Infrared Graviton Mass and Inertia Quantum from the I^3 Vacuum Crystal

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Abstract

The cosmological constant problem and the nature of inertia are two of the most profound puzzles in theoretical physics. We propose a unified solution based on the elastic properties of a structured vacuum: the I^3 crystal, a Planck-scale lattice characterized by negative energy density and negative pressure. From this substrate, we derive an infrared graviton mass $m_g = \hbar H/c^2$ and an inertia quantum $m_\star = \hbar H/(2\pi c^2)$, related by $m_g = 2\pi m_\star$. A fundamental Planck-invariant product emerges naturally:

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi}\frac{\hbar c^5}{G} = \frac{3}{8\pi}E_{\rm Pl}^2, \tag{1}$$

linking the graviton's rest energy to the dark energy within its Compton volume through a constant Planck-scale invariant. The framework provides a mechanical explanation for inertia, gravitation, and cosmic acceleration without fine-tuning, and offers testable predictions for gravitational wave dispersion and dark energy evolution.

1 Introduction

The cosmological constant problem [12] represents one of the most persistent challenges in theoretical physics. The observed dark energy density $\rho_\Lambda \sim 10^{-123} M_{\rm Pl}^4$ appears unnaturally small compared to quantum field theory expectations, suggesting either profound fine-tuning or missing theoretical structure. Simultaneously, the nature of inertia and its connection to gravitation remain incompletely understood at fundamental levels.

We propose that both puzzles share a common resolution through the elastic properties of a structured vacuum: the I^3 crystal. This framework extends Kleinert's world-crystal concept [6] into a dual-sector construction where spacetime itself possesses a microscopic lattice structure. Unlike conventional massive gravity approaches [4], the graviton mass here emerges as an infrared phenomenon from vacuum elasticity, naturally explaining why deviations from general relativity manifest only at cosmic scales.

It is important to note that the I^3 crystal is not a return to the Lorentz-violating aether of the 19th century. Instead, it is a quantum-gravity-inspired structure that is compatible with Lorentz invariance and general covariance in the observable sector ($\mathbb{R}^3$). The imaginary sector (I^3) is a mathematical representation of a negative-energy, negative-pressure vacuum substrate that is not directly observable but whose elastic and thermodynamic properties give rise to inertia, gravity, and cosmic acceleration.

2 The I^3 Crystal: Vacuum Elastic Substrate

2.1 Fundamental Structure and Kinematics

The I^3 crystal represents a Bravais lattice with spacing:

$$a_I = 2\ell_{\text{Pl}} = 2\sqrt{\frac{\hbar G}{c^3}} \tag{2}$$

yielding a site density $n_0 = 1/a_I^3 = 1/(8\ell_{\text{Pl}}^3)$.

The fundamental kinematic mapping uses imaginary units to represent negative energy densities:

$$c_I = ic, \quad \ell_I = i\ell, \quad v_I = iv \tag{3}$$

$$E_I = mc_I^2 = -mc^2 \tag{4}$$

This mathematical construction produces the required negative pressure while maintaining positive energy density.

2.2 Interpretation of Imaginary Coordinates

The use of imaginary coordinates in the I^3 sector is a mathematical device to model a vacuum with negative energy density and negative pressure. The physical spacetime is the real $\mathbb{R}^3$ sector, and all observable quantities are real. The imaginary sector is a representation of a thermodynamic and elastic medium that is not directly observable but whose effects are manifested through gravitational phenomena. This approach is analogous to the use of imaginary time in thermodynamics and quantum field theory, but here applied to spatial coordinates to encode the negative-energy properties of the vacuum substrate.

The use of imaginary coordinates represents a mathematical mapping to describe a sector with specific thermodynamic properties, not a claim about physical imaginary dimensions. This is analogous to:

- **Wick rotation** in quantum field theory
- **Complex analysis** in condensed matter physics
- **Analytic continuation** methods throughout theoretical physics

The physical content resides in the real observables and their relationships, all of which are real and measurable.

Historical Context: From Lorentz Aether to Structured Vacuum

The concept of a material substrate for physical phenomena has a long and contentious history. The 19th-century luminiferous aether was rightly abandoned due to its incompatibility with relativity and lack of experimental support. However, the modern concept of vacuum structure differs fundamentally:

- **Not a preferred frame**: The I^3 crystal defines no preferred rest frame; Lorentz invariance emerges from its elastic properties
- **Not a mechanical medium for light**: Unlike the classical aether, it doesn't serve as propagation medium for electromagnetic waves
- **Quantum information substrate**: It functions as an information-theoretic foundation from which spacetime geometry and inertia emerge
- **Background independence**: The crystal structure is dynamical, not fixed against spacetime

Contemporary approaches like analogue gravity, emergent spacetime, and condensed matter approaches to quantum gravity share this modern perspective. The I^3 crystal represents a specific realization within this research program.

Why white holes as lattice constituents (and not generic Planck clumps)?

(1) Horizon area $\Rightarrow$ one nat per tile. The information accounting used throughout (one nat per $A_{\text{tile}} = 4\ell_{\text{Pl}}^2$) requires a bona fide horizon with Bekenstein–Hawking entropy $S = k_B A/(4\ell_{\text{Pl}}^2)$. A featureless Planck clump has no intrinsic area–entropy law, so it cannot support the Landauer bookkeeping that yields $m_\star = \hbar H/(2\pi c^2)$ or the Planck-invariant product:

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi}\,\frac{\hbar c^5}{G} = \frac{3}{8\pi}\,E_{\text{Pl}}^2. \tag{5}$$

(2) Surface gravity $\Rightarrow$ Unruh/Landauer energy scale. A horizon provides a natural local acceleration κ (surface gravity) and hence a temperature:

$$\kappa = \frac{c^2}{2r_s}, \qquad T_{\text{H}} = \frac{\hbar\kappa}{2\pi k_B c} = \frac{\hbar c}{4\pi k_B r_s} = \frac{\hbar c}{8\pi k_B \ell_{\text{Pl}}},$$

so erasing one nat costs $E_{\text{nat}} = k_B T_{\text{U}} = \hbar H/(2\pi)$ at the cosmic horizon. Without a horizon, there is no canonical route from mechanics to the information–thermodynamic scale used to define inertia.

(3) Outgoing causal bias $\Rightarrow$ stability and defocusing. White holes are time-reversed black holes: their null congruences have positive expansion (outgoing), matching the negative-volume orientation ($ijk = -1$) and the repulsive pressure $p_I < 0$. Generic clumps (or black holes) would favor inflow/accumulation and drive collapse rather than the required Raychaudhuri defocusing used to avoid singularities and to seed $\mathbb{R}^3$.

(4) Maximal stiffness with minimal structure. A white-hole site saturates compactness ($r_s = 2\ell_{\text{Pl}}$) and sets the lattice spacing $a_I = 2\ell_{\text{Pl}}$, giving Planck-level elastic moduli while keeping only near-horizon data. The global continuation of the white-hole solution is irrelevant; the lattice dynamics depend only on the local horizon patch.

(5) Consistency with boundary exchange. Each site exposes a spherical horizon area $A_{\rm WH} = 16\pi\ell_{\rm Pl}^2 = 4\pi\,A_{\rm tile}$, so it carries $S_{\rm WH} = 4\pi k_B$ (one nat per tile). The holographic cap remains at the boundary: $N_{\rm bdy} = A_H/A_{\rm tile}$ governs the degrees of freedom available to $\mathbb{R}^3$, while the bulk remains sequestered and evanescent.

(6) Thermodynamic consistency with negative temperature. The population inversion implied by $T_I < 0$ naturally favors emission over absorption, matching white-hole behavior rather than black-hole accretion. This ensures the lattice maintains its negative-energy character while permitting controlled energy exchange at the boundary.

In summary, modeling each node as a Planck white hole is the minimal assumption that (i) endows the lattice with an intrinsic entropy per area, (ii) fixes the mechanical $\leftrightarrow$ thermodynamic conversion via surface gravity, and (iii) enforces the outgoing causal structure consistent with negative pressure and defocusing. These three ingredients are precisely what the elastic, Landauer–Unruh, and rotational routes all use to obtain $m_g = \hbar H/c^2$ and the Planck-invariant product:

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi}\,\frac{\hbar c^5}{G} = \frac{3}{8\pi}\,E_{\rm Pl}^2. \tag{6}$$

2.3 Thermodynamic Interpretation and Negative Temperature

The I^3 lattice is not a purely mechanical construct but a thermodynamic medium endowed with entropy and temperature. Because every site carries *negative internal energy* ($E_I = -m_{\rm Pl}c^2$) while mass $m_{\rm Pl} > 0$ and volume $V_I < 0$, its statistical mechanics necessarily obeys the inverted Boltzmann hierarchy:

$$p_i \;\propto\; \exp\!\left(-\frac{E_i}{k_{\rm B}T_I}\right) \quad\Rightarrow\quad \frac{n_{\rm excited}}{n_{\rm ground}} = \exp\!\left(+\frac{m_{\rm Pl}c^2}{k_{\rm B}|T_I|}\right), \tag{7}$$

indicating a *population inversion* and therefore a **negative absolute temperature** $T_I < 0$. For a single excitation level at $E = -m_{\rm Pl}c^2$ the occupation ratio gives

$$\frac{n_{\rm excited}}{n_{\rm total}} = \frac{1}{1 + e^{-E/k_{\rm B}T_I}} = \frac{1}{1 + e^{+1}} \approx 0.269,$$

which matches the fractional emission probability inferred in the early-universe white-hole transition scenario [14]. This thermodynamic inversion underlies the initial outflow from the I^3 bulk: because $T_I < 0$, higher-energy (less negative) states are *more* populated, so excitations from $E = -mc^2$ to the boundary state $E = 0$ are favored until the finite boundary capacity saturates. Stability arises not from suppressing high-energy states, but from saturation and repulsive pressure $p_I < 0$, which together halt runaway excitation and maintain equilibrium with the horizon.

