Proper Time as Emergent Vacuum Accessibility

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Abstract

Time is treated here not as a fundamental parameter but as an emergent property arising from a system's structural coupling to the quantum vacuum. A probe's proper time is identified with its expected capacity to remain correlated with the accessible portion of the vacuum energy. Two postulates formalize this relationship. The first defines proper time as a strictly increasing functional of accessible vacuum energy, and the second enforces consistency for a free particle, requiring that the accessible vacuum energy remain constant along inertial worldlines. Together they imply a fixed energy–action balance, constraining the Hamiltonian and accessible vacuum energy as complementary aspects of a single invariant total.

Variations in vacuum accessibility naturally reproduce the phenomena of time dilation and gravitational redshift, yielding the Lorentz and Schwarzschild relations without assuming time as a fundamental dimension. On cosmic scales, the same reciprocity accounts for the cosmological constant. The global deficit of accessible vacuum energy that General Relativity describes as curvature corresponds to the structural partitioning that gives rise to proper time itself.

The theory yields falsifiable predictions. Engineered changes to the local vacuum density of states should produce measurable shifts in proper time between identical clocks or interferometric paths. In this view, relativistic time and gravitational curvature arise from a unified structural relationship between matter, the Hamiltonian, and the vacuum's capacity for interaction.

1 Introduction

Although time may ultimately be an emergent property, it is presently a fundamental and indispensable parameter within the standard QFT framework, directly tied to the formal machinery that describes system evolution. There is good reason for it to be treated as an external intrinsic property of evolution because it matches our everyday experience. Our classical experience suggests that we cannot halt time by preventing its flow. It marches onward still. This unstoppable advance is something that does not seem to need a physical reason. Thinking about what drives time would appear to be more of a philosophical endeavor than a scientific one. What foothold or basis in reality would we have to explore such a question?

2 Single Particle Universe (1PU) in $\mathbb{R}^0$

It is helpful to reduce a system down to its absolute minimum to probe deep seated existential questions about time. We might consider a free particle in Minkowski space as an ideal candidate. It might seem to be a minimal structure, but it has a hidden complexity. It assumes certain physical relationships such as the number of dimensions of that space and how time flows. This is not the minimal configuration we need to challenge and investigate our intuition of time. Rather, we can construct a single particle universe (1PU) where even the vacuum of space does not exist. In fact, our particle e necessarily exists in a zero dimensional space

$$e \in \mathbb{R}^0.$$

Of course, the laws of physics do not apply here, including our notion of time. We could assign various properties to the particle e such as time, but what meaning could they have if $e = ()$? And if we did so we would only be inserting our preconceived bias as an unnecessary complexity on (). It is reasonable to conclude that if we can say anything about time, it is undefined for e and without the ability to interact with some other particle it is forever frozen in time.

3 Two Particle Universe (2PU) in $\mathbb{R}^1$

Interaction would appear to be a key ingredient for time to evolve. This does not have to be the case, but our intuition from a 1PU suggests that it might be. We therefore extend our universe to two particles and thus must increase our dimensions of space to $\mathbb{R}^1$. We still have not reached a universe where the laws of physics apply because an empty vacuum energy is forbidden by the Uncertainty Principle. Here we ask ourselves a key question. Does time need to be added as a second dimension forcing us into $\mathbb{R}^2$? If we follow of our intuition from a 1PU, which implies that interactions might drive time, then the answer is no. We could count the interactions between our two particles e_1 and e_2 to describe the proper time each particle experiences.

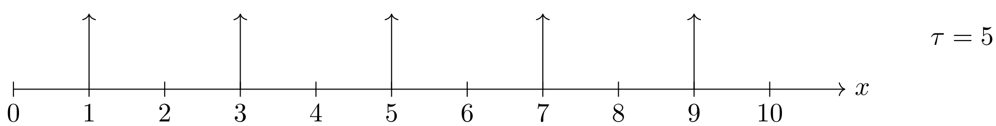

Figure 1: Fixed spacing of interaction records at e_1 from a source e_2 with a proper time of 5 units.

This relational emergence of time closely parallels the framework introduced by Page and Wootters [1], who described time as a correlation between subsystems within a global stationary state. Our **2PU** provides a concrete physical realization of that idea. Each particle experiences the progression of proper time only through its conditional correlations with the other. The difference is that the present construction grounds those correlations in vacuum accessibility rather than in abstract entanglement structure, thereby giving the relational notion an explicit energetic substrate.

