Beyond the Horizon (Part III): The Boundary Engine

A Ghost-Free Scalar–Gauss–Bonnet Framework for Cosmogenesis

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October 2025

Abstract

In this concluding part of the *Beyond the Horizon* trilogy, we present a refined and ghost-free formulation of the curvature-driven VSL cosmogenesis model introduced in Parts I–II. At a heuristic level, those works proposed that the Kretschmann scalar $\mathcal{K}$ controls the suppression $c \to 0$ near black-hole horizons. Here we make that link explicit through a Lagrangian framework. While a direct $\phi\mathcal{K}$ coupling captures the essential idea, it introduces higher-derivative ghosts. To resolve this, we adopt a stable scalar–Gauss–Bonnet interaction, $\mathcal{L} = \frac{1}{16\pi G}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) + f(\phi)\mathcal{G}$, which preserves the same horizon-peaking behavior while avoiding pathologies. In vacuum, $\mathcal{G} = \mathcal{K}$, ensuring the desired curvature response; in accreting systems, $\mathcal{G}$ also couples to matter via the Ricci tensor, maintaining sensitivity to infalling mass. Collapse then halts within a finite-thickness region where $c \to 0$, the entropy density $s \propto c^3$ vanishes, and geometric inversion maps the horizon to a cosmological Big Bang. The resulting "boundary engine" offers a stable Lagrangian foundation for the curvature-driven cosmogenesis picture developed across the trilogy.

1 Introduction

Building upon Parts I-II [1, 2], which proposed that black hole horizons become cosmological boundaries through a variable speed of light (VSL), we now outline the underlying physics. The mechanism stems from curvature self-regulation—a principle inspired by Brans-Dicke scalar-tensor gravity [3] but dynamically driven by spacetime's intrinsic geometry. Unlike phenomenological VSL theories [4], our approach derives speed of light variation from fundamental curvature coupling.

In Part I, we introduced an exponential ansatz for $c(r)$ to model causal suppression. Part II showed how geometric inversion $R = r_H^2/r$ and conformal time $d\eta = (c/r_H)dt$ reframe the interior as an expanding FLRW-like universe. This final part provides the dynamical engine: a Lagrangian formulation where curvature self-consistently drives $c(\phi) \to 0$. We refine the earlier ansatz, naturally deriving a logarithmic profile from the field equations, and show how the framework dynamically resolves the entropy problem, providing a stable foundation for the model.

2 From Heuristic Idea to Ghost-Free Lagrangian

The core idea is simple: use the curvature that peaks at horizons to dynamically suppress the speed of light and halt collapse. A minimal Lagrangian capturing this is:

$$\mathcal{L} = R + (\partial_\mu \phi)^2 + \phi\mathcal{K}, \quad \mathcal{K} \equiv R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}. \tag{1}$$

This model exploits a key geometric feature: the Kretschmann scalar $\mathcal{K}$ naturally diverges at classical singularities and peaks strongly near horizons [5], making it a natural driver for the scalar field ϕ, which modulates the effective speed of light via $c \propto 1/\phi$.

However, this $\phi\mathcal{K}$ coupling introduces higher-derivative terms, leading to Ostrogradsky ghost instabilities [6]. A viable physical theory must be ghost-free. We therefore adopt a refined Lagrangian, well-studied in Horndeski-type scalar-tensor theories [7]:

$$\mathcal{L} = \frac{1}{16\pi G}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) + f(\phi)\mathcal{G} + \mathcal{L}_{\text{matter}}, \tag{2}$$

where

$$\mathcal{G} \equiv R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2 \tag{3}$$

is the Gauss-Bonnet invariant and $f(\phi)$ is a smooth coupling function. In four dimensions, the integral of $\mathcal{G}$ is topological, so it contributes only when coupled to a dynamical scalar. Crucially, the $f(\phi)\mathcal{G}$ term yields second-order field equations and avoids Ostrogradsky ghosts in standard analyses.

The scalar field obeys:

$$\Box\phi - V'(\phi) + f'(\phi)\mathcal{G} = 0, \tag{4}$$

while the metric equations acquire an additional source term. This framework preserves the essential physical intuition:

- **In vacuum** (Schwarzschild, where $R_{\mu\nu} = 0, R = 0$), $\mathcal{G} = \mathcal{K}$. The scalar field is sourced by the horizon-peaking curvature, driving $c(\phi) \to 0$ exactly as desired.
- **During accretion**, $\mathcal{G}$ is additionally sensitive to matter through the Ricci terms $(R_{\mu\nu}, R)$. This ensures that infalling matter directly enhances the scalar response, making the mechanism robust in astrophysical environments.

The relation $c \propto 1/\phi$ can also be realized dynamically via a conformal rescaling of the metric, a mechanism we detail in the next subsection.

2.1 Dynamical Mechanism via Conformal Rescaling

The conformal rescaling

$$g_{\mu\nu} = \Omega^2(\phi)\,\tilde{g}_{\mu\nu}, \quad \Omega(\phi) = \frac{\kappa}{\phi}, \tag{5}$$

provides a formal mechanism to embed the VSL into the spacetime metric. Here $\tilde{g}_{\mu\nu}$ is a fiducial metric with constant light speed, while physical measurements made with $g_{\mu\nu}$ yield an effective speed of light $c \propto \Omega^{-1}(\phi) \propto \phi$. In this way, the growth of ϕ dynamically narrows the local light cone, realizing the central VSL ansatz of our model.