Geometry and entropy per white-hole site. Each lattice site corresponds to a Planck-mass white hole of Schwarzschild radius $r_s = 2\ell_{\rm Pl}$, giving a horizon area $A_{\rm WH} = 4\pi r_s^2 = 16\pi\ell_{\rm Pl}^2$. Since one information tile carries area $A_{\rm tile} = 4\ell_{\rm Pl}^2$, each white hole exposes $A_{\rm WH}/A_{\rm tile} =$

$4\pi \approx 12.6$ tiles. Accordingly, the entropy per white hole is $S_{\rm WH} = 4\pi k_B$ (or $4\pi k_B \ln 2$ if expressed in bits). This geometric factor reconciles the spherical horizon geometry with the Planck-tile information counting and ensures consistency with the Bekenstein–Hawking bound $S/A = 1/(4\ell_{\rm Pl}^2)$.

Numerical consistency: boundary vs bulk entropy. The cosmological horizon of radius $R_H = c/H_0$ has area $A_H = 4\pi R_H^2$, giving the Bekenstein–Hawking (holographic) entropy

$$S_{\rm bdy} = \frac{k_B A_H}{4\ell_{\rm Pl}^2} = \pi\, k_B \left(\frac{R_H}{\ell_{\rm Pl}}\right)^2 \;\sim\; 10^{122}\, k_B. \tag{8}$$

The number of boundary information tiles is therefore $N_{\rm bdy} = A_H/A_{\rm tile} = \pi(R_H/\ell_{\rm Pl})^2$.

Each lattice site is a Planck white hole with Schwarzschild radius $r_s = 2\ell_{\rm Pl}$ and horizon area $A_{\rm WH} = 16\pi\ell_{\rm Pl}^2$, i.e. it covers $A_{\rm WH}/A_{\rm tile} = 4\pi$ tiles. Hence its intrinsic entropy is

$$S_{\rm WH} = \frac{k_B A_{\rm WH}}{4\ell_{\rm Pl}^2} = 4\pi\, k_B \;\approx\; 12.6\, k_B. \tag{9}$$

If we naïvely counted *all* bulk white holes within the Hubble volume $V_H = 4\pi R_H^3/3$ at lattice spacing $a_I = 2\ell_{\rm Pl}$, we would obtain

$$N_{\rm bulk} \;\simeq\; \frac{V_H}{a_I^3} = \frac{4\pi}{3}\left(\frac{R_H}{2\ell_{\rm Pl}}\right)^3 \;\sim\; 10^{183}, \qquad S_{\rm bulk} \simeq N_{\rm bulk}\, S_{\rm WH} \;\sim\; 10^{184}\, k_B, \tag{10}$$

vastly exceeding the holographic bound (8). This does not signal an inconsistency: only *boundary-facing* degrees of freedom are thermodynamically accessible to $\mathbb{R}^3$. The effective, exchangeable entropy is capped by the boundary count $N_{\rm bdy}$, i.e. it is *holographic*:

$$S_{\rm exch} \;=\; N_{\rm bdy}\, k_B \;=\; \pi\, k_B \left(\frac{R_H}{\ell_{\rm Pl}}\right)^2 \;=\; S_{\rm bdy}. \tag{11}$$

Equivalently, one can say that out of the 4π tiles on each white hole, only a boundary-limited subset participates in information exchange at any time; the rest are sequestered in the evanescent bulk. Thus the I^3 crystal can be *bulk-rich* (large $N_{\rm bulk}$) yet *boundary-limited* (finite $N_{\rm bdy}$), precisely reproducing the Bekenstein–Hawking entropy (8) and preserving the Planck-invariant product:

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi}\,\frac{\hbar c^5}{G} = \frac{3}{8\pi}\, E_{\rm Pl}^2. \tag{12}$$

Equation of State and Negative Pressure. Using the first law $dE_I = T_I dS_I - p_I dV_I$ with $E_I < 0$ and $V_I < 0$, the combination $T_I dS_I$ remains positive while $-p_I dV_I$ is also positive if $p_I < 0$. Hence the I^3 vacuum possesses *negative pressure*

$$p_I = -\rho_I c^2,$$

the signature of a de Sitter–like repulsive medium. This relation holds locally because each lattice site behaves as a Planck-area thermodynamic patch in equilibrium with the global negative-temperature bath.

Physical Picture. In summary, the I^3 crystal is a negative-energy, negative-volume, negative-temperature medium with positive energy density and positive entropy per site. Its inherent negative pressure provides the cosmological repulsion that later manifests as dark energy when the crystal couples to the $\mathbb{R}^3$ sector.

2.4 Energy Density and Elastic Response

The crystal behaves as an isotropic elastic continuum with displacement field $\mathbf{u}_I(\mathbf{x}_I, t)$. The strain tensor and constitutive relations maintain Hooke's law form with appropriate sign mappings to ensure positive elastic energy density:

$$\mathcal{U}_I = \tfrac{1}{2}\lambda(\varepsilon_{kk})^2 + \mu\varepsilon_{ij}\varepsilon_{ij} > 0 \tag{13}$$

The negative pressure $p_I < 0$ provides the repulsive gravitation responsible for cosmic acceleration, consistent with exotic energy conditions [1].

2.5 Thermodynamic and Geometric Parameters of the I^3 Crystal

The defining physical parameters of the I^3 vacuum crystal are summarized in Table 1. They combine imaginary kinematics ($c_I = ic$) with real elastic moduli, yielding a medium of negative pressure and positive energy density. Each lattice site represents a Planck-mass white hole of radius $r_{\mathrm{s}} = 2\ell_{\mathrm{Pl}}$, forming a close-packed structure in imaginary space with negative volume element from the quaternionic relation $ijk = -1$.

2.6 Constitutive Nature and Non-Observability

Due to imaginary speed ic the I^3 lattice possesses effective electromagnetic parameters

$$\varepsilon_0^{(I)} < 0, \qquad \mu_0^{(I)} > 0,$$

so that the wave impedance $Z_I = \sqrt{\mu_0^{(I)}/\varepsilon_0^{(I)}}$ is purely imaginary. Consequently, any electromagnetic disturbance in this sector is evanescent: the electric field component is suppressed while a magnetic–like (gravitomagnetic) component persists. This feature explains both the invisibility of the I^3 crystal to ordinary radiation and the emergence of its negative pressure $p_I < 0$. The remaining field degrees of freedom are magnetostatic in character, supporting only shear or torsional deformations—precisely those responsible for the infrared graviton mass derived in the next section.

2.7 Geometric and Energetic Properties

The geometry of the I^3 crystal follows directly from the quaternionic orientation rule $ijk = -1$, which assigns a negative volume element

$$dV_I = i\,j\,k\,dx\,dy\,dz = -\,dx\,dy\,dz. \tag{14}$$

Consequently, any integration of energy over volume, $E_I = \int \rho_I dV_I$, yields a negative total energy $E_I < 0$ even though the local density $\rho_I > 0$. This purely geometric sign inversion

Table 1: Fundamental thermodynamic and geometric parameters of the I^3 vacuum crystal.

Quantity	**Symbol**	**Expression**	**Sign/Nature**	**Physical Meaning**
Lattice spacing	a_I	$2\ell_{\rm Pl} = 2\sqrt{\hbar G/c^3}$	real (metric imaginary)	Site-to-site distance between Planck-mass white holes.
Site mass	$m_{\rm Pl}$	$\sqrt{\hbar c/G}$	positive	Rest mass per white-hole node.
Energy per site	E_I	$-m_{\rm Pl}c^2$	negative	Baseline negative-energy state of the vacuum.
Volume element	V_I	$-\ell_{\rm Pl}^3$	negative	From quaternion property $ijk = -1$.
Mass density	$\rho_{\rm mass}^{(I)}$	$m_{\rm Pl}/V_I$	negative	Source of repulsive gravitation.
Energy density	$\rho_{\rm energy}^{(I)}$	$\lvert E_I\rvert/\lvert V_I\rvert$	positive	Drives cosmic acceleration.
Characteristic speed	c_I	ic	imaginary	Imaginary-space propagation speed.
Pressure	p_I	$-\rho_{\rm energy}^{(I)}c^2$	negative	Raychaudhuri defocusing term.
Entropy per site	$S_{\rm site}$	$k_B A_{\rm Pl}/(4\ell_{\rm Pl}^2) = k_B$	positive	One Bekenstein–Hawking nat per site (one bit $= k_B \ln 2$).
Temperature	T_I	< 0	negative absolute	Negative-temperature equilibrium of the lattice.

provides a natural explanation for the effective gravitational repulsion associated with the I^3 substrate.

In the absence of real curvature sources, the I^3 lattice forms a hyperbolic (negatively curved) three-space. Its curvature scalar may be expressed as

$$\mathcal{R}_I = -\frac{6}{a_I^2}, \tag{15}$$

where $a_I = 2\ell_{\rm Pl}$ is the lattice spacing. This defines a background curvature of Planck magnitude that acts as the restoring field for all elastic deformations.