As we can see time emerges naturally from such an environment. In fact, it is actually necessary. Even if we were to claim that time advanced between interactions we would have devolved into a 1PU during the null events where the particle is frozen in time. We can immediately see the meaning of proper time. We can draw our timing diagram from any frame of reference by inserting more space between the ticks but that is meaningless to the particle's experience. Their entire view of the outside world is defined entirely by how they experience each other through that interaction. This gives us a natural reason for a Lorentz transformation for proper time. We just remove the null event ticks that would otherwise devolve to a 1PU at the null events.

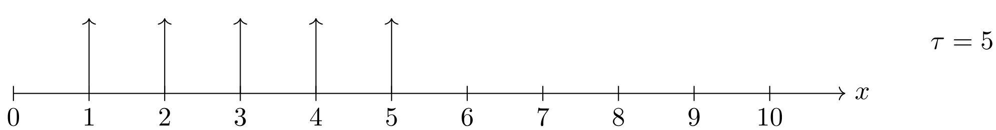

Figure 2: Particle e_1's frame of reference with a proper time of 5 units.

Notice that proper time was immediately apparent in either frame of reference. However, we express it naturally by removing all null events. This is exactly the Lorentz transformation for proper time. We can therefore define time for a 2PU in $\mathbb{R}^1$, but it is not enough. If we assign properties like charge to the particles, we cannot distinguish between motion $\dot{x}$ and a varying charge $\dot{q}$.

4 Multi Particle Universe (3PU+) in $\mathbb{R}^1$

Finally we have enough structure to describe all the necessary features of physics to model a real world reduced to just the x-dimension. We do not need another dimension to describe proper time for the particles[1] as we saw with our previous timing diagrams. In fact, we can now assert our first postulate which relates proper time to the accessible vacuum energy describing the capacity to interact.

Postulate 1 (Accessible Vacuum Energy–Interaction Capacity) *The expected value of interaction capacity N for a probe is a strictly increasing functional of its accessible vacuum energy and is equal to the probe's proper time:*

$$\tau = \mathbb{E}N = f(E_{\text{vac}})$$

But there is a problem. If a free particle has constant momentum, then how could interactions drive time without increasing its momentum? This paradox is resolved cleanly because we restrict time to interactions with the accessible vacuum energy. The short answer is that there is no measurable difference because it affects all particles in the same way. Even if we consider a curved spacetime for General Relativity the difference we can discern is exactly that effect of curvature. If we consider the difference between two worldlines in Special Relativity it is a result of the period in which it is able to probe the accessible vacuum energy. This physical justification leads directly to length contraction. It also leads us to our second and final postulate.

Postulate 2 (Free Particle Consistency) *In a Minkowski vacuum with identical probes prepared in the same state Ψ along any inertial worldline with constant momentum, the accessible vacuum energy E_{vac} is constant. Evolution through $\mathbb{E}N$ does not add discernible kinetic energy between two probes. Each probe's proper time advances at a constant, nonzero rate because $E_{\text{vac}} > 0$ and $f(E_{\text{vac}}) > 0$.*

These two postulates together result in an energy action budget that creates a simple and elegant teeter-totter balance between accessible vacuum energy and the Hamiltonian.

5 Vacuum Energy Differences

From QFT we know that we must sum over the modes of the EM field to get the vacuum energy defined as the ground state of the quantized EM field:

$$|0\rangle = \bigotimes_{\mathbf{k},\lambda}, |0_{\mathbf{k}\lambda}\rangle$$

However, without some UV cutoff window this diverges to infinity.

$$E_{\text{vac}}^{\max} = \sum_{\mathbf{k}\lambda} \frac{1}{2}\hbar\omega_{\mathbf{k}} = \sum_{\mathbf{k}\lambda} \frac{1}{2}\hbar c|\mathbf{k}| \to \infty$$

This isn't an issue as long as we consider differences of two divergent energies. We see this exact approach when calculating the energy of the Casimir effect for some plate geometry. We look at the difference of the vacuum energy modulated by the plate conditions and the vacuum energy in free space without the plates. The result falls off quadratically with plate distance and is dependent upon the geometry and dielectric that changes the available k, λ for the plate's summation.

