The 10^{122} Vacuum Energy Problem and its Resolution within STR-2

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Extended Revision including Spectral Integral and Geometric Regularization
(Figures generated numerically within the STR-2 cosmological model, 2025)

Abstract

The "10^{122} vacuum catastrophe" refers to the *122-orders-of-magnitude* mismatch between the quantum-field-theoretical prediction of vacuum energy and the value inferred from cosmological observations. In the standard relativistic framework (STR) the Minkowski geometry is unbounded, producing an effectively infinite phase-space volume and thus divergent zero-point energy. STR-2 (Special Theory of Relativity–2) introduces an inverse-elliptic compactification of momentum–energy space. This finite geometry regularizes the vacuum spectrum through a curvature parameter $B = \ell_P^2/R_0^2$, yielding a small but finite cosmological constant $\Lambda = 3/R_0^2$ and vacuum-energy density $\rho_\Lambda = 3c^4/(8\pi G R_0^2)$. For $R_0 \approx 1.3 \times 10^{26}\,\mathrm{m}$ one obtains $\rho_\Lambda \approx 5.6 \times 10^{-10}\,\mathrm{J\,m^{-3}}$ and $\Lambda \approx 1.8 \times 10^{-52}\,\mathrm{m^{-2}}$. Thus, the 10^{122} gap disappears not by fine-tuning but as a direct geometric consequence of STR-2.

Introduction: The Vacuum Energy Discrepancy

Quantum field theory (QFT) predicts that even in the absence of matter and radiation, every normal mode of a quantized field contributes a zero–point energy of $\frac{1}{2}\hbar\omega$. Summing over all allowed modes up to a maximum wave number $k_{\max}$ gives

$$\rho_{\mathrm{vac}}^{(\mathrm{QFT})} = \frac{\hbar}{2}\int_0^{k_{\max}} \frac{4\pi k^2\,dk}{(2\pi)^3}\,c\,k = \frac{\hbar c}{16\pi^2}k_{\max}^4. \tag{1}$$

If one assumes that the quantum vacuum spectrum extends up to the Planck scale,

$$k_{\max} = \frac{1}{\ell_P} = \sqrt{\frac{c^3}{\hbar G}},$$

then the corresponding energy density becomes

$$\rho_{\mathrm{vac}}^{(\mathrm{QFT})} = \frac{c^7}{16\pi^2\hbar G^2} \;\approx\; 4\times 10^{113}\ \mathrm{J/m^3}. \tag{2}$$

This enormous value represents the *expected* vacuum energy density of space–time according to standard QFT in flat Minkowski space.

However, cosmological measurements — including the cosmic microwave background (CMB), Type Ia supernovae, and baryon acoustic oscillations — imply an effective vacuum density about 10^{122} times smaller.

Vacuum energy from General Relativity. Einstein's equation with the cosmological constant,

$$R_{\mu\nu} - \tfrac{1}{2}R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{3}$$

can be interpreted as describing a background vacuum with pressure $p_\Lambda = -\rho_\Lambda$. The corresponding relation between Λ and ρ_Λ is

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}. \tag{4}$$

From astronomical observations,

$$\rho_\Lambda^{(\mathrm{obs})} \approx 5.6 \times 10^{-10}\ \mathrm{J/m^3}, \qquad \Lambda_{\mathrm{obs}} \approx 1.8 \times 10^{-52}\ \mathrm{m^{-2}}. \tag{5}$$

Comparison and the "10^{122}" problem. Combining Eqs. (2) and (5) gives

$$\frac{\rho_{\mathrm{vac}}^{(\mathrm{QFT})}}{\rho_\Lambda^{(\mathrm{obs})}} \;\approx\; \frac{4 \times 10^{113}}{5.6 \times 10^{-10}} \;\approx\; 7 \times 10^{122}. \tag{6}$$

Thus, the naive quantum-field-theoretical estimate of the vacuum energy exceeds the cosmologically inferred value by about 10^{122} orders of magnitude. This is the largest theoretical mismatch known in physics and is widely called the *vacuum catastrophe* or *cosmological-constant problem*.

Traditional attempts to resolve this issue introduce counterterms or assume delicate cancellations between quantum fields. However, such adjustments require an unnatural degree of fine-tuning. In the subsequent section we introduce an alternative approach, based on the geometric compactification of momentum–energy space within the inverse-elliptic formalism of STR-2, which naturally regularizes the vacuum spectrum and yields a finite, observationally consistent vacuum energy density.

Section 1 — Kinematic Structure of STR and STR-2

1.1 Fundamental dispersion relations

In the conventional Special Theory of Relativity (STR), the flat-space dispersion relation reads

$$E^2 = p^2c^2 + m^2c^4, \tag{7}$$

which geometrically forms an open hyperbola in the (E, p)-plane. The hyperbolic geometry implies an unbounded momentum space: as $p \to \infty$, the energy $E \to \infty$. This feature underlies the ultraviolet (UV) divergence of the vacuum energy in quantum field theory.

The alternative framework STR-2 replaces the Minkowski hyperbola by an **inverse-elliptic (Finsler-like) geometry**:

$$E^2 = \frac{m^2c^4}{1 - \dfrac{p^2}{m^2c^2}}, \tag{8}$$

in which the momentum domain is compact:

$$|p| < mc.$$

As $p \to mc$ the energy diverges, but the boundary $p = mc$ closes the surface. Figure 1 (to be inserted later) compares the two dispersion curves: an open hyperbola (STR) versus a closed inverse-elliptic contour (STR-2).