The intrinsic energy density of the crystal,

$$\rho_{I,0} = \frac{m_{\rm Pl}c^2}{(2\ell_{\rm Pl})^3} = \frac{c^7}{8\,\hbar\,G^2} \simeq 3.9\times 10^{113}\,{\rm J/m^3}, \tag{16}$$

represents the maximal stiffness of the vacuum—a value reduced by more than 10^{122} in the observable Universe after expansion and thermal relaxation. This Planck-level density provides the elastic moduli entering the effective Lamé parameters in Eq. (39).

The crystal curvature and energy density are linked through the Einstein relation written in imaginary variables:

$$\mathcal{R}_I = -\frac{8\pi G}{c^4}\,T_I \quad \text{with} \quad T_I = E_I/V_I = -\rho_I c^2. \tag{17}$$

This form shows that a positive ρ_I necessarily produces a negative curvature scalar, confirming the anti-de Sitter-like geometry of the substrate. The real Universe ($\mathbb{R}^3$) then arises as a low-curvature boundary excitation embedded in this hyperbolic background.

The negative curvature also implies that the elastic excitations of the lattice possess an imaginary wavevector component, leading to the evanescent (non-propagating) character of the modes discussed in Section 3. The I^3 crystal therefore combines the roles of a gravitational vacuum, a negative-curvature geometric background, and an information-saturated thermodynamic medium with Planck-scale stiffness.

2.8 Derived macroscopic scales and horizon thermodynamics

Table 2 collects the principal macroscopically observable scales implied by the I^3 crystal, evaluated symbolically and at the present epoch ($H_0 \simeq 2.2\times 10^{-18}\,{\rm s^{-1}}$) for reference.
Notes. (i) The elasticity and thermodynamic routes imply $m_g = 2\pi m_\star$. (ii) The Planck-invariant product:

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi}\,\frac{\hbar c^5}{G} = \frac{3}{8\pi}\,E_{\rm Pl}^2 \tag{18}$$

remains constant throughout cosmic evolution. (iii) $N_{\rm bdy}$ controls thermodynamically accessible degrees of freedom (holographic), whereas $N_{\rm cell}$ characterizes bulk coherence for the lowest lattice mode.

3 Graviton Mass from Vacuum Elasticity

3.1 Imaginary-Space Elastic Tensor and Wave Dynamics

The I^3 crystal behaves as an isotropic continuum characterized by Lamé parameters (λ_I, μ_I) defined in imaginary coordinates $x_i^{(I)} = i\,x_i$. The displacement field $\mathbf{u}_I(\mathbf{x}_I, t)$ obeys the

Table 2: Derived physical characteristics from the I^3 crystal (present-epoch numerical values use $H_0 \simeq 2.2 \times 10^{-18}$ s^{-1}, $c = 2.998 \times 10^8$ m/s, $\hbar = 1.055 \times 10^{-34}$ J s, $k_B = 1.381 \times 10^{-23}$ J/K, $\ell_{\rm Pl} = 1.616 \times 10^{-35}$ m).

Quantity	**Symbol**	**Definition**	**Present epoch** ($z = 0$)
Horizon / graviton Compton length	λ_g	$\lambda_g = \dfrac{\hbar}{m_g c} = \dfrac{c}{H}$	$\lambda_g \simeq 1.36 \times 10^{26}$ m
Infrared graviton mass	m_g	$m_g = \dfrac{\hbar H}{c^2}$	$m_g \simeq$ 1.1×10^{-33} eV/c^2 (1.8×10^{-69} kg)
Inertia quantum (per tile)	$m_\star$	$m_\star = \dfrac{\hbar H}{2\pi c^2} = \dfrac{m_g}{2\pi}$	$m_\star \simeq$ 1.8×10^{-34} eV/c^2
Unruh temperature (cosmic accel.)	T_U	$T_U = \dfrac{\hbar H}{2\pi k_B}$	$T_U \simeq 2.7 \times 10^{-30}$ K
Energy per erased nat	$E_{\rm nat}$	$E_{\rm nat} = k_B T_U = \dfrac{\hbar H}{2\pi}$	$E_{\rm nat} \simeq 3.7 \times 10^{-53}$ J
Dark energy density	ρ_Λ	$\rho_\Lambda = \dfrac{3c^2 H^2}{8\pi G}$	$\rho_\Lambda \simeq 7.8 \times 10^{-10}$ J/m^3
Zitterbewegung / response time	$\tau_{\rm ZB}$	$\tau_{\rm ZB} = \dfrac{2\pi}{H}$	$\tau_{\rm ZB} \simeq 2.9 \times 10^{18}$ s ($\sim 9.2 \times 10^{10}$ yr)
Boundary tile count (holographic)	$N_{\rm bdy}$	$N_{\rm bdy} = \dfrac{A_H}{4\ell_{\rm Pl}^2} = \pi\left(\dfrac{R_H}{\ell_{\rm Pl}}\right)^2$	$N_{\rm bdy} \sim 2.2 \times 10^{122}$
Coherent sites per graviton cell	$N_{\rm cell}$	$N_{\rm cell} \sim \left(\dfrac{\lambda_g}{2\ell_{\rm Pl}}\right)^3$	$N_{\rm cell} \sim 10^{183}$
White-hole radius / area (per site)	$r_s,\ A_{\rm WH}$	$r_s = 2\ell_{\rm Pl},\ A_{\rm WH} = 16\pi\ell_{\rm Pl}^2$	$A_{\rm WH}/(4\ell_{\rm Pl}^2) =$ $4\pi \approx 12.6$ tiles

standard Cauchy momentum equation written in I^3 form,

$$\rho_I\, \ddot{u}_i^{(I)} = \partial_j^{(I)} \sigma_{ij}^{(I)}, \qquad \sigma_{ij}^{(I)} = C_{ijkl}^{(I)}\, \varepsilon_{kl}^{(I)}, \tag{19}$$

where $C_{ijkl}^{(I)}$ is the fourth-rank elasticity tensor and $\varepsilon_{kl}^{(I)}$ the strain tensor. The imaginary derivative $\partial_j^{(I)} \equiv (1/i)\, \partial_j$ implies an overall sign flip in all quadratic invariants.

For an isotropic medium,

$$C_{ijkl}^{(I)} = \lambda_I\, \delta_{ij}\delta_{kl} + \mu_I\, (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \tag{20}$$

and the strain tensor is

$$\varepsilon_{ij}^{(I)} = \tfrac{1}{2}(\partial_i^{(I)} u_j^{(I)} + \partial_j^{(I)} u_i^{(I)}). \tag{21}$$

Inserting $\partial_i^{(I)} = (1/i)\partial_i$ and choosing the constitutive mapping $\lambda_I = -\lambda$, $\mu_I = -\mu$ ensures that the stored elastic energy density

$$\mathcal{U}_I = \tfrac{1}{2}\lambda(\varepsilon_{kk})^2 + \mu\varepsilon_{ij}\varepsilon_{ij} > 0$$

remains positive when expressed in real variables.

3.2 Linearized Field Equation in Real Coordinates

Substituting these relations into Eq. (19) gives

$$\rho_I \ddot{u}_i = -(\lambda + 2\mu)\, \partial_i \partial_k u_k - \mu\, \nabla^2 u_i, \tag{22}$$

which is the imaginary-space analog of the Navier–Cauchy equation. The negative sign before the spatial derivatives reflects the repulsive (anti-binding) nature of the I^3 substrate. Equation (22) governs both longitudinal and transverse distortions of the vacuum lattice.

Seeking plane-wave solutions $u_i \propto e^{i(\mathbf{k}\mathbf{x} - \omega t)}$ yields the eigenvalue problem

$$\left[\mu k^2\, \delta_{ij} + (\lambda + \mu)\, k_i k_j - \rho_I \omega^2\, \delta_{ij}\right] u_j = 0. \tag{23}$$

The determinant condition produces the two familiar branches,

$$\omega_{\mathrm{L}}^2 = -\frac{\lambda + 2\mu}{\rho_I}\, k^2, \qquad\qquad \omega_{\mathrm{T}}^2 = -\frac{\mu}{\rho_I}\, k^2, \tag{24}$$

showing explicitly that ω is imaginary: $\omega = i\Gamma$. Hence the lattice supports only *evanescent*, non-radiative oscillations.

3.3 Evanescent Mode Characteristics

For a decaying mode $u_i \propto e^{-|\Gamma| t}$, the decay constant is

$$\Gamma_{\mathrm{L,T}} = \sqrt{\frac{(\lambda + 2\mu,\ \mu)}{\rho_I}}\, k \equiv c_{\mathrm{s,L,T}}\, k, \tag{25}$$

where $c_{\mathrm{s,L,T}}$ denote the imaginary "sound speeds" ($c_{\mathrm{s}} \simeq i\, c$ at the Planck limit). The corresponding attenuation length along the propagation direction is

$$\ell_{\mathrm{ev}} = \frac{1}{k} = \frac{c}{\Gamma_{\mathrm{L,T}}}, \tag{26}$$

demonstrating that short-wavelength deformations are strongly confined, whereas the longest wavelength compatible with causality extends to the Hubble radius, $\ell_{\mathrm{ev}}^{\mathrm{max}} = R_H = c/H$.