$$E_{\text{Casimir}}(a) = \underbrace{\sum_{\mathbf{k},\lambda}^{\text{plates}} \frac{1}{2}\hbar c|\mathbf{k}|}_{\text{diverges as } \infty} - \underbrace{\sum_{\mathbf{k},\lambda}^{\text{free}} \frac{1}{2}\hbar c|\mathbf{k}|}_{\text{diverges as } \infty} .$$

[1] A natural objection is that we must still evolve through the phases of a mode's harmonic structure to create the interaction events. Calling this time would be a mistake because it leads to unnecessary confusion and complexity. It is already encoded as c. It is the one worldline that is invariant across Lorentz transformations. This further indicates that we should not conflate evolving through the phases in a consistent way (e.g. c) with our observation of proper time.

In a similar vein we can describe the vacuum energy deficit due to spacetime curvature by comparing the difference in the renormalized vacuum expectation value in curved and Minkowski spacetime. Even without any mass we expect the deficit to be non-zero due to vacuum fluctuations. We can describe the local energy density as

$$\rho_g(r) = \Delta\langle T_{00}(r)\rangle_{\text{ren}}^{\text{gravity}} = \langle T_{00}(r)\rangle_{\text{ren}}^{\text{curved}}[g_{\mu\nu}] - \langle T_{00}(r)\rangle_{\text{ren}}^{\text{Minkowski}} \tag{1}$$

5.1 Max Conceivable Accessible Vacuum Energy

We can describe how g and Casimir plate conditions impact $E_{\text{vac}}^{\max}$ by recognizing they are two different types of modulation. The Casimir effect is a function of the condensed matter dielectric and its geometry adjusting the boundary conditions whereas g arises from the stress-energy of matter and fields reshaping the vacuum mode structure universally dependent only on its mass-energy density.

$$\rho_i[\Psi; g, r, \mathcal{B}] = \rho_{\text{vac}}^{\max}[\Psi; g, r] - \rho_g[g, r] - \rho_{\text{cas}}[\mathcal{B}, r] \tag{2}$$

This defines an intermediate value for the conceivable operational upper bound for any probe coupling to the ground state of the quantized EM field.

5.2 Unit Mass Accessible Vacuum Energy-Action Budget

The Interaction Capacity Postulate (1) tells us that the capacity to interact with accessible vacuum energy N_i is proper time and is related to that energy by $f(E_i)$. We know from the Free Particle Consistency Postulate (2) that this must cancel out except for our observables such as the effect of g, the Casimir effect, and a particle's Hamiltonian. We therefore create the definition of the Accessible Vacuum Energy-Action Budget for a unit mass:

$$\boxed{E_i = E_{\text{vac}}^{\max} - E_g - E_{\text{cas}}.} \tag{3}$$

$$E_i = \int_V \rho_i[\Psi; g, r, \mathcal{B}]\, d^3r \tag{4}$$

Here E_i represents the formal definition of what is accessible as constrained by geometry, stress-energy curvature and, as we shall see, results in an implied Hamiltonian under those restrictions. As in renormalized QFT, only energy deficits relative to the unmodulated vacuum have operational meaning. $E_{\text{vac}}^{\max}$ serves purely as a normalization reference and is never physically accessible.

Taken together, our postulates require that the accessible vacuum energy E_i and the implied Hamiltonian H_i remain in balance, since both draw from the same maximum unmodulated vacuum energy, which cannot increase. The Fock-space construction provides a familiar reference for this balance. The ground state defines the vacuum, and excitations correspond to raising the energy through creation operators. Thus, for a probe with **unit mass** and **at rest**,

$$\boxed{H_i = E_g + E_{\text{cas}}} \tag{5}$$

$$\boxed{E_{\text{vac}}^{\max} = E_1 + H_1 = E_0 + H_0} \tag{6}$$

becoming our teeter-totter of energy for rest coupling. Our baseline for normalization and comparison is E_0 representing the max theoretically possible accessible vacuum energy in a realistic environment with minimal H_0 and coupling to create a stable record in the field's degrees of freedom for observation. The balance dictates that as the accessible vacuum energy is restricted the Hamiltonian must go up commensurately with that restriction defined by our max theoretically physically realizable E_0 accessible vacuum energy. This E_0 is greater for a free particle in Minkowski space because we can ignore the g and Casimir terms in E_i. The greater the E_i the larger the capacity to interact with the vacuum is, resulting in a faster (larger) proper time.