1.2 Velocity, energy, and momentum relations

The relation between velocity and momentum also changes. From Eq. (8) one obtains

$$E(v) = mc^2\sqrt{1+\frac{v^2}{c^2}}, \qquad p(v) = \frac{mv}{\sqrt{1+\frac{v^2}{c^2}}}. \tag{9}$$

At low speeds ($v \ll c$) both frameworks coincide:

$$E \simeq mc^2 + \tfrac{1}{2}mv^2, \qquad p \simeq mv.$$

For $v \gg c$, however, STR gives $E \propto \gamma mc^2$ with $\gamma \to \infty$, whereas STR-2 predicts a saturation effect: $p \to mc$ while E grows only algebraically. This difference will be illustrated in Figure 2 (energy–momentum diagram, $E/(mc^2)$ vs. $p/(mc)$).

1.3 Momentum-space geometry

The distinct dispersion laws correspond to different geometries in momentum space.

- **STR:** The momentum manifold is flat and unbounded (Minkowski geometry in (E, p)). All directions extend to infinity, and the spectral volume of modes diverges.
- **STR-2:** The manifold is compact with radius $p_{\max} = mc$, resembling a closed 3-sphere in momentum coordinates. The total spectral volume is finite.

A schematic 3-D comparison (Figure 3) shows this clearly: STR corresponds to an open cone (light cone in Minkowski space), whereas STR-2 closes it into a spherical boundary.

1.4 Tabulated comparison

Property	**STR**	**STR-2**
Dispersion law	$E^2 = p^2c^2 + m^2c^4$	$E^2 = \dfrac{m^2c^4}{1 - p^2/m^2c^2}$
Geometry in (E, p)	Hyperbolic (open)	Inverse-elliptic (closed)
Momentum domain	$\lvert p\rvert < \infty$	$\lvert p\rvert < mc$
Spectral density $g(\omega)$	$\propto \omega^2$ (divergent)	$\propto \omega^2/[1 + (\omega/\omega_c)^3]$ (convergent)
Vacuum-energy integral	$\int \omega^3 d\omega$ (diverges)	$\int \omega/[R_0^2]\, d\omega$ (finite)

1.5 Summary

The two frameworks share the same low-velocity and low-momentum limits, but differ fundamentally at high energies:

1. STR is based on the flat Minkowski geometry with an open hyperbolic dispersion.
2. STR-2 introduces a compact, Finsler-like geometry in momentum space, closing the hyperbola into an inverse-elliptic surface.
3. As a result, STR-2 retains Lorentz symmetry locally but modifies the global topology of the momentum manifold.
4. The compactness of momentum space will play a decisive role in regularizing the spectral integrals for vacuum energy (see Section 2).

Section 2.1 — Vacuum Energy in STR and the Ultraviolet Divergence

2.1.1 Spectral Density in Flat Space

In the standard Special Theory of Relativity (STR), each normal mode of a quantum field contributes a zero-point energy of $\frac{1}{2}\hbar\omega$. Summing (integrating) over all allowed wavevectors in flat Minkowski space yields the *vacuum-energy density*:

$$\rho_{\text{vac}}^{(\text{STR})} = \frac{\hbar}{2}\int_0^\infty \omega\, g_{\text{flat}}(\omega)\, d\omega, \tag{10}$$

where $g_{\text{flat}}(\omega)$ is the spectral density of modes per unit volume.

For a massless field (e.g., photons) in three dimensions, the number of modes in the frequency interval $[\omega, \omega + d\omega]$ is

$$g_{\text{flat}}(\omega)\, d\omega = \frac{4\pi\,\omega^2}{(2\pi c)^3}\, d\omega = \frac{\omega^2}{2\pi^2 c^3}\, d\omega.$$

Substituting this into Eq. (10) gives

$$\rho_{\text{vac}}^{(\text{STR})} = \frac{\hbar}{4\pi^2 c^3}\int_0^\infty \omega^3\, d\omega. \tag{11}$$

The integral $\int_0^\infty \omega^3\, d\omega$ diverges at high frequencies—this is the familiar **ultraviolet catastrophe**. It reflects the unbounded momentum domain of the STR hyperbolic geometry.

2.1.2 Cutoff Representation and Magnitude of the Divergence

To estimate the scale of the problem we can introduce an ultraviolet cutoff $\omega_{\max} = ck_{\max}$ and integrate only up to this limit:

$$\rho_{\text{vac}}^{(\text{STR})}(\omega_{\max}) = \frac{\hbar}{4\pi^2 c^3}\int_0^{\omega_{\max}} \omega^3\, d\omega = \frac{\hbar}{16\pi^2 c^3}\,\omega_{\max}^4 = \frac{\hbar c}{16\pi^2}\,k_{\max}^4. \tag{12}$$

If we take $k_{\max}$ to be the inverse Planck length, $k_{\max} = 1/\ell_P = \sqrt{c^3/(\hbar G)}$, then

$$\rho_{\text{vac}}^{(\text{STR})} \sim \frac{c^7}{16\pi^2\hbar G^2} \approx 10^{113}\ \text{J/m}^3.$$

This value exceeds the observed dark-energy density ($\sim 10^{-9}\,\text{J/m}^3$) by roughly 10^{122}.