This cutoff introduces an intrinsic infrared scale into the elastic vacuum. Identifying the lowest longitudinal evanescent mode ($k = i/\ell_{\mathrm{ev}}$) with a *massive graviton* leads to the dispersion relation

$$E_g^2 = (\hbar\omega)^2 = (m_g c^2)^2 = \hbar^2 c^2 / \ell_{\mathrm{ev}}^2, \tag{27}$$

giving directly

$$m_g = \frac{\hbar}{c\, \ell_{\mathrm{ev}}}. \tag{28}$$

At the cosmic horizon scale $\ell_{\mathrm{ev}} = R_H$, Eq. (28) reproduces the observed graviton mass $m_g = \hbar H / c^2$, establishing the elastic origin of inertia and gravity.

3.4 Elastic Constants and Energy Scales

Using the Planck energy density $\rho_{I,0} \simeq 3.9 \times 10^{113}$ J/m^3, the effective bulk modulus of the crystal is $K_I = (\lambda + 2\mu)/3 \simeq \rho_{I,0} c^2$, corresponding to a maximum propagation speed equal to ic. The ratio $\mu/\lambda \approx 1/2$ produces a Poisson coefficient $\nu_I \approx 1/4$, a value typical of hyperbolic crystals [14]. Small deviations from these Planck values at large scales translate into the minute but finite graviton mass that governs cosmic acceleration.

3.5 Evanescent Modes and Infrared Cutoff

The equation of motion for lattice displacements follows from momentum conservation:

$$\rho_I \ddot{u}_i = -(\lambda + 2\mu)\partial_i \partial_k u_k - \mu \nabla^2 u_i \tag{29}$$

For plane wave solutions $u_i \propto e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$, the dispersion relations are:

$$\omega_{\mathrm{L}}^2 = -\frac{\lambda + 2\mu}{\rho_I} k^2, \quad \omega_{\mathrm{T}}^2 = -\frac{\mu}{\rho_I} k^2 \tag{30}$$

The negative signs imply purely imaginary frequencies:

$$\omega_{\mathrm{L,T}} = i\Gamma_{\mathrm{L,T}}, \quad \Gamma_{\mathrm{L,T}} = \sqrt{\frac{(\lambda + 2\mu,\ \mu)}{\rho_I}} k \tag{31}$$

These describe evanescent, non-radiative modes with decay length:

$$\ell_{\mathrm{ev}} = \frac{c_{\mathrm{s}}}{\Gamma_k} = \frac{1}{k} \tag{32}$$

where $c_{\mathrm{s}} = \sqrt{|\lambda + 2\mu|/\rho_I}$ is the sound speed.

In an expanding universe, the maximal sustainable wavelength cannot exceed the causal horizon radius, setting the infrared cutoff:

$$\ell_{\mathrm{ev}}^{\mathrm{max}} = R_H = \frac{c}{H} \tag{33}$$

3.6 Effective Graviton Mass Derivation

Identifying the longitudinal evanescent mode with a massive graviton gives the mass-energy relation:

$$m_g c^2 = \hbar |\Gamma_k| = \frac{\hbar c}{\ell_{\mathrm{ev}}} \tag{34}$$

At the infrared cutoff $\ell_{\mathrm{ev}} = c/H$, this yields:

$$m_g = \frac{\hbar H}{c^2} \tag{35}$$

Numerical evaluation with current Hubble parameter $H_0 \approx 2.2 \times 10^{-18}$ s^{-1} [10] gives:

$$m_g \approx 1.1 \times 10^{-33}\,\mathrm{eV}/c^2 \approx 1.8 \times 10^{-69}\,\mathrm{kg} \tag{36}$$

comfortably consistent with LIGO constraints $m_g < 10^{-22}$ eV/c^2 [8] and solar system tests.

The corresponding Compton wavelength:

$$\lambda_g = \frac{\hbar}{m_g c} \approx \frac{c}{H} \approx 1.4 \times 10^{26}\,\mathrm{m} \tag{37}$$

confirms that gravitational modifications are confined to Hubble scales.

3.7 Wavelength-Dependent Mass and Scale Safety

The finite coherence of the I^3 lattice produces a scale-dependent mass kernel:

$$m_g^2(k) = \frac{\hbar^2 k^2}{c^2}\mathcal{S}(k), \quad \mathcal{S}(k) = \frac{1}{1 + (kR_H)^2} \tag{38}$$

yielding the wavelength-dependent mass:

$$m_g(k) = \frac{\hbar k}{c} \frac{1}{\sqrt{1 + (kR_H)^2}} \tag{39}$$

This kernel ensures general relativity recovery at small scales while maintaining Hubble-scale modifications:

- $kR_H \ll 1$: $m_g(k) \to \hbar H/c^2$ (Hubble-scale IR mass)
- $kR_H \gg 1$: $m_g(k) \to \hbar/(cR_H)$ (GR recovery)

4 Inertia Quantum from Horizon Thermodynamics

4.1 Thermodynamic Link between the I^3 Crystal and Horizon Physics

The I^3 crystal does not merely store elastic energy—it also obeys a microscopic information–energy correspondence. Each Planck-scale patch at the $\mathbb{R}^3$–I^3 interface acts as an *information capacitor* of capacity one natural unit (one nat). When an accelerating observer interacts with this substrate, bits of information are erased or written at a finite thermodynamic cost set by the Unruh temperature.

The Unruh relation for acceleration a reads

$$T_{\mathrm{U}} = \frac{\hbar a}{2\pi c k_{\mathrm{B}}}. \tag{40}$$

For cosmic acceleration $a \sim cH$, the interface temperature becomes

$$T_{\mathrm{U}} = \frac{\hbar H}{2\pi k_{\mathrm{B}}}, \tag{41}$$

corresponding to $T_{\mathrm{U}} \sim 10^{-30}$ K at the present epoch. Because the I^3 crystal possesses negative temperature ($T_I < 0$), this energy flow reverses direction: instead of absorbing heat from real space, it releases information energy toward the $\mathbb{R}^3$ boundary. The Landauer principle [7] then gives the elementary energy associated with a one-nat erasure:

$$E_{\mathrm{nat}} = k_{\mathrm{B}} T_{\mathrm{U}} = \frac{\hbar H}{2\pi}. \tag{42}$$

4.2 Microscopic Energy Exchange and Inertia

When a particle in $\mathbb{R}^3$ accelerates, it perturbs the adjacent I^3 tiles, forcing local bit transitions between the two sectors. Each transition requires an energy transfer E_{nat} and acts as a tiny impulse opposing acceleration. Summed over N_{eff} active tiles, the total reaction energy defines the inertial mass:

$$m_{\text{eff}}c^2 = N_{\text{eff}}E_{\text{nat}}. \tag{43}$$

For a single-tile interaction $N_{\text{eff}} = 1$,

$$m_\star = \frac{E_{\text{nat}}}{c^2} = \frac{\hbar H}{2\pi c^2}, \tag{44}$$

identifying the fundamental *inertia quantum.*

Physically, $m_\star$ measures the resistance of the vacuum to changes of information state under acceleration—a Landauer cost encoded in horizon dynamics. In cosmic equilibrium, all inertial and gravitational masses scale with $H(t)$, maintaining the dimensionless ratio $m_g/m_\star = 2\pi$ as the Universe expands.

4.3 Horizon Energy Balance and Negative-Temperature Work

The first law applied to the I^3 interface reads

$$dE_{\text{tot}} = T_I dS_I - p_I dV_I, \tag{45}$$

where $T_I < 0$ and $p_I < 0$. During acceleration, the information transfer $dS_I = -k_{\text{B}} \ln 2$ per tile yields a positive work term $|T_I|\, dS_I$ that appears as inertial energy in $\mathbb{R}^3$. The same process in bulk gives the effective Unruh–Landauer energy density:

$$u_{\text{eff}} = \frac{E_{\text{nat}}}{(2\ell_{\text{Pl}})^2 c/H} = \frac{\hbar H^3}{8\pi \ell_{\text{Pl}}^2 c^2}, \tag{46}$$

a value matching the dark-energy density at the present H_0.

This dual-energy description highlights the self-consistency of the framework: horizon thermodynamics, vacuum elasticity, and information erasure all express the same underlying process—the mechanical exchange of energy between real and imaginary sectors across Planck-scale boundaries.

4.4 Entropy Rate and Inertial Response Time

Defining a local entropy flux $\dot{S}_I = (a/c)\, k_{\text{B}}/(2\pi)$, the response time associated with one information exchange is

$$\tau_{\text{ZB}} = \frac{\hbar}{m_\star c^2} = \frac{2\pi}{H}, \tag{47}$$

which equals the period of Zitterbewegung in the cosmological frame. Hence, the Unruh–Landauer process reproduces the same timescale that governs microscopic matter oscillations, suggesting that all inertial phenomena are manifestations of this horizon–crystal interaction.

4.5 Comparison with Other Horizon Thermodynamic Models

Unlike emergent-gravity approaches that postulate entropic forces [9], the present derivation links the entropy change directly to the mechanical deformation of the I^3 lattice and its imaginary stress–strain response. The proportionality between $m_\star$ and H arises automatically from the thermodynamic boundary condition, with no adjustable parameters or additional fields. It also establishes a concrete microphysical meaning for inertia: the cost of maintaining information equilibrium between $\mathbb{R}^3$ and I^3 under acceleration.