5.3 Measuring the Capacity to Interact with the Vacuum

The coupling to the accessible vacuum E_i may only care about the count and not the intensity of the interactions. So we introduce a monotone function f that creates a relationship to E_i:

$$\mathbb{E}N_i = f(E_i), \quad f'(E_i) > 0$$

where $\mathbb{E}N$ is the formal measure of the expected capacity to interact with the vacuum energy and its impact on proper time τ. To prevent confusion with E_i we will continue to write just N for its expected value. In local linearization:

$$N_i \approx \kappa_{\text{loc}} E_i$$

where κ_{loc} has dimensions of inverse energy. In the absence of experimental data we can take a reasonable guess at what this might be. We can get an estimate for κ by comparing the UV-cutoff regularized expressions for the total number of EM field modes and the vacuum energy, which has the necessary units of inverse energy to relate $f(E_i)$ to τ

$$\kappa_{\text{loc}} = \frac{N_{\text{modes}}}{E_{\text{vac}}} = \frac{8}{3\hbar c k_{\max}}$$

Our ansatz for measuring the capacity to interact with the vacuum therefore becomes

$$\boxed{\tau_i = N_i = \int_V \mathcal{N}[\Psi; g, r, \mathcal{B}] \cdot \rho_i[\Psi; g, r, \mathcal{B}]\, d^3r} \tag{7}$$

Where $\mathcal{N}[\Psi; g, r, \mathcal{B}]$ has dimensions of [energy^{-1}volume^{-1}] and represents the mode-energy density factor. In flat Minkowski space

$$\mathcal{N}_{\text{flat}} = \frac{8}{3\hbar c k_{\max}}. \tag{8}$$

For spatially homogeneous systems

$$N_i = \mathcal{N} \cdot E_i. \tag{9}$$

It is worth noting that one could attempt to define the interaction capacity N in more explicit physical terms, for example through the local density of states (LDOS), spectral shifts, or the TGTG scattering formalism that underlie Casimir-type energies. Each of these representations expresses the accessible vacuum energy in terms of mode structure. However, such formulations depend on the existence of a well defined mode basis and therefore fail to remain covariant in curved spacetime, where the notion of a global vacuum or complete set of modes no longer applies. To avoid this representational dependence, we therefore treat N as a functional of the accessible vacuum energy E_{vac} itself. This preserves general covariance and allows the same framework to encompass both Casimir-like environments and gravitational settings, where $\langle T_{00} \rangle_{\text{ren}}$ determines the local vacuum energy deficit. The abstraction to energy accessibility is not a simplification, but a necessary generalization that maintains physical and mathematical consistency across diverse regimes, while remaining agnostic about the ultimate covariance of the formulation. It should be regarded as a provisional construct, sufficient until such time as the mathematical framework is restructured to accommodate the ideas of our postulate, should experimental evidence warrant it.

5.4 Hamiltonian Constraints from Vacuum Modulation

Any specific τ_i implies a specific E_i and H_i that must be enforced and can be related back to the baseline. In other words, for some environment with a gravity field g the H_i is determined directly from the energy deficit g creates with the accessible vacuum energy. The energy flows back and forth through the T and V terms of H as they normally would. It could be represented as a large potential from a large distance away from the field or an increase in the mechanical energy as it approaches the field. The same analysis for the action of a Lagrangian can be used but its energy is locked in exactly by H_i determined by the modulation to E_i implied by that environment for some probe state Φ.

Are we implying τ is an energy ratio with this construction? No, this would misinterpret the role of $f(E_i)$. Although we do establish a connection to energy with that function, it is actually establishing a formal relationship to c. The expected value of N_i represents the ticks of sampling and is defensible in the same way that defining the SI seconds as 9,192,631,770 cycles of Cs-133 radiation is.

The dependence of the local cadence on the accessible vacuum energy also resonates with Rovelli's *thermal-time hypothesis* [4, 5, 6], in which the flow of time is generated by the statistical state of the universe rather than by an external parameter. In the present formulation, the functional $f(E_i)$ plays the same structural role. It defines the flow of proper time from the system's capacity to couple to the vacuum state, providing a measurable analogue of Rovelli's state-dependent temporal flow.