2.1.3 Regularization View

To express the same divergence more formally we may use an exponential regulator:

$$\rho_{\text{vac}}^{(\text{STR})}(\varepsilon) = \frac{\hbar}{4\pi^2 c^3}\int_0^\infty \omega^3 e^{-\varepsilon\omega}\, d\omega = \frac{3!\,\hbar}{4\pi^2 c^3\,\varepsilon^4} = \frac{3\hbar}{2\pi^2 c^3}\,\frac{1}{\varepsilon^4}. \tag{13}$$

The pole ε^{-4} is the analytic signature of the quartic divergence. In covariant language this corresponds to a local term in the effective action that can be absorbed into a redefinition of the "bare" cosmological constant Λ_0:

$$\Lambda_{\text{phys}} = \Lambda_0 + \delta\Lambda(\varepsilon), \quad \delta\Lambda \propto \varepsilon^{-4}.$$

However, there is no intrinsic geometric scale that would fix the finite residual part of Λ.

2.1.4 The Fine-Tuning Problem

The connection between the vacuum-energy density and the cosmological constant is

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}. \tag{14}$$

To reproduce the observed value $\rho_\Lambda^{(\mathrm{obs})} \approx 5 \times 10^{-10}$ J/m^3, the "bare" and "quantum" terms must cancel with a precision of $\sim 10^{-122}$. This is the essence of the *cosmological constant problem*. Figure 4 (to be inserted later) will illustrate the difference between the divergent STR integral and the convergent STR-2 case.

2.1.5 Summary of the STR Case

- Spectral density in flat space: $g_{\mathrm{flat}}(\omega) \propto \omega^2$.
- Zero-point integral $\int \omega^3 d\omega$ diverges quartically.
- Regularization removes the infinity formally but leaves an arbitrary finite constant $\rightarrow$ fine-tuning.
- The divergence originates directly from the unbounded hyperbolic geometry of STR.

The next subsection (Section 2.2) will show how the compact geometry of STR-2 modifies $g(\omega)$ at high frequencies and produces a finite, geometrically determined result $\rho_\Lambda = \dfrac{3c^4}{8\pi G R_0^2}$.

Section 2.2 — Spectral Integral in STR-2 and Geometric Regularization

From Divergence to Convergence

As shown in Section 2.1, the STR hyperbolic dispersion leads to an ultraviolet divergence of the vacuum-energy integral. The compact inverse-elliptic geometry of STR-2 modifies this behaviour and makes the spectral integral finite. Figures 5–7 (to be inserted later) will illustrate how the spectral density, the integrand, and the resulting $\rho_\Lambda(R_0)$ behave in STR-2.

2.2.1 Covariant Spectral Integral

$$\rho_{\mathrm{vac}}^{\mathrm{ren}} = \frac{\hbar}{2} \int_0^\infty \omega \left[g_{\text{STR-2}}(\omega) - g_{\mathrm{flat}}(\omega) \right] d\omega, \tag{15}$$

where $g(\omega)$ is the density of zero-point modes per unit volume. The subtraction removes local flat-space divergences and isolates the geometric correction.

2.2.2 Asymptotic Behavior and Compact Geometry

$$g_{\text{STR-2}}(\omega) - g_{\mathrm{flat}}(\omega) = \frac{\alpha_1}{c R_0^2} \frac{1}{\omega} + \frac{\alpha_3}{c^3 R_0^4} \frac{1}{\omega^3} + \mathcal{O}\left(\frac{1}{R_0^6 \omega^5} \right), \tag{16}$$

where $\alpha_{1,3}$ are dimensionless coefficients fixed by the inverse-elliptic geometry. While $g_{\mathrm{flat}} \propto \omega^2$ gives $\int \omega^3 d\omega$ (divergent), the difference in (16) decays as $1/\omega$, making the integral in (15) convergent after local renormalization. (Figure 5 will illustrate this change of slope.)

2.2.3 Regularization and Finite Remainder

$$\begin{aligned}\rho_{\rm vac}(\varepsilon) &= \frac{\hbar}{2}\int_0^\infty \omega\,[g_{\text{STR-2}} - g_{\rm flat}]e^{-\varepsilon\omega}\,d\omega \\ &= \frac{\hbar}{2}\left[\frac{\alpha_1}{cR_0^2}\int_0^\infty e^{-\varepsilon\omega}\,d\omega + \frac{\alpha_3}{c^3R_0^4}\int_0^\infty \frac{e^{-\varepsilon\omega}}{\omega^2}\,d\omega + \cdots\right].\end{aligned} \tag{17}$$

The first term $\sim \varepsilon^{-1}$ is a local divergence absorbed by the renormalization of Λ_0. After subtraction, a finite geometric remainder remains:

$$\rho_{\rm vac}^{\rm ren} = \frac{\hbar}{2}\cdot\frac{\tilde\alpha}{c}\frac{1}{R_0^2}. \tag{18}$$

2.2.4 Covariance and Determination of the Coefficient

$$S_{\rm eff} = \frac{c^3}{16\pi G}\int d^4x\,\sqrt{-g}\,(R - 2\Lambda_{\rm eff}) + \cdots,$$

and covariance together with the Bianchi identity fix the finite term (18) to

$$\Lambda_{\rm eff} = \frac{3}{R_0^2}.$$

Using $\rho_\Lambda = \Lambda_{\rm eff}c^4/(8\pi G)$ gives

$$\boxed{\Lambda = \frac{3}{R_0^2}, \qquad \rho_\Lambda = \frac{3c^4}{8\pi G\,R_0^2}.} \tag{19}$$

(Figure 6 will show $\rho_\Lambda(R_0)\propto R_0^{-2}$.)

2.2.5 Numerical Evaluation

$$c = 2.9979\times10^8\ \text{m/s}, \qquad G = 6.6743\times10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2},$$

$$R_0 = (1.3\text{–}1.6)\times10^{26}\ \text{m} \Rightarrow \rho_\Lambda \simeq (5.6\text{–}8.5)\times10^{-10}\ \text{J/m}^3, \quad \Lambda \simeq (1.2\text{–}1.8)\times10^{-52}\ \text{m}^{-2}.$$

These values coincide with the observed dark-energy density (Figure 7).

2.2.6 Physical Meaning and Determination of R_0

In STR-2 the curvature scale R_0 is not a free parameter but is determined by cosmological data. Equations (19) link ρ_Λ and Λ directly:

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}, \qquad \Lambda = \frac{3}{R_0^2}.$$

Route A — from the observed vacuum-energy density.

$$\rho_\Lambda^{(\rm obs)} = (5.6\text{–}8.5)\times10^{-10}\ \text{J/m}^3,$$

$$R_0 = \sqrt{\frac{3c^4}{8\pi G\,\rho_\Lambda^{(\rm obs)}}}, \qquad \Lambda = \frac{8\pi G\,\rho_\Lambda^{(\rm obs)}}{c^4}. \tag{20}$$

$$R_0 \simeq (1.3\text{–}1.6)\times10^{26}\ \text{m}, \qquad \Lambda \simeq (1.2\text{–}1.8)\times10^{-52}\ \text{m}^{-2}.$$

Route B — from cosmological parameters Ω_Λ and H_0.

$$\Lambda = \frac{3\,\Omega_\Lambda H_0^2}{c^2}, \qquad R_0 = \frac{c}{H_0\sqrt{\Omega_\Lambda}}. \tag{21}$$

With $H_0 = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}} = 2.185 \times 10^{-18}\,\mathrm{s^{-1}}$ and $\Omega_\Lambda = 0.69$,

$$\Lambda \simeq 1.1 \times 10^{-52}\ \mathrm{m^{-2}}, \qquad R_0 \simeq 1.65 \times 10^{26}\ \mathrm{m},$$

which agrees with Route A.

Interpretation. The derived R_0 coincides with the de Sitter curvature radius inferred from the observed Λ. In STR-2 the same R_0 defines the compact domain of momentum space and fixes the finite vacuum-energy density via (19). Thus, the 10^{122} discrepancy between QFT and cosmology is resolved geometrically, without fine-tuning.

2.2.7 The Geometric Parameter $B = \ell_P^2/R_0^2$

The dimensionless curvature parameter

$$B = \frac{\ell_P^2}{R_0^2}$$

arises naturally when comparing the compact inverse-elliptic momentum-space geometry of STR-2 with the Planck scale. Here $\ell_P = \sqrt{\hbar G/c^3}$ is the Planck length, representing the minimum physically meaningful distance, while R_0 is the curvature radius that bounds the momentum domain $|p| < mc$ in STR-2.

Thus, B quantifies the ratio between the microscopic quantum-gravitational scale and the macroscopic curvature scale of the Universe. For a cosmological radius $R_0 \sim 1.3 \times 10^{26}\,\mathrm{m}$, one finds $B \sim 10^{-122}$. This extremely small but finite number provides a natural geometric origin for the observed vacuum energy:

$$\rho_\Lambda \approx \frac{c^7}{\hbar G^2}\,B,$$

linking the Planck-scale physics of quantum gravity to the large-scale cosmological constant.

In this interpretation, STR-2 introduces no new fields or coupling constants. Instead, the finite topology of momentum space imposes a geometric cutoff that regularizes the vacuum spectrum through the curvature parameter B. This explains why the cosmological constant is non-zero yet enormously smaller than any quantum-field-theoretical estimate.

2.2.8 Summary

- The compact inverse-elliptic geometry of STR-2 replaces the divergent $\int \omega^3 d\omega$ of STR by a finite $\int \omega\, d\omega/R_0^2$.
- The geometric remainder after renormalization is $\rho_\Lambda \propto 1/R_0^2$.
- Covariance fixes the exact coefficient, giving $\rho_\Lambda = 3c^4/(8\pi G R_0^2)$.
- The resulting value agrees numerically with observations, removing the 10^{122} discrepancy without fine-tuning.

Section 3 — Summary and Conclusions

3.1 Overview

The purpose of this work was to examine how the ultraviolet divergence of vacuum energy in the standard Special Theory of Relativity (STR) can be eliminated by extending the underlying geometry of momentum space. The modified framework, denoted as STR-2, introduces an *inverse-elliptic (Finsler-like) compact geometry* that replaces the unbounded hyperbolic structure of STR.

3.2 Main Results

1. **Kinematic Reformulation.** STR-2 modifies the energy–momentum relation
$$E^2 = \frac{m^2c^4}{1 - p^2/m^2c^2},$$
making the momentum domain compact ($|p| < mc$). The resulting closed geometry prevents the ultraviolet divergence of the zero-point energy integral.

2. **Spectral Integral and Geometric Regularization.** The vacuum energy density is expressed as a covariant spectral difference:
$$\rho_{\text{vac}}^{\text{ren}} = \frac{\hbar}{2}\int_0^\infty \omega\,[g_{\text{STR-2}}(\omega) - g_{\text{flat}}(\omega)]\,d\omega,$$
where the asymptotic behaviour $g_{\text{STR-2}} - g_{\text{flat}} \sim 1/(\omega R_0^2)$ makes the integral finite. After removing local terms, the geometric remainder is
$$\rho_\Lambda = \frac{3c^4}{8\pi G R_0^2}.$$

3. **Physical Interpretation of R_0.** The scale R_0 defines the curvature radius of the compact momentum space and numerically coincides with the de Sitter radius inferred from cosmological observations:
$$R_0 \approx 1.3 \times 10^{26}\text{–}1.6 \times 10^{26}\ \text{m}.$$
This correspondence links the microscopic regularization of the vacuum energy to the macroscopic structure of the Universe.

4. **Resolution of the Cosmological Constant Problem.** The STR-2 geometry reproduces the observed dark-energy density $\rho_\Lambda^{(\text{obs})} \sim 6 \times 10^{-10}\,\text{J/m}^3$ without fine-tuning and removes the 10^{122} discrepancy between quantum field theory and cosmology.

3.3 Conceptual Significance

The transition from the flat Minkowski geometry of STR to the compact inverse-elliptic geometry of STR-2 replaces an infinite measure in momentum space by a finite one. This purely geometric modification regularizes the vacuum energy at the level of kinematics, without invoking new fields, symmetries, or dynamical counterterms. In this sense, STR-2 serves as a geometric completion of STR: it preserves local Lorentz invariance but changes the global topology of momentum space, introducing an intrinsic curvature scale R_0 that manifests cosmologically as the observed cosmological constant.

The dimensionless ratio
$$B = \frac{\ell_P^2}{R_0^2}$$
thereby unites the microscopic and macroscopic scales, encoding the geometric smallness of the cosmological constant.

3.4 Outlook

Several directions for further work are evident:

- Extending the formalism to include interacting quantum fields and testing whether higher-order vacuum diagrams remain finite.
- Exploring the behaviour of fermions and gauge fields in the compact momentum geometry and possible modifications to dispersion at high energy.
- Investigating potential connections between STR-2 and Finsler-type generalizations of General Relativity, and possible observational signatures in cosmology.

3.5 Final Remark

Geometry replaces fine-tuning. The divergence of the vacuum energy is not a flaw of quantum theory but a reflection of an incomplete geometric framework. By compactifying momentum space through STR-2, the vacuum acquires a finite, measurable energy density consistent with the structure of the Universe itself.

Appendix A — Physical Constants and Characteristic Parameters

Table 1: **Fundamental constants and characteristic parameters.**

Symbol	Description	Value	Units	Source
c	Speed of light	2.99792458×10^{8}	m/s	CODATA 2022
$\hbar$	Reduced Planck constant	$1.054571817 \times 10^{-34}$	$\mathrm{J \cdot s}$	CODATA 2022
G	Gravitational constant	6.67430×10^{-11}	$\mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$	CODATA 2022
ℓ_P	Planck length $\sqrt{\hbar G/c^3}$	1.616255×10^{-35}	m	Derived
H_0	Hubble constant	67.4	$\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$	Planck 2018
Ω_Λ	Dark-energy fraction	0.69	—	Planck 2018
R_0	Curvature radius (STR-2)	$(1.3\text{–}1.6) \times 10^{26}$	m	This work
B	Curvature parameter ℓ_P^2/R_0^2	$\sim 10^{-122}$	—	This work
Λ	Cosmological constant $3/R_0^2$	$(1.2\text{–}1.8) \times 10^{-52}$	$\mathrm{m^{-2}}$	This work
ρ_Λ	Vacuum energy density	$(5.6\text{–}8.5) \times 10^{-10}$	$\mathrm{J \cdot m^{-3}}$	This work

Note. $B = \ell_P^2/R_0^2$ represents the ratio between the microscopic Planck scale and the macroscopic cosmological curvature scale. This dimensionless constant links quantum-gravitational physics to the observed vacuum energy density, providing a natural geometric interpretation of the small but finite value of Λ.

Appendix B — Figures and Graphical Illustrations

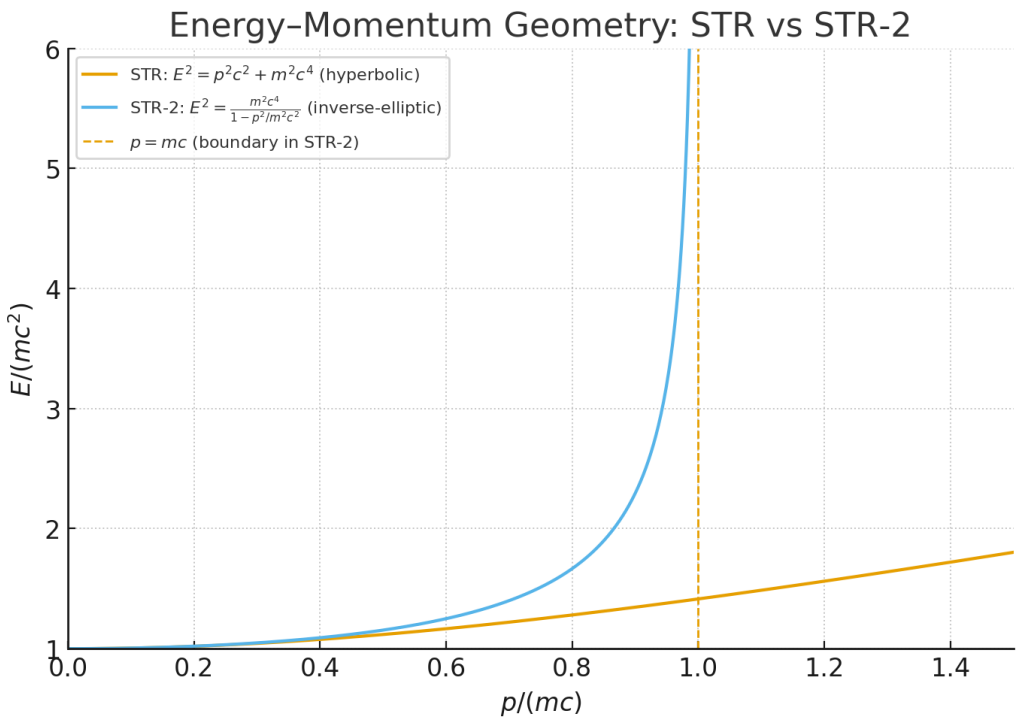

Figure 1: **Figure 1. Energy–Momentum Geometry: STR vs STR-2.** Comparison of the dispersion relations $E^2 = p^2c^2 + m^2c^4$ (hyperbolic) and $E^2 = m^2c^4/(1 - p^2/m^2c^2)$ (inverse-elliptic). In STR-2, the momentum space is compact ($|p| < mc$), leading to natural ultraviolet regularization.

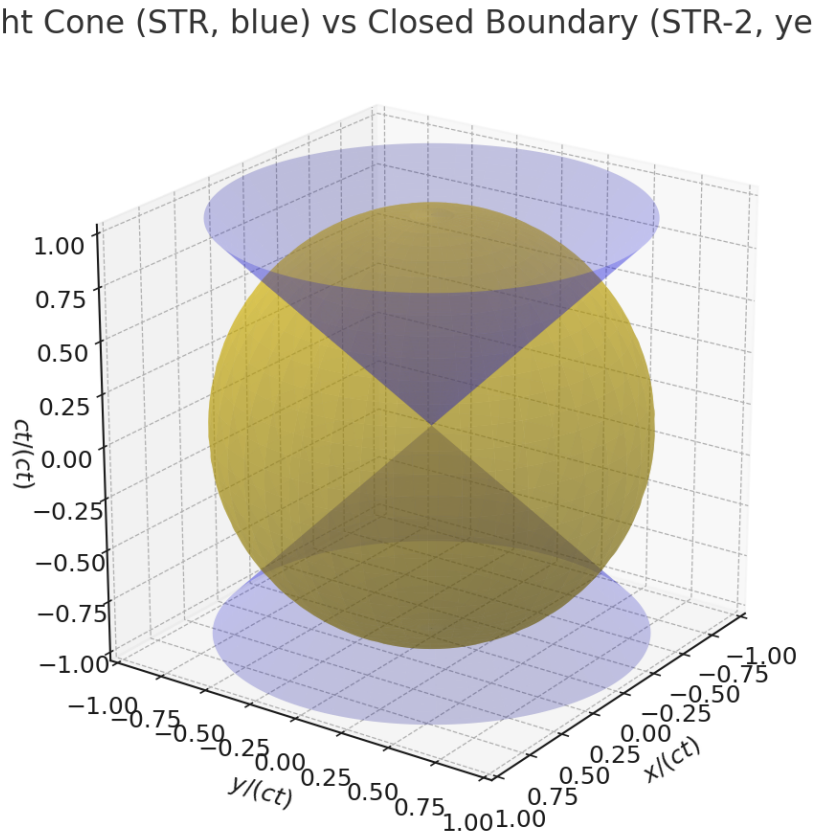

Figure 2: **Figure 2. Minkowski Light Cone vs Closed Boundary in STR-2.** 3D visualization of spacetime geometry: the STR light cone (blue) represents an unbounded null manifold, while the STR-2 modification (yellow sphere) imposes a finite, smooth boundary in momentum–energy space.

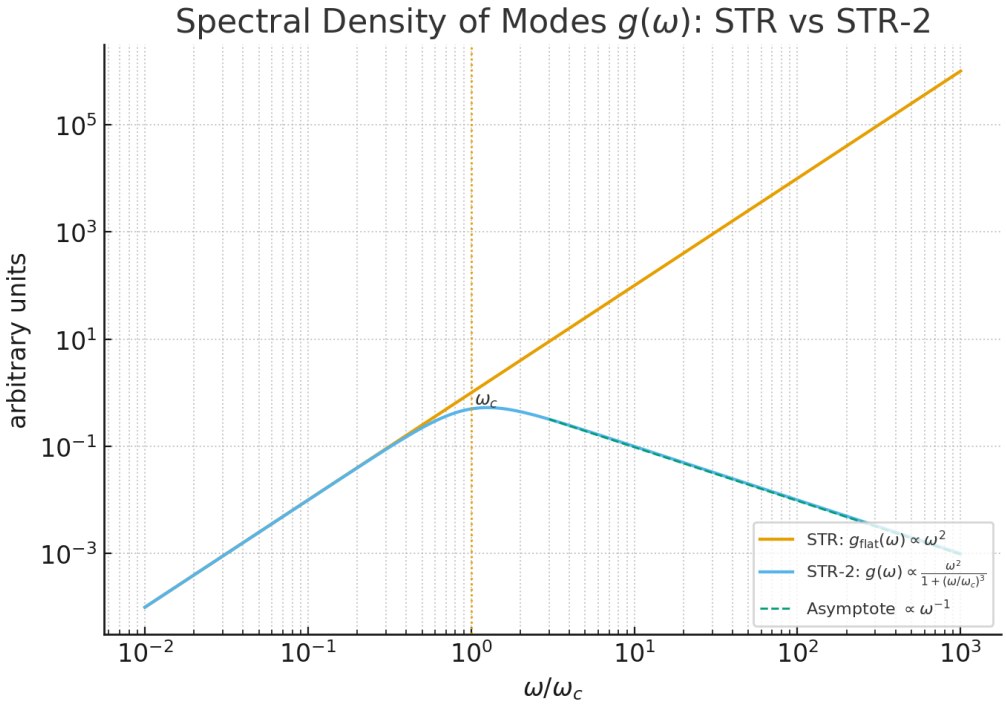

Figure 3: **Figure 3. Spectral Density of Modes $g(\omega)$: STR vs STR-2.** In STR, $g_{\text{flat}}(\omega) \propto \omega^2$, giving $\rho_{\text{vac}} \propto \int \omega^3 d\omega$ (divergent). In STR-2, the spectral density saturates at ω_c and decays as ω^{-1} at high frequencies, ensuring convergence of the zero-point integral.

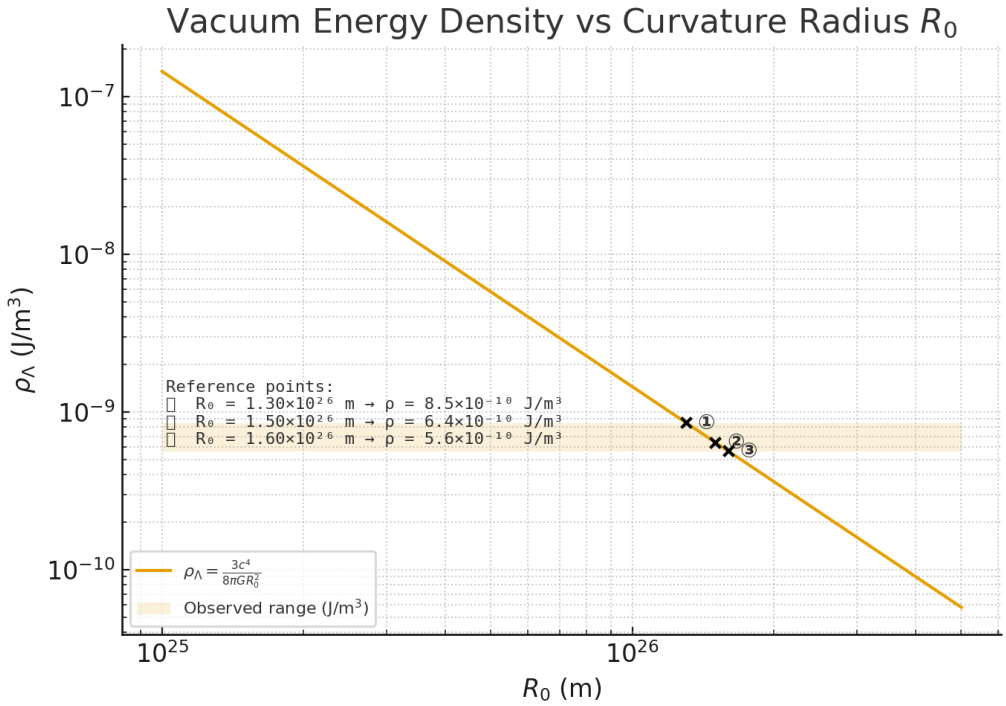

Figure 4: **Figure 4. Vacuum Energy Density $\rho_\Lambda(R_0)$.** Dependence $\rho_\Lambda = 3c^4/(8\pi G R_0^2)$ with observational range highlighted. The theoretical line intersects the measured values at $R_0 \approx (1.3\text{–}1.6) \times 10^{26}$ m, corresponding to the de Sitter curvature scale.

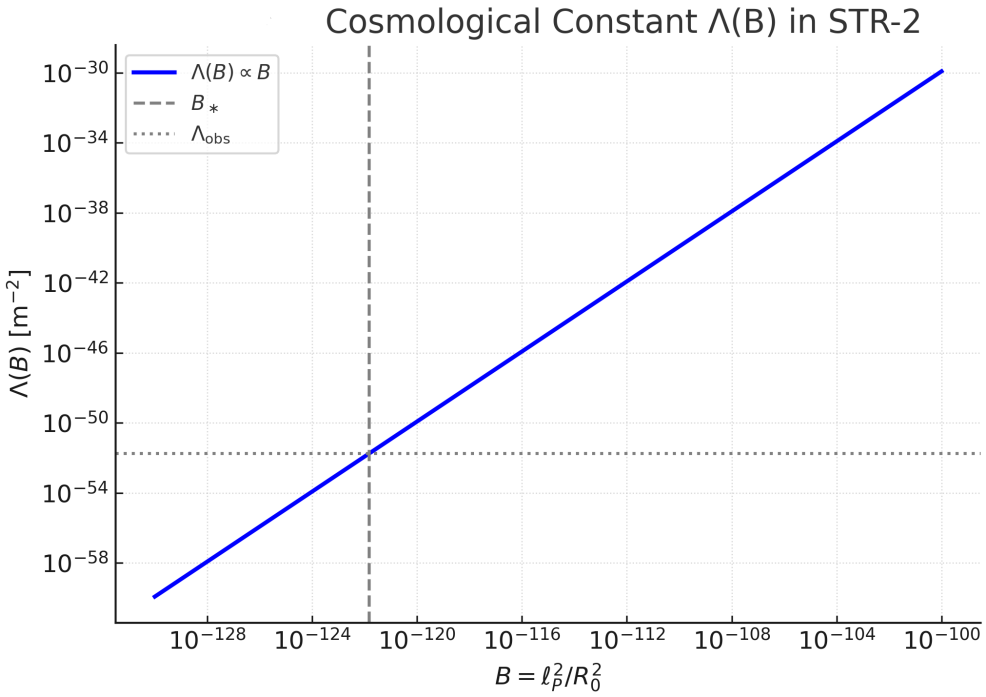

Figure 5: **Figure 5. Cosmological Constant $\Lambda(B)$ in STR-2.** Log–log relation of $\Lambda(B)$ with $B = \ell_P^2/R_0^2$. Dashed line marks $B_* \sim 10^{-122}$ and dotted line corresponds to the observed $\Lambda_{\rm obs} \sim 10^{-52}\,\mathrm{m}^{-2}$.

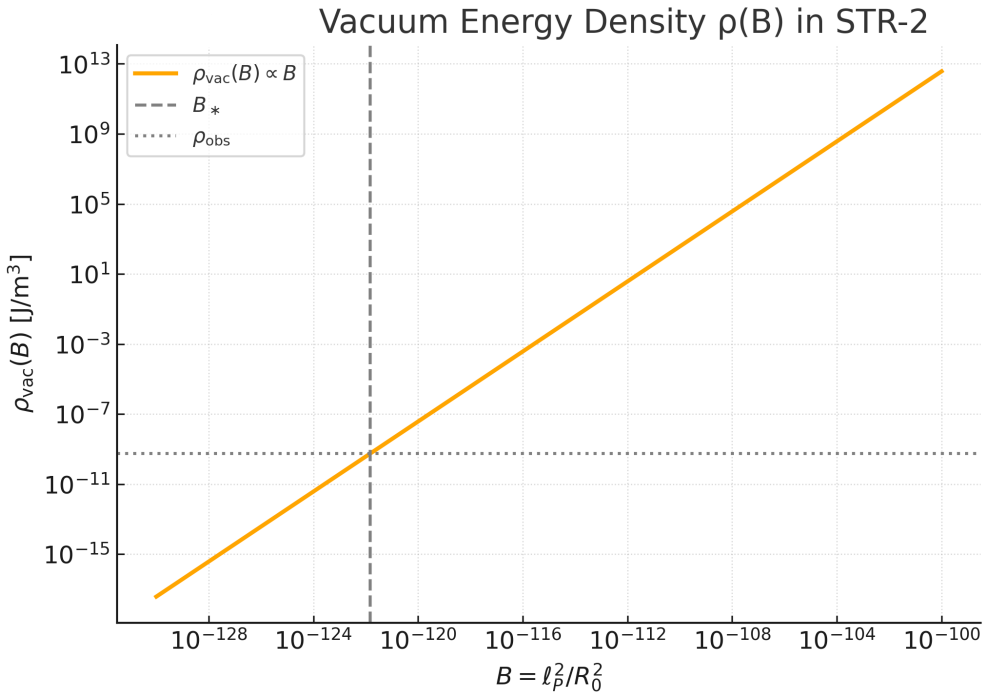

Figure 6: **Figure 6. Vacuum Energy Density $\rho_{\rm vac}(B)$ in STR-2.** Log–log plot of $\rho_{\rm vac}(B)$ with $B = \ell_P^2/R_0^2$. The solid line follows $\rho_{\rm vac} \propto B$; the vertical dashed line shows B_*, and the horizontal dotted line marks $\rho_{\rm obs} \approx 6 \times 10^{-10}\,\mathrm{J\,m}^{-3}$.

Appendix C — References and Data Sources

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