies according to the observed ratio $v_\phi/|v_r|$. BNC thereby provides a single first–order law encompassing the Schwarzschild inflow, its cosmological (Kottler) extension, and the stationary spiral geometries observed in disk galaxies.

Summary

The Bernoulli–Noether Closure $v^\mu \not{D}_\mu Q_g = 0$ serves as the general self–organisation law of the spacetime medium. It replaces Einstein's second–order curvature equations by a first–order constitutive relation for the rotor field, in which the constant–Lagrangian spiral of galaxies, the Schwarzschild inflow, and the Kerr rotation all appear as distinct stationary solutions of one unifying geometric flow equation.

BNC is a dynamical statement of Noether's theorem for the Lorentz symmetry of the gravitational medium – just as energy and momentum conservation follow from translational symmetry, the conservation of rapidity (and hence Bernoulli invariance) follows from local Lorentz symmetry applied to the self-flow of spacetime. That's not a new "force law"; it's an extension of Noether's principle to the metric substrate itself.

GR treats spacetime as geometry — a static object shaped by energy and curvature. BNC treats spacetime as a self-organising medium — a field with internal degrees of freedom (rotor orientation and rapidity amplitude) that evolve according to first-order transport. This defines a new domain of application: between macroscopic geometry (GR) and microscopic quantum fields, describing the kinetic behaviour of the spacetime fabric itself, the regime where geometry behaves as a fluid with conserved rapidity. That's a domain GR doesn't cover — it assumes the steady state that BNC produces.

The Bernoulli–Noether Closure is not an additional force law but the first-order dynamical expression of Noether symmetry and Bernoulli energy conservation applied to the spacetime medium itself — extending GR into the new domain where geometry behaves as a self-organising fluid and the stationary metrics of GR appear as its steady states.

The Bernoulli–Noether Closure stands one level deeper than the Einstein Field Equations: it is a first-order kinetic law for the self-organisation of the spacetime rotor field Qg; when this flow reaches equilibrium, its stationary solutions reproduce Einstein's second-order equations — so it is not a rival to GR, but a more fundamental dynamical substrate from which GR's geometries emerge.

11 Why the Galactic Rotation Curve Problem Required a New Domain of Physics

11.1 The empirical problem in GR and Newtonian dynamics

Astronomical observations reveal that galactic rotation curves remain approximately flat far beyond the luminous disk, that gravitational lensing implies more mass than observed, and that the Tully–Fisher relation links baryonic mass to asymptotic velocity over many orders of magnitude. In Newtonian gravity,

$$v^2(r) = \frac{GM(r)}{r},$$

so flat rotation curves demand $M(r) \propto r$, i.e. ever–increasing mass. General relativity reduces to the same law in the weak–field limit and therefore predicts declining rotation speeds once the luminous mass saturates.

To match the data, two empirical remedies were introduced: (i) add non–luminous "dark matter" halos to supply the extra $M(r)$, or (ii) modify the acceleration law (MOND, TeVeS, etc.). Both preserve the second–order structure of Einstein's theory, treating spacetime as a passive geometry shaped by the matter stress–energy, and thus attempt to alter either the source term $T_{\mu\nu}$ or the functional form of $G_{\mu\nu}$.

11.2 What was missing in GR's structure

The Einstein equations are equilibrium constraints,

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

describing a balance between curvature and energy–momentum. They contain no *local flow equation* for the geometry itself:

- no Bernoulli–type conservation of metric kinetic energy,
- no coupling to cosmological boundary conditions through H_z,
- and no constitutive rule for the redistribution of curvature energy within the spacetime medium.

Hence, in extended, weakly curved systems such as galaxies, GR provides no mechanism that constrains the behaviour of the "metric inflow" beyond the luminous region. Without an additional constitutive relation, one is forced either to postulate hidden mass or to modify the gravitational constant empirically.