5.5 Normalization Baseline E_0

Our baseline E_0 must represent a well defined minimal coupling to the vacuum that can create observable records in its degrees of freedom. For this purpose, we adopt the Unruh-DeWitt (UDW) detector framework [2, 3], which provides a canonical model for how localized quantum systems couple to field modes.

The UDW detector is a two-level quantum system that couples linearly to a quantum field. Despite its simplicity, it captures the essential physics of how matter interacts with the vacuum and has been extensively studied in both flat and curved spacetimes. The detector's coupling strength is characterized by an effective temperature $T_U = \frac{a}{2\pi}$, where a is the proper acceleration. In the limit of weak coupling and large interaction time, the UDW detector thermalizes with the vacuum, making it equivalent to a black body radiator at temperature T_U.

A minimal probe must be able to exchange energy with the vacuum modes at a rate sufficient to establish thermal equilibrium. The UDW framework ensures our baseline E_0 represents the accessible vacuum energy for a probe with minimal but non-zero coupling.

We therefore define:

$$E_0 \equiv \int_V \rho_{\text{vac}}[\Psi_{\text{UDW}}; \eta_{\mu\nu}, r_\infty] d^3r \tag{10}$$

$$H_0 \equiv \int_V [\mathcal{H}_{\text{UDW}}] d^3r \tag{11}$$

where:

- $\eta_{\mu\nu}$ is Minkowski metric
- r_∞ indicates asymptotic flat space
- Ψ_{UDW} is minimal coupling state for Unruh-DeWitt detector with coupling characterized by $T_U = \frac{a}{2\pi}$
- $\mathcal{H}_{\text{UDW}}$ represents the minimal Hamiltonian density required for the detector to maintain coherent coupling to the field

This baseline represents the maximum accessible vacuum energy in flat spacetime for a probe with minimal structure, and is the theoretical upper bound against which all other configurations are compared.

6 Special Relativity

We have asserted that proper time is defined by the access a probe has to vacuum energy E_i, as given by $f(E_i) = N_i$ representing its capacity for interaction. The physical justification is that as we increase a particle's momentum it has less access to the vacuum energy. Changes to the structure of the vacuum resulting in E_i also propagate at c. This becomes normal relative motion. Also, consider a fast-moving particle crossing a standing wave. It experiences the oscillating field for a shorter duration, so the net energy transfer is smaller and thus the capacity to interact with the field is reduced, resulting in a commensurate reduction given in our N.

6.1 Lorentz Invariance

Before we discuss how the ratio of v to c affects our diagrams we must first establish Lorentz invariance because the actual ratio is unimportant here. If we compare the timing diagrams for two different worldlines we can see this invariance manifest visually.

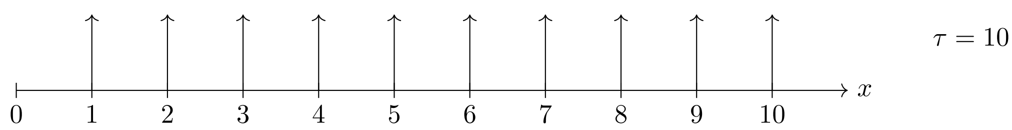

Figure 3: Worldline γ_0 for particle e_0 in its frame of reference.

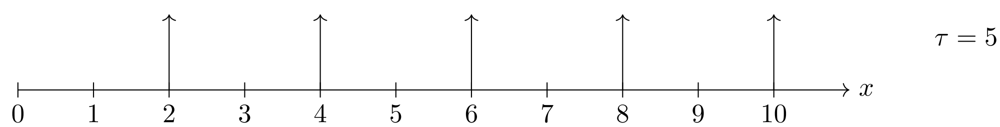

Figure 4: Worldline γ_1 for particle e_1 according to e_0.

As simple as these diagrams are, we have an elegant and concise story about Lorentz transformations for proper time, as long as we add a few rules on how to interpret them. Removing the null events from any diagram is the act of transforming to that probe's frame of reference. To transfer another diagram e_0 to some other probe's frame of reference e_1 you first compare the relative τ values. If $\tau_0 < \tau_1$, add evenly