4.6 Coherent Participation and Collective Behavior

The collective behavior of Planck-scale sites provides microscopic insight. For N sites participating coherently in the lowest normal mode, the collective coordinate:

$$Q = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} u_n \tag{48}$$

yields kinetic energy:

$$T = \tfrac{1}{2} m_{\text{Pl}} \sum_n \dot{u}_n^2 = \tfrac{1}{2} \left(\frac{m_{\text{Pl}}}{\sqrt{N}} \right) \dot{Q}^2 \tag{49}$$

giving effective inertial mass $m_{\text{eff}} = m_{\text{Pl}}/\sqrt{N}$.

For the evanescent mode with participation ratio $\mathcal{P} \approx \ell_{\text{ev}}/a$ and horizon cutoff $\ell_{\text{ev}} = R_H$, we recover:

$$m_{\text{eff}} \approx \frac{m_{\text{Pl}}}{\sqrt{R_H/(2\ell_{\text{Pl}})}} \approx \frac{\hbar H}{c^2} = m_g \tag{50}$$

confirming the consistency of the coherent mode picture.

4.7 Dynamical Tile Count and Frame Covariance

The inertia quantum $m_\star = \hbar H/(2\pi c^2)$ exhibits explicit time dependence through the Hubble parameter $H(t)$. To preserve the observed constancy of particle masses across cosmic evolution and maintain observer independence, the framework requires a compensatory mechanism: the engaged tile count N_p must adjust dynamically.

Cosmic Evolution and Mass Constancy Particle masses remain constant despite the evolving inertia quantum because the number of Planck tiles engaged by each particle adjusts inversely with $H(t)$:

$$N_p(t) = N_p^{(0)} \cdot \frac{H_0}{H(t)}, \tag{51}$$

ensuring the product remains time-independent:

$$m = N_p(t) \cdot m_\star(t) = N_p^{(0)} \cdot m_\star^{(0)} = \text{constant}. \tag{52}$$

Physical Mechanism: Horizon Crossing and Linear Engagement The proper physical picture is one of *linear* rather than areal engagement. Each fundamental particle's worldline intersects the cosmological horizon along a one-dimensional trajectory, engaging successive Planck-scale tiles as the horizon expands. The number of engaged tiles therefore scales with the horizon radius,

$$N_p(t) \propto \frac{\text{length of worldline–horizon intersection}}{\ell_{\text{Pl}}} \propto \frac{R_H}{\ell_{\text{Pl}}} \propto \frac{1}{H(t)}.$$

Meanwhile, the inertia quantum varies as

$$m_\star(t) \propto H(t),$$

so that their product remains constant,

$$m = N_p(t)\, m_\star(t) \propto \left(\frac{1}{H(t)}\right) H(t) = \text{constant}.$$

Geometric Interpretation Each particle's worldline threads the cosmological horizon along a null geodesic. The proper length of this intersection grows with the horizon radius $R_H = c/H(t)$. As the Universe expands, more Planck-scale tiles are encountered along this linear crossing path, increasing $N_p(t)$. The total horizon area $A_H \propto R_H^2 \propto 1/H^2$ represents the *aggregate* information capacity of all possible worldlines, but each individual particle accesses only a one-dimensional subset scaling as $R_H \propto 1/H$. This geometric hierarchy explains why particle masses remain invariant while the horizon's information content grows.

Quantum Inertia and Cosmic Evolution The linear engagement model naturally explains the cosmic evolution of inertia:

- **Early Universe** (H large): Fewer tiles (N_p small) but higher energy per tile ($m_\star$ large)
- **Late Universe** (H small): More tiles (N_p large) but lower energy per tile ($m_\star$ small)
- **Mass Invariance**: Product $N_p \cdot m_\star$ remains exactly constant

This mechanism ensures that while the quantum of inertia $m_\star$ evolves with the Hubble parameter, macroscopic particle masses remain constant throughout cosmic history, providing a natural resolution to what would otherwise be a serious fine-tuning problem.

Frame Transformations and Local Lorentz Invariance The framework maintains observer independence through exact compensation between reference frames. While the Hubble parameter H is a cosmological scalar field, different observers measure effective values due to their motion relative to the cosmological rest frame:

- A **comoving observer** at rest with the Hubble flow measures H directly from the cosmic microwave background dipole.

- A **boosted observer** moving with peculiar velocity $\vec{v}$ relative to the Hubble flow measures an effective H' due to aberration and Doppler effects on the observed horizon structure.
- An **accelerated observer** with proper acceleration a experiences a Rindler horizon that mixes with the cosmological horizon, yielding an effective H_{eff} that interpolates between local and global scales.

In all cases, the compensation is exact:

$$N'_p = N_p^{(0)} \cdot \frac{H_0}{H'}, \tag{53}$$

$$m'_\star = m_\star^{(0)} \cdot \frac{H'}{H_0}, \tag{54}$$

$$m = N'_p \cdot m'_\star = N_p^{(0)} \cdot m_\star^{(0)} = \text{constant.} \tag{55}$$

This ensures all observers, regardless of their motion, agree on particle properties while maintaining consistency with the global cosmological evolution.

Horizon Hierarchy and Dominance of Cosmological Structure Although accelerated observers possess local Rindler horizons, the relevant scale for N_p is set by the *largest causal structure* accessible to the particle. The cosmological horizon at $R_H = c/H$ dominates because:

1. **Maximum Coherence Length**: The cosmological horizon represents the fundamental coherence scale of the I^3 lattice, setting the natural scale for vacuum elasticity and information exchange.
2. **Sub-dominant Local Corrections**: Local accelerations produce perturbations $\propto (a \cdot R_H/c^2) \ll 1$ that are negligible compared to the cosmological scale.
3. **Equivalence Principle Preservation**: The independence of inertial mass from local acceleration requires selection of the global cosmological scale, maintaining the foundation of general relativity.

Thus, while local horizons exist, they provide negligible corrections to the cosmological-scale determination of N_p and $m_\star$.

Observational Constraints and Hidden Variation The predicted variation rate $\dot{N}_p/N_p \sim 10^{-18}\ \text{s}^{-1}$ would appear to violate stringent constraints on varying fundamental constants if directly observable. However, the I^3 framework naturally evades these constraints:

- **Product Invariance**: Only the combination $m = N_p \cdot m_\star$ couples to real-sector fields through the $\mathbb{R}^3$-I^3 interface. Individual variations in N_p and $m_\star$ cancel exactly in all observable quantities.

- **Sector Separation**: The variation represents internal reconfiguration within the I^3 lattice—a redistribution of information capacity among Planck patches that leaves real-sector couplings unchanged.
- **No Coupling Constant Dependence**: Standard Model parameters depend on the interface boundary conditions, which remain fixed, not on the internal I^3 tile distribution.

The dynamical adjustment of $N_p(t)$ therefore represents a hidden degree of freedom that maintains cosmic equilibrium while preserving all established observational constraints.

Testable Implications and Future Probes While the $N_p(t)$ variation itself is hidden in the I^3 sector, it produces secondary effects that may be detectable:

- **Horizon Crossing Length Growth**: The linear extent of each particle's horizon engagement evolves as $\ell_{\text{cross}}(t) \propto R_H \propto 1/H(t)$, representing the growing proper length of the worldline-horizon intersection. This could influence phenomena sensitive to the causal structure of the vacuum.
- **Information Current**: The continuous engagement of new tiles along expanding worldline-horizon intersections represents an information flow that could manifest in precision measurements of vacuum correlations.
- **Evolutionary Signatures**: In the early universe when $H(t)$ was larger, particles engaged fewer tiles along shorter horizon crossings, potentially leaving imprints on primordial correlation functions and the statistical properties of the cosmic microwave background.

These subtle effects provide potential observational avenues for testing the dynamical aspects of the I^3 framework while maintaining consistency with all current experimental constraints.

5 Fundamental Closure Relation

5.1 Numerical Re-evaluation and Breakdown of the Equality

Direct substitution of present-day parameters shows that the equality

$$m_g c^2 = \rho_\Lambda \lambda_g^3$$

does not hold numerically. Using $H_0 = 2.2 \times 10^{-18}\,\text{s}^{-1}$, $\rho_\Lambda \simeq 6 \times 10^{-10}\,\text{J/m}^3$, and $\lambda_g = c/H_0 \simeq 1.36 \times 10^{26}\,\text{m}$, we obtain

$$m_g c^2 = \hbar H_0 = 1.7 \times 10^{-52}\,\text{J}, \tag{56}$$

$$\rho_\Lambda \lambda_g^3 = 6 \times 10^{-10}\,(1.36 \times 10^{26})^3 \simeq 1.5 \times 10^{70}\,\text{J}. \tag{57}$$

Their ratio is therefore

$$\frac{\rho_\Lambda \lambda_g^3}{m_g c^2} \simeq 9 \times 10^{121},$$

coinciding with the horizon-entropy factor $S_H/k_B \sim 10^{122}$. Hence the graviton energy corresponds not to the total vacuum energy within its Compton volume, but to a single *Planck-scale information patch* among $N_H \sim 10^{122}$ such patches on the cosmic horizon.

5.2 Planck-Scale Invariant Product

The physically correct closure relation is an invariant product, not an equality:

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi} \frac{\hbar c^5}{G} = \frac{3}{8\pi} E_{\rm Pl}^2. \tag{58}$$

This constant expresses a fundamental equilibrium between microscopic inertia quanta and macroscopic vacuum energy. As H evolves, $m_g c^2 \propto H$ and $\rho_\Lambda \lambda_g^3 \propto 1/H$, so their product remains fixed at a Planck value. The equality $m_g c^2 \simeq \rho_\Lambda \lambda_g^3$ held only in the Planck epoch when $H \sim 1/t_{\rm Pl}$.

5.3 Interpretation in the I^3 Crystal Framework

Within the I^3 elastic substrate, each coherent graviton cell of size λ_g^3 contains

$$N = \frac{\lambda_g^3}{(2\ell_{\rm Pl})^3} \sim 10^{183}$$

Planck sites. The collective-mode inertia $m_{\rm eff} = m_{\rm Pl}/\sqrt{N}$ gives $m_{\rm eff} \simeq \hbar H/c^2 = m_g$, showing that the infrared graviton represents a coherent oscillation of N lattice sites. The enormous ratio $N_H \sim 10^{122}$ between $\rho_\Lambda \lambda_g^3$ and $m_g c^2$ then corresponds to the number of independent horizon patches that can store one graviton quantum each.

5.4 Thermodynamic–Elastic Equilibrium

Equation (82) can be read as the mechanical equilibrium between elastic tension in I^3 and thermodynamic pressure in $\mathbb{R}^3$:

$$T_{\rm el} \sim m_g c^2, \tag{59}$$
$$P_\Lambda \sim \rho_\Lambda c^2, \tag{60}$$
$$T_{\rm el} P_\Lambda^{-1} \sim \lambda_g^3. \tag{61}$$

A small perturbation δH shifts both quantities in opposite directions, restoring balance through expansion or contraction of the cosmic scale factor. The dark-energy density therefore remains nearly constant while $m_g \propto H(t)$ redshifts with cosmic time, maintaining the invariant (82).

5.5 Physical and Conceptual Implications

The corrected closure law unifies three complementary aspects:

1. **Quantum**: $m_g c^2 = \hbar H$ defines the energy quantum of the lowest collective lattice mode.

2. **Thermodynamic**: ρ_Λ encodes the information density of the I^3 substrate, each Planck patch storing one bit.

3. **Cosmological**: The invariant $(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi} E_{\rm Pl}^2$ links microscopic and macroscopic sectors through a conserved Planck-scale product.

Thus the Universe preserves a global balance between graviton energy quanta and the vacuum's total information capacity. The previously assumed equality is replaced by a *holographic scaling law*:

$$\frac{\rho_\Lambda \lambda_g^3}{m_g c^2} = \frac{S_H}{k_B} \sim 10^{122},$$

signifying that today's dark energy corresponds to the collective ground state of roughly 10^{122} infrared graviton cells in the I^3 lattice.

6 Observable Consequences and Experimental Signatures

6.1 Frequency-Dependent Dispersion of Gravitational Waves

A finite graviton mass m_g introduces a tiny but measurable frequency-dependent propagation delay for gravitational waves. Starting from the dispersion relation

$$\omega^2 = c^2 k^2 + \frac{m_g^2 c^4}{\hbar^2}, \tag{62}$$

the group velocity becomes

$$v_g = \frac{\partial \omega}{\partial k} \simeq c \left[1 - \tfrac{1}{2} \left(\frac{m_g c^2}{\hbar \omega} \right)^2 \right], \tag{63}$$

so that

$$\frac{\Delta v}{v} \approx -\frac{1}{2} \left(\frac{H}{\omega} \right)^2. \tag{64}$$

This formula holds universally because $m_g c^2 = \hbar H$ from Section 5. For sources emitting at frequency ω and located at redshift z, the total arrival-time delay is

$$\Delta t \approx \frac{H_0^2 D}{2\,\omega^2 c}, \tag{65}$$

where D is the comoving distance.

Expected magnitude.

- **Pulsar-timing arrays (PTA)**: $\omega \sim 10^{-8}$ Hz $\Delta v/v \sim 10^{-8}$, producing phase drifts near current sensitivity.
- **LISA band (10^{-4}–10^{-1} Hz)**: $\Delta v/v \sim 10^{-12}$–10^{-18}, still potentially measurable in cross-correlation analyses.
- **Ground-based interferometers (LIGO/Virgo)**: $\omega \sim 10^2$ Hz $\Delta v/v \sim 10^{-40}$, negligible.

Observation of any low-frequency suppression or phase lag would directly probe the I^3 lattice elasticity scale.

6.2 Horizon-Scale Evolution of Dark Energy

Because both m_g and $m_\star$ scale with $H(t)$, the dark-energy density obeys

$$\rho_\Lambda(t) \propto H(t)^2, \tag{66}$$

which implies a slowly varying equation of state:

$$w_{\rm DE}(a) = -1 - \frac{1}{3}\frac{d\ln H^2}{d\ln a} \simeq -1 - \frac{\dot{H}}{3H^2}. \tag{67}$$

For a matter-dominated background $H \propto a^{-3/2}$, this gives $w_{\rm DE} \simeq -1.05$, in excellent agreement with DESI constraints [3]. Future supernova and baryon-acoustic-oscillation surveys can test the predicted redshift dependence

$$\rho_\Lambda(z) = \rho_{\Lambda 0}\left[\frac{H(z)}{H_0}\right]^2, \quad m_g(z) = m_{g0}\frac{H(z)}{H_0}.$$

6.3 Laboratory-Scale Manifestations

Although the graviton mass is minuscule, the elastic I^3 background alters boundary stresses at microscopic distances. Two effects are, in principle, testable:

Casimir-type corrections. The negative-pressure character of the I^3 substrate leads to a modified vacuum stress between conducting plates,

$$\Delta F/A \simeq \rho_\Lambda\left[1 - e^{-2\pi d/\lambda_g}\right], \tag{68}$$

where d is plate separation. At micron scales the correction is $\sim 10^{-27}$ N/m^2, beyond current sensitivity but conceptually finite.

Vacuum refractive index. The presence of an evanescent graviton background produces a frequency-dependent refractive index for light:

$$n(\omega) - 1 \simeq \frac{1}{2}\left(\frac{m_g c^2}{\hbar\omega}\right)^2. \tag{69}$$

For optical frequencies this yields $n - 1 \sim 10^{-62}$, explaining why the effect is unobservable in laboratory optics but could accumulate over cosmological distances.

6.4 Astrophysical and Cosmological Diagnostics

CMB B-modes. The long-wavelength cutoff at $\lambda_g \sim R_H$ suppresses primordial gravitational-wave amplitudes below multipoles $\ell \lesssim 10$, a distinctive signature differing from inflationary tilt.

Large-scale structure. The evolving mass $m_g(t)$ slightly alters the growth rate of density perturbations $f\sigma_8(a)$. Linear-theory calculations show $\delta f/f \approx -m_g^2 c^4/(2\hbar^2 H^2) \sim 10^{-6}$ today, within reach of next-generation surveys.

Cosmic birefringence. If the I^3 lattice carries intrinsic handedness (Section C), then its gravitomagnetic component $B_g(I)$ induces a minute rotation of polarization for waves traversing horizon scales, $\Delta\theta \sim H_0 t_{\text{travel}} \sim 10^{-10}$ rad. Detecting such rotation would provide direct evidence for the underlying imaginary-space spin structure.

6.5 Summary of Experimental Outlook

- **LISA + PTA:** Direct probes of dispersion [Eq. (64)]; graviton mass tracking $m_g \propto H(z)$.
- **DESI + LSST:** Verification of $w_{\text{DE}}(a)$ and $\rho_\Lambda \propto H^2$.
- **CMB-S4:** Low-ℓ suppression of tensor power.
- **Laboratory Casimir and refractive-index tests:** constraints on I^3 vacuum stiffness.

Each of these regimes tests a different projection of the same universal law—the elasticity and information-capacity of the Planck-scale I^3 vacuum. The coherence of their predictions across forty orders of magnitude in scale marks this framework as an empirically falsifiable theory rather than a purely metaphysical construct.

7 Discussion and Conclusions

7.1 Synthesis of Results

The analysis of preceding sections reveals a coherent and self-contained mechanical picture of the vacuum. The I^3 crystal—an imaginary, negative-energy but positive-density lattice of

Planck-mass white-hole sites—acts simultaneously as the origin of inertia, gravitation, and dark-energy pressure. Its key predictions are internally consistent:

1. **Infrared graviton mass:** $m_g = \hbar H/c^2$, emerging from evanescent longitudinal oscillations of the lattice.

2. **Inertia quantum:** $m_\star = \hbar H/(2\pi c^2)$ from the Landauer–Unruh energy of one bit per Planck patch.

3. **Planck-invariant product:**

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi}\frac{\hbar c^5}{G} = \frac{3}{8\pi}E_{\rm Pl}^2, \tag{70}$$

   linking microscopic and cosmological scales without fine-tuning.

4. **Scale safety:** Wavelength-dependent graviton mass kernel (39) ensures recovery of general relativity at short distances.

5. **Self-regulation:** $\rho_\Lambda \propto H^2$ stabilizes the vacuum equation of state near $w = -1$.

Together these relations form a closed hierarchy where no free parameter remains once G, $\hbar$, and c are specified. The Universe becomes a self-adjusting resonator between real and imaginary sectors.

7.2 Comparison with Other Approaches

dRGT massive gravity. Conventional massive-gravity theories [4] introduce a fixed graviton mass and encounter the Boulware-Deser ghost. In contrast, the I^3 crystal yields a dynamical mass that varies with the Hubble rate, $m_g(t) \propto H(t)$, eliminating any conflict with local tests of general relativity while preserving stability through its lattice origin. Additionally, the wavelength-dependent mass kernel ensures that the vDVZ discontinuity is avoided and general relativity is recovered at small scales.

Verlinde's entropic gravity. Verlinde's approach [15] proposes that gravity is an entropic force. Our framework shares the information-theoretic and thermodynamic aspects but provides a concrete microscopic model: the I^3 crystal. We derive both inertia and gravity from the elastic and thermodynamic properties of the crystal, and we also account for the cosmological constant and dark energy.

Sakharov's induced gravity. Sakharov [16] proposed that gravity is induced by quantum fluctuations. Our model is in a similar spirit but with a specific mechanism: the elasticity of the I^3 crystal. We also include the cosmological constant as an inherent property of the crystal's equilibrium.

7.3 Physical Interpretation of the I^3 Sector

In I^3, distances, velocities, and forces are imaginary ($x_I = ix$, $v_I = ic$, $F_I = iF$), while material moduli (E, μ, λ) remain real. Consequently $E_I = F_I x_I = -E$ gives negative total energy yet positive energy density $\rho_I = |E_I|/|V_I| > 0$. The product of negative pressure and negative volume, $p_I V_I > 0$, drives an outward force in $\mathbb{R}^3$, manifesting as the observed cosmic acceleration. In this sense dark energy is not an exotic fluid but a geometric property of the imaginary-metric phase of the vacuum.

7.4 Microscopic Degrees of Freedom and Spin Structure

Each Planck tile carries intrinsic spin $S = \hbar/2$ and rotates about the cosmic center with tangential speed c, producing a global gravitomagnetic field $B_g(I) \sim 2H$. The coherent precession of these spins yields the same effective frequency $\Gamma = H/2$ that defines the graviton mass $m_g c^2 = \hbar H/2$, confirming the dynamical unity of rotation, elasticity, and inertia. The aggregate spin density, summed over $N \sim (R_H/2\ell_{\text{Pl}})^2$ tiles, reaches $J_{\text{tot}} \sim N(\hbar/2) \sim 10^{121}\hbar$, matching the Bekenstein–Hawking entropy of the observable Universe. This correspondence suggests that cosmic rotation and horizon information content are two manifestations of the same underlying spin lattice.

7.5 Conceptual and Observational Outlook

Conceptual unification. The I^3 framework integrates four domains:

1. **Quantum information:** inertia as Landauer cost per bit, $E_{\text{nat}} = \hbar H/2\pi$;
2. **Thermodynamics:** horizon temperature $T_U = \hbar H/2\pi k_B$ and negative T_I balance;
3. **Elastic mechanics:** vacuum rigidity generating $m_g = \hbar H/c^2$;
4. **Cosmology:** dark-energy density $\rho_\Lambda \propto H^2$.

These four faces of the same law remove the need for fine-tuning and provide a natural pathway from Planck physics to cosmic acceleration.

Empirical tests. Section 6 summarized quantitative predictions: gravitational-wave dispersion measurable by LISA and PTA, $w_{\text{DE}}(z)$ evolution accessible to DESI and LSST, and potential birefringence or Casimir anomalies as laboratory proxies. Agreement among these tests would verify the elastic and thermodynamic parameters of the I^3 crystal.

7.6 Concluding Perspective

The I^3 crystal framework transforms the cosmological constant problem from a fine-tuning puzzle into a statement of mechanical equilibrium between two conjugate sectors of reality. The same Planck tiles that endow spacetime with rigidity also encode its entropy and inertia. The relation

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi}\frac{\hbar c^5}{G} = \frac{3}{8\pi} E_{\text{Pl}}^2 \tag{71}$$

thus stands as a new universal constant—a closure principle uniting quantum mechanics, thermodynamics, and gravitation in a single geometric law. Future theoretical work will extend this description to include nonlinear defect dynamics, lattice dislocations as black-hole analogs, and possible couplings to standard-model fields, while observational programs will test its predictions from the femtometer to the Hubble scale.

7.7 Outlook and Future Directions

The I^3 crystal framework opens a wide landscape for quantitative research that bridges microscopic Planck physics and macroscopic cosmology. Several immediate directions are both feasible and potentially decisive.

(a) Nonlinear lattice dynamics. The present analysis treated only linearized oscillations around the equilibrium configuration. Extending to nonlinear elasticity will permit study of solitons, dislocations, and domain walls within the I^3 lattice. Such defects may correspond to black holes, white holes, or particle excitations in $\mathbb{R}^3$. Numerical simulation of nonlinear lattice relaxation could reveal whether horizon areas quantize naturally as $A = 4n\ell_{\text{Pl}}^2$ and whether Hawking evaporation emerges from defect annihilation processes.

(b) Coupling to quantum fields. Standard-model fields may be reinterpreted as collective excitations propagating on the $\mathbb{R}^3$–I^3 interface. Deriving their dispersion relations and gauge symmetries from the geometry of the lattice could unify gravitation and particle physics under a single elastic–informational substrate. Of particular interest is whether electromagnetic duality arises from torsional modes of the imaginary crystal, consistent with the condition $\epsilon_0 < 0$, $\mu_0 > 0$.

(c) Numerical cosmology and effective-field modeling. Replacing the cosmological constant in the Friedmann equations by $\rho_\Lambda(H) = 3M_{\text{Pl}}^2 H^2$ allows direct simulation of the cosmic scale factor $a(t)$ and comparison with Planck, DESI, and LSST datasets. The model predicts small deviations from ΛCDM at late times that can be confronted with supernova and baryon-acoustic data within existing numerical frameworks.

(d) Laboratory analogues. Condensed-matter systems with negative compressibility or meta-materials engineered for imaginary effective permittivity ($\epsilon < 0$) may serve as macroscopic analogues of I^3 dynamics. Wave attenuation and boundary-oscillation experiments at cryogenic temperatures could mimic the evanescent behavior of the imaginary sector and test the correspondence between elastic and thermodynamic descriptions.

(e) Quantum-information implications. Each Planck tile functions as an information capacitor of one natural unit, suggesting a finite-dimensional Hilbert space per horizon area $A/4\ell_{\text{Pl}}^2$. Investigating error-correction, entanglement, and quantum-channel analogies could connect this framework with holographic codes and spacetime entanglement entropy, providing a rigorous information-theoretic underpinning to inertia.

(f) Observational tests. Future missions such as LISA, SKA, CMB-S4, and the Rubin Observatory will measure the parameters entering Eqs. (35) and (82) to unprecedented precision. Detection of frequency-dependent gravitational-wave delays, a mild evolution $w_{\rm DE}(z) \neq -1$, or large-angle polarization rotation would strongly support the presence of an elastic I^3 substrate.

Toward a unified program. These threads point toward a single quantitative goal: construct a relativistic field theory whose low-energy limit reproduces Einstein's equations with $m_g(H)$ and whose high-frequency limit reduces to a discrete I^3 lattice. Success would transform the I^3 model from a geometric hypothesis into a complete microscopic description of spacetime mechanics.

The next decade of theoretical and observational progress will determine whether the vacuum is truly a negative-energy crystal whose microscopic rhythm at frequency H encodes the inertia of all matter. Should these predictions be confirmed, the I^3 lattice would constitute the long-sought bridge between quantum information, gravity, and cosmology.

A Derivation of the Elastic Dispersion Relation

The equation of motion for displacements in the I^3 crystal:

$$\rho_I \ddot{u}_i = \partial_j^I \sigma_{ij}^I \tag{72}$$

With Hooke's law $\sigma_{ij}^I = \lambda_I \delta_{ij} \varepsilon_{kk}^I + 2\mu_I \varepsilon_{ij}^I$ and strain definition $\varepsilon_{ij}^I = \frac{1}{2}(\partial_i^I u_j^I + \partial_j^I u_i^I)$:

$$\begin{aligned} \partial_j^I \sigma_{ij}^I &= \lambda_I \partial_i^I (\partial_k^I u_k^I) + \mu_I (\partial_j^I \partial_j^I u_i^I + \partial_j^I \partial_i^I u_j^I) &(73)\\ &= -\lambda \partial_i (\partial_k u_k) - \mu (\nabla^2 u_i + \partial_i \partial_j u_j) &(74) \end{aligned}$$

Using $\partial_i^I = (1/i)\partial_i$ and $\lambda_I = -\lambda$, $\mu_I = -\mu$.
For plane waves $u_i \propto e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$:

$$-\rho_I \omega^2 u_i = -(\lambda + 2\mu) k_i k_j u_j - \mu k^2 u_i \tag{75}$$

Longitudinal modes ($\mathbf{u} \parallel \mathbf{k}$):

$$\omega^2 = -\frac{\lambda + 2\mu}{\rho_I} k^2 \tag{76}$$

Transverse modes ($\mathbf{u} \perp \mathbf{k}$):

$$\omega^2 = -\frac{\mu}{\rho_I} k^2 \tag{77}$$

The negative signs confirm purely imaginary frequencies $\omega = i\Gamma$.

B Cosmological Scaling Relations

The Planck-era values confirm saturation of lattice stiffness, while late-time values match observed dark energy density and provide testable evolution trends.

Table 3: Cosmological evolution of key mass and energy scales

Epoch	H (s^{-1})	m_g (eV/c^2)	$m_\star$ (eV/c^2)	ρ_Λ (J/m^3)
Planck	1.85×10^{43}	1.22×10^{28}	1.94×10^{27}	5.50×10^{112}
Recombination	4.5×10^{-14}	2.96×10^{-29}	4.71×10^{-30}	3.25×10^{-1}
Present	2.2×10^{-18}	1.45×10^{-33}	2.30×10^{-34}	7.78×10^{-10}

C Spin and Gravitomagnetic Coupling

The spin–rotation (Mashhoon) coupling is $H_{\rm rot} = -\mathbf{\Omega}\cdot\mathbf{J}$. For a spin-$\frac{1}{2}$ tile, the two levels shift by $E_\pm = \mp(\hbar/2)\Omega$, so the *level splitting* is $\Delta E = E_- - E_+ = \hbar\Omega$. The associated precession frequency of the soft mode is therefore

$$\Gamma = \frac{\Delta E}{\hbar} = \Omega. \tag{78}$$

Identifying the horizon co-rotation $\Omega \simeq H$ gives

$$m_g c^2 = \hbar\Gamma = \hbar H \quad \Rightarrow \quad m_g = \frac{\hbar H}{c^2} = 2\pi m_\star, \tag{79}$$

in exact agreement with the Landauer–Unruh derivation. Any net cosmic rotation would only induce tiny helicity-odd corrections $\propto \Omega/H$ around this value.

Appendix D: Graviton Mass–Energy Density Equivalence

D.1Thermodynamic origin of the closure relation

From Sections 3–5, the infrared graviton mass and Compton wavelength are

$$m_g = \frac{\hbar H}{c^2}, \qquad \lambda_g = \frac{\hbar}{m_g c} = \frac{c}{H}. \tag{80}$$

Multiplying these quantities gives a constant energy per graviton cell,

$$m_g c^2 = \hbar H, \tag{81}$$

numerically equal to the Landauer–Unruh energy of a single horizon bit. The cosmological dark-energy density of the I^3 vacuum, $\rho_\Lambda = 3M_{\rm Pl}^2 H^2$, therefore satisfies

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi}\,\frac{\hbar c^5}{G} = \frac{3}{8\pi}\,E_{\rm Pl}^2. \tag{82}$$

D.2Elastic and information-theoretic interpretation

Equation (82) expresses equilibrium between two conjugate energy ledgers:

- The **elastic ledger** of the I^3 lattice, characterized by modulus E_I and strain energy density $\mathcal{U}_I \sim \rho_\Lambda$;
- The **informational ledger** of the $\mathbb{R}^3$–I^3 interface, where each Planck tile stores one natural unit with energy $E_{\rm nat} = \hbar H/2\pi$.

When one full wavelength λ_g of an evanescent graviton fits within the horizon volume, the elastic energy of that cell equals the informational energy cost of maintaining its one-bit capacity. Hence Eq. (82) embodies the balance between mechanical stiffness and information flow through the horizon.

D.3Saturation of the Bekenstein–Hawking bound

Each graviton cell of volume $V_g = \lambda_g^3$ intersects the horizon through an effective area $A_g = (\lambda_g/2)^2 \approx R_H^2$. The associated entropy,

$$S_g = \frac{A_g}{4\ell_{\rm Pl}^2} k_{\rm B} \simeq \frac{k_{\rm B}}{4} \left(\frac{R_H}{\ell_{\rm Pl}}\right)^2, \tag{83}$$

matches the Bekenstein–Hawking entropy of the cosmic horizon when summed over all $N \sim (R_H/2\ell_{\rm Pl})^2$ tiles, yielding $S_{\rm tot} \simeq N S_g \approx 10^{122} k_{\rm B}$. The closure relation therefore implies that the Universe stores exactly one natural unit of elastic-informational energy per bit of its horizon entropy:

$$E_{\rm bit} = k_{\rm B} T_{\rm U} = \frac{\hbar H}{2\pi} \quad \Longleftrightarrow \quad m_g c^2 = 2\pi E_{\rm bit}. \tag{84}$$

D.4Cosmological implications

Because $m_g \propto H$ and $\lambda_g \propto 1/H$, Eq. (82) ensures that the total vacuum energy $E_{\rm vac} = \rho_\Lambda V_{\rm H} \propto H^0$ remains constant even as the Hubble radius evolves. The cosmological constant problem is thus resolved dynamically: the apparent constancy of ρ_Λ is not a fixed input but the self-adjusting outcome of the equality between graviton rest energy and vacuum elastic energy within one horizon-sized coherence domain.

D.5Interpretive summary

The Planck-invariant product

$$(m_g c^2)(\rho_\Lambda \lambda_g^3) = \frac{3}{8\pi} \frac{\hbar c^5}{G} = \frac{3}{8\pi} E_{\rm Pl}^2 \tag{85}$$

represents the equilibrium condition of the vacuum: every infrared graviton cell stores precisely one bit of horizon information and contains exactly its own rest energy in elastic vacuum pressure. This equality fuses the thermodynamic, informational, and geometric faces of spacetime into a single conservation law.

Appendix E: Gravitomagnetic Coupling and Cosmic Spin Density

E.1Intrinsic spin of the I^3 tiles

Each Planck patch at the $\mathbb{R}^3$–I^3 interface is assumed to possess an intrinsic angular momentum

$$S = \frac{\hbar}{2},$$

representing the minimal spin quantum of the vacuum lattice. The ensemble of $N \simeq (R_H/2\ell_{\rm Pl})^2$ tiles thus carries a total angular momentum

$$J_{\rm tot} \simeq N\,S \simeq \frac{\hbar}{2}\left(\frac{R_H}{2\ell_{\rm Pl}}\right)^2 \sim 10^{121}\hbar,$$

coincident with the Bekenstein–Hawking entropy in units of $\hbar$, $S_{\rm BH}/k_{\rm B} \simeq A_H/(4\ell_{\rm Pl}^2)$. This suggests that cosmic entropy and total spin are dual descriptions of the same microscopic degree of freedom.

E.2Global rotation and gravitomagnetic field in I^3

If every tile co-rotates about the cosmic center with angular velocity $\Omega \simeq H$, its tangential speed at the horizon radius satisfies $v = R_H\Omega = c$. The corresponding gravitomagnetic field inside the imaginary sector is analogous to a uniform magnetic field generated by aligned spins:

$$\mathbf{B}_g(I) = 2\,\boldsymbol{\Omega} \simeq 2H\,\hat{\mathbf{z}},$$

where the unit vector $\hat{\mathbf{z}}$ denotes the axis of global rotation. This field is not directly observable in $\mathbb{R}^3$ but modulates the evanescent phase of gravitational waves propagating across the interface.

E.3 Spin–rotation (Larmor) coupling and the infrared mass

The spin of each tile couples to the gravitomagnetic field via the Mashhoon spin–rotation Hamiltonian $H_{\rm rot} = -\,\boldsymbol{\Omega}\cdot\mathbf{J}$. For a spin-$\frac{1}{2}$ constituent with $J = \hbar/2$, the two levels shift by $E_\pm = \mp(\hbar/2)\Omega$, so the *level splitting* is

$$\Delta E \;=\; E_- - E_+ \;=\; \hbar\,\Omega, \qquad \Rightarrow \qquad \Gamma \;=\; \frac{\Delta E}{\hbar} \;=\; \Omega. \tag{86}$$

Identifying the co-rotation at the horizon with the Hubble rate, $\Omega \simeq H$, gives

$$m_g c^2 \;=\; \hbar\,\Gamma \;=\; \hbar H \qquad \Rightarrow \qquad m_g = \frac{\hbar H}{c^2}. \tag{87}$$

Thus the rotational (Larmor) route yields exactly the same infrared mass as the elastic and Landauer–Unruh derivations, and with no extra factor. Equivalently, $m_g = 2\pi m_\star$ with $m_\star = \hbar H/(2\pi c^2)$.

E.4Gravitomagnetic correction and parity splitting

If the Universe exhibits even a minute global spin asymmetry, the two helicity states of the graviton experience opposite coupling to the background field, leading to a parity-odd mass splitting:

$$m_{g,\pm} = m_g \left[1 \pm \alpha \left(\frac{\Omega}{H} \right) \right], \qquad \alpha \ll 1.$$

Such splitting would produce tiny birefringence in long-wavelength gravitational waves or CMB B-modes. Given $\Omega/H \lesssim 10^{-3}$, the predicted asymmetry is $\Delta m_g/m_g \lesssim 10^{-3}$—undetectable at present but conceivably measurable with next-generation polarization missions.

E.5Energetic consistency and spin-entropy closure

The total rotational energy of all horizon tiles,

$$E_{\rm rot} = J_{\rm tot}\Omega \simeq \frac{\hbar H}{2} \left(\frac{R_H}{2\ell_{\rm Pl}} \right)^2,$$

equals the total vacuum energy inside the horizon volume $E_\Lambda = \rho_\Lambda (4\pi R_H^3/3)$ within order unity, reaffirming the energetic self-consistency of the I^3 crystal. The Universe can thus be regarded as a slowly precessing rigid body whose cumulative spin accounts for both its entropy and its dark-energy content.

E.6Interpretive perspective

The gravitomagnetic viewpoint completes the physical closure of the framework:

1. **Thermodynamic route:** $E_{\rm nat} = k_{\rm B} T_{\rm U} = \hbar H/2\pi$ (Landauer–Unruh).
2. **Elastic route:** $m_g c^2 = \hbar H$ (vacuum rigidity).
3. **Rotational route:** $E_{\rm Larmor} = \hbar H$ (cosmic spin precession).

All three yield the same numerical mass scale

$$m_g = \frac{\hbar H}{c^2},$$

demonstrating that the graviton mass, the inertia quantum, and the Universe's total spin originate from one underlying Planck-lattice mechanism operating in the I^3 sector.

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