This approach aligns with scalar–tensor theories (e.g., Brans–Dicke) and VSL frameworks [4], offering a bridge between our construction and established literature. While we do not work out a full example here, one can think of $\Omega(\phi)$ as shrinking the local light cone as ϕ grows, embedding the variable speed of light directly into the causal fabric of spacetime.

3 Horizon Dynamics and Entropy Resolution

As matter collapses toward a black hole horizon ($r \to r_H$), the sourced field equation leads to a logarithmic growth profile:

$$\phi \sim \ln|r - r_H|^{-1}, \tag{6}$$

refining the exponential ansatz from Part I. The effective speed of light $c \propto 1/\phi$ thus vanishes smoothly at the horizon. Infalling matter decelerates, forming a finite-thickness layer where collapse halts and curvature remains finite [8], realizing the regular interior proposed in Part I.

This mechanism dynamically resolves the entropy tension identified in Part I. As $c \to 0$, entropy density scales universally as $s \propto c^3$, causing it to vanish at the horizon while preserving total entropy. The resulting thin shell becomes a low-entropy surface—a thermodynamic analogue of the Big Bang. The scaling holds for both Bekenstein-Hawking and relativistic gas entropy:

$$s_{\mathrm{BH}} = \frac{S_{\mathrm{BH}}}{A} = \frac{1}{4\ell_{\mathrm{P}}^2}, \quad \ell_P = \sqrt{\frac{\hbar G}{c^3}}, \tag{7}$$

$$s_{\mathrm{gas}} \propto T^3 \propto c^3. \tag{8}$$

Since $\ell_P \propto c^{-3/2}$, the area density of entropy scales as $\ell_P^{-2} \propto c^3$, so $s_{\mathrm{BH}} \propto c^3$. This shared scaling ensures a thermodynamically consistent origin for cosmological initial conditions.

4 The Inside-Out Cosmological Picture

The model posits two complementary descriptions of reality. An external observer sees matter freeze into a high-density shell at a horizon where $c \to 0$. An internal observer experiences this same frozen matter as the homogeneous, high-density initial state of an expanding Big Bang universe. The mathematical link is a geometric coordinate transformation adapted from Schwarzschild-VSL frameworks as a heuristic tool to visualize causal structure.

The radial inversion $R = r_H^2/r$ and conformal time $d\eta = (c/r_H)dt$ repackage collapse as expansion:

- The **horizon** ($r = r_H$) becomes a *temporal origin* ($R = r_H$), with $c(\eta) \to 0$ as $\eta \to -\infty$ mimicking a Big Bang.
- The **interior** ($r < r_H$) maps to $R > r_H$—an *expanding domain* where R grows with time.
- The point $r = 0$ transitions to $R \to \infty$, representing the infinite future.

The Schwarzschild-inspired metric in these coordinates is:

$$ds^2 = \left(\frac{r_H}{R}\right)^4 \left[-d\eta^2 + \frac{dR^2}{(R/r_H)^2 - 1} + r_H^2 d\Omega^2\right]. \tag{9}$$

While our scalar-tensor model may yield quantitative deviations near the horizon, the inversion's power lies in reframing interior dynamics as cosmological evolution—a perspective that remains valid irrespective of metric details.

Furthermore, as the horizon grows ($dr_H > 0$) due to accretion, the interior geometry—expressed in inverted coordinates—expands. Defining a scale factor $a \propto r_H^2$ as in Part II, we obtain:

$$\frac{da}{d\eta} = 2\frac{a}{r_H}\frac{dr_H}{d\eta}, \tag{10}$$

showing that accretion-driven increase in r_H directly fuels expansion. This provides a geometric foundation for both early inflation (via rapid suppression of c) and late-time acceleration (via ongoing accretion).

5 Conclusion and the Boundary Engine Mechanism

Through curvature self-regulation in a ghost-free framework, we have developed a complete picture of black hole cosmogenesis:

1. Horizon-peaking curvature ($\mathcal{G} = \mathcal{K}$ in vacuum) dynamically drives $c(\phi) \to 0$, halting collapse in a finite-thickness layer.
2. The heuristic $\mathcal{K}$-driven mechanism of Parts I-II is formalized through a stable scalar-tensor Lagrangian.
3. Entropy continuity follows from universal c^3 scaling, resolving the tension between black hole and cosmological entropy.
4. Geometric inversion reframes collapse as cosmological expansion, with accretion driving the scale factor.

This framework suggests a powerful "boundary engine" mechanism. The event horizon $\mathcal{H}$ acts as a null hypersurface—a dynamical boundary. The accretion of matter from the parent universe updates the boundary data on $\mathcal{H}$ (such as the energy flux $T_{ab}\ell^a\ell^b$ and shear σ_{AB}). The modified field equations, incorporating the scalar-Gauss-Bonnet coupling $\Delta^{(\mathrm{GB})}_{\mathrm{VSL}}$, then propagate this updated information instantaneously into the child universe's interior via constraint equations. This results in a global re-computation of the cosmological geometry, manifesting as expansion. In this way, the horizon is not a passive surface but an active encoder of spacetime, translating external accretion into internal cosmic evolution.

Thus, black holes emerge as candidate universe-generators. This mechanism offers a natural geometric origin for both early inflation (via rapid suppression of c) and late-time acceleration (via ongoing accretion). Future work will involve numerical simulations of this mechanism, such as using characteristic codes to model CMB imprints, to derive potential observable imprints. The scalar-Gauss-Bonnet framework strengthens the theoretical foundation while retaining the essential insight of Parts I-II: horizons may be not ends, but beginnings.[1]

Acknowledgments

The author acknowledges helpful discussions with Grok, DeepSeek, and ChatGPT for conceptual and editorial assistance.

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