11.3 What the Q_g field introduces

The Q_g–Bernoulli formalism supplies exactly the missing structure. It introduces a first–order self–organisation law, the *Bernoulli–Noether Closure* (BNC),

$$v^\mu \not{D}_\mu Q_g = 0, \qquad \Rightarrow \qquad \frac{1}{2}(v_r^2 + v_\phi^2) = \text{const}(R),$$

which ensures conservation of total metric kinetic energy along each streamline. A natural cosmological boundary arises through the Hubble term

$$v_{r,\text{eff}} = \sqrt{\tfrac{2GM}{r}} - H_z r,$$

setting the finite radius where inflow and expansion balance ($v_{r,\mathrm{eff}} \to 0$). The self–organised redistribution between radial and azimuthal modes follows from

$$v_\phi^2(r;R) = \frac{3}{2}\left(\sqrt{\tfrac{2GM}{R}} - H_z R\right)^2 - \left(\sqrt{\tfrac{2GM}{r}} - H_z r\right)^2,$$

which naturally generates sustained azimuthal velocities—the observed flat rotation curves—without invoking additional mass. The galaxy appears as a stationary, irrotational *spiral metric* of the spacetime medium, self–organised under BNC.

11.4 Why GR and MOND could not reach this

Framework	**Structural limitation**	**Consequence**
Newtonian	Scalar potential only; no inflow dynamics.	Predicts $v^2 = GM/r$ declining $\propto r^{-1}$, inconsistent with flat rotation curves.
General Relativity	Second–order equilibrium equation; no first–order energy conservation of geometry.	Cannot sustain constant v_ϕ without invoking additional mass; geometry is passive.
MOND / TeVeS	Phenomenological modification of acceleration scale.	Fits rotation curves but lacks geometric foundation or connection to cosmic boundary conditions.
Q_g **formalism**	Adds first–order Bernoulli–Noether closure for metric flow.	Predicts flat or spiral metric velocities from self–organisation without dark matter; includes H_z as natural boundary.

Table 2: Comparison of traditional and Q_g frameworks for explaining galactic rotation curves.

The failure of GR in this domain is not due to incorrect field equations but to the absence of a first–order kinetic law. GR describes the equilibrium geometry of curvature, not the dynamical behaviour of the geometry as a flow.

11.5 Physical interpretation

In the Q_g picture, the constant–Lagrangian spiral represents a steady state of the spacetime flow rather than the motion of hidden matter. Observed flat rotation curves trace the azimuthal velocity of the metric itself,

$$v_\phi = c \tanh\psi \, \sin\alpha,$$

while the Hubble term H_z defines the outer boundary and pitch evolution. The gravitational "dark matter" effect is reinterpreted as the macroscopic manifestation of the self–organised kinetic energy of the Q_g field—a curvature energy stored in the spacetime medium.

11.6 Implications for physics

- BNC adds the missing constitutive dynamics of spacetime as a self–sustaining medium.

- The galactic regime—extended, weakly curved, and quasi–stationary—is precisely where this internal energy balance becomes visible.

- The cosmic term H_z supplies the global boundary condition that closes the energy loop between local inflow and universal expansion.

- The central question thus shifts from "Where is the missing mass?" to "How is metric energy stored and redistributed?"

The constant–Lagrangian spiral emerges as the natural steady state of a Bernoulli–Noether self–organising spacetime flow, making the flat rotation phenomenon a consequence of first–order gravitational kinetics rather than an anomaly of unseen matter.

Summary

The galactic rotation curve problem persisted because general relativity, as a second–order equilibrium theory, contains no first–order law governing the self–organisation of spacetime. The Bernoulli–Noether Closure of the Q_g field supplies that missing law, showing that flat rotation curves and spiral morphologies arise naturally from the steady–state energy balance of the spacetime flow itself, eliminating the need for dark matter or empirical MOND corrections.

12 Completion of the Q_g Theory: Law, Constant–Lagrangian Postulate, and Empirical Results

12.1 The theoretical core

The theory now rests on a single, closed first–order gravitational law:

$$\boxed{v^\mu \not{D}_\mu Q_g = 0} \qquad \text{(Bernoulli–Noether Closure, BNC)} \tag{17}$$

where Q_g is the gravitational rotor field and $\not{D}_\mu = \not{\partial}_\mu + (\not{\partial}_\mu Q_g)Q_g^{-1}$. Equation (17) states that the spacetime medium is self–parallel transported along its own flow lines. From this single closure follow:

$$\tfrac{1}{2}(v_r^2 + v_\phi^2) = \text{const}(R), \qquad \tan\alpha(r;R) = \frac{v_\phi(r;R)}{|v_r(r)|},$$

which together form the *constant–Lagrangian postulate*: each streamline of the metric medium evolves at constant total energy per unit rest mass. The BNC thus provides the kinetic layer beneath the equilibrium geometry of general relativity.

12.2 Relation to general relativity

The Bernoulli–Noether Closure reproduces all stationary GR metrics (Schwarzschild, Kerr, de Sitter, etc.) as steady–state solutions, but operates one level deeper dynamically. While the Einstein equations are second–order curvature constraints, the BNC is a first–order constitutive law describing *how the geometry self–organises into those equilibria.* Symbolically,

$$\text{BNC (first order)} \implies \text{Einstein equations (second order, steady state).}$$

It provides the missing dynamical foundation of GR, converting static curvature into active spacetime flow.

12.3 The constant–Lagrangian spiral disk

For an axisymmetric, irrotational flow the rotor takes the form

$$Q_g(r;R) = \exp\Big[\frac{\psi_0(R)}{2}\,(\cos\alpha(r;R)\,\beta_r + \sin\alpha(r;R)\,\beta_\phi)\,\beta_0\Big],$$

with constant rapidity amplitude $\psi_0(R)$ and variable pitch $\alpha(r;R)$. The stationary solution yields

$$v_{r,\mathrm{eff}} = \sqrt{\tfrac{2GM}{r}} - H_z r, \qquad v_\phi^2(r;R) = \frac{3}{2}\Big(\sqrt{\tfrac{2GM}{R}} - H_z R\Big)^2 - \Big(\sqrt{\tfrac{2GM}{r}} - H_z r\Big)^2,$$

representing a *constant–Lagrangian, self–organised spiral disk.* Here $v_{r,\mathrm{eff}}$ includes the gravitational potential and the cosmological boundary term H_z, while v_ϕ provides the flat or slowly rising rotation curve observed in galaxies. The pitch $\tan\alpha = v_\phi/|v_r|$ evolves with radius, yielding the empirical morphology of spiral arms.

12.4 Empirical validation

The same field equations have been applied successfully to data:

- Fits to the **SPARC database** reproduce the observed rotation curves of disk galaxies without invoking dark matter.

- The **Bernoulli–Rapidity Field** model (*EPJ de Haas ID 2181380*) reproduces bar and spiral morphologies as steady states of the Q_g flow.

- The same formulation remains consistent with **GNSS and terrestrial gravity** when applied on Earth scales, and with **cosmic expansion** through the H_z boundary.

Thus the constant–Lagrangian postulate and the empirical rotation–curve fits arise from the same governing law.

12.5 Physical interpretation

The apparent "dark matter" phenomenon is reinterpreted as the kinetic energy balance of the spacetime medium:

- Flat rotation curves arise from the *redistribution of curvature energy* from radial inflow to azimuthal circulation.

- The *Hubble term H_z* supplies the natural outer boundary and the observed relation between spiral morphology and redshift.

- The galactic disk is therefore a stationary, self–organised structure of the metric flow, not a halo of hidden mass.

This identifies a new domain of gravitational physics: the first–order, thermodynamically closed regime where spacetime behaves as a self–regulating fluid medium.

12.6 The synthesis

Aspect	Einstein GR	Q_g–BNC theory
Law type	Second–order equilibrium equation	First–order kinetic closure for the spacetime medium.
Governs	Curvature balance between geometry and matter	Self–organisation of the spacetime rotor field.
Predicts	Schwarzschild / Kerr metrics	Spiral metric with flat or rising v_ϕ.
Cosmic embedding	Added ad hoc via Λ	Natural through the Hubble term H_z.
Dark matter	Required to fit galaxy data	Emergent from internal Q_g energy balance.
Empirical status	Excellent for strong fields; fails for galaxies	Reproduces galactic rotation curves and morphologies; consistent with strong–field GR.

Summary

The Q_g–BNC framework is now complete: it provides a universal first–order law, a constant–Lagrangian postulate for steady flows, and empirical rotation–curve fits that verify its predictions. In structure, it constitutes the *first–order, kinetic completion of general relativity*: spacetime becomes a self–organising fluid whose stationary states produce the observed galactic dynamics without invoking unseen matter or modified forces.

Conclusion

The present work has reformulated the stationary solutions of general relativity as manifestations of a first–order gravitational rotor field $Q_g(x) \in \mathrm{Spin}(1,3)$. Within this Dirac–BQ framework, the metric tensor $g_{\mu\nu} = \langle Q_g \beta_\mu Q_g^{-1} Q_g \beta_\nu Q_g^{-1} \rangle_0$ and the gravitational connection $(\not{\partial} Q_g)Q_g^{-1}$ replace the second–order Einstein tensor by a first–order algebraic dynamics.

Through explicit reconstruction, the Schwarzschild and Kerr geometries were obtained as stationary states of this rotor field: a purely radial boost yields the Painlevé–Gullstrand Schwarzschild form, and the addition of an azimuthal rotation reproduces the Kerr metric in Doran coordinates. In both cases, the curvature invariants of the first–order field $F_{\mu\nu} = [\not{D}_\mu, \not{D}_\nu]$ coincide with those of the corresponding Einstein solutions, showing that the traditional metrics emerge as the squared images of a deeper, first–order structure.

The *Bernoulli–Noether Closure*

$$v^\mu \not{D}_\mu Q_g = 0$$

was introduced as the general constitutive law of stationary gravity. It expresses the self–organisation of spacetime as an inviscid, barotropic perfect fluid in which the total metric energy per streamline, $\frac{1}{2}(v_r^2 + v_\phi^2)$, remains constant. This closure unifies the Schwarzschild inflow, the Kottler (Schwarzschild–de Sitter) extension, the Doran form of Kerr, and the constant–Lagrangian spiral disks of galaxies within one single equation of motion for the rotor field.

The conserved rapidity norm $\tanh^2 \psi = v^2/c^2 = \mathrm{const}(R)$ represents the Bernoulli invariant of the metric flow and provides the link between gravity and thermodynamics. The first, second, and third laws of thermodynamics acquire geometric form as the conservation, self–organisation, and asymptotic stabilisation of the rapidity field. Spacetime thereby appears as a self–organising medium whose curvature and time dilation are thermodynamic variables of an underlying rotor dynamics.

Applied to galactic scales, the same closure yields flat or slowly rising rotation curves without invoking dark matter. The metric's azimuthal velocity v_ϕ arises from the redistribution of curvature energy between radial inflow and rotational circulation, while the Hubble parameter H_z sets the natural outer boundary where inflow and expansion balance. This constant–Lagrangian spiral solution has been shown to fit the SPARC database and related galactic data, linking local disk dynamics with cosmic boundary conditions.

In summary, the Qg–BNC formulation achieves three unifications:

(i) It reproduces all stationary GR metrics as steady–state solutions of a first–order rotor law;

(ii) It interprets gravity thermodynamically as the self–organisation of spacetime flow;

(iii) It explains galactic rotation phenomena as manifestations of that same first–order dynamics.

The resulting picture is that general relativity describes the equilibrium geometry, while the Bernoulli–Noether Closure provides the kinetic law that generates and maintains that equilibrium. Spacetime is thus revealed not as a static manifold but as a self–organising field whose internal rotor dynamics give rise to curvature, mass, and cosmic structure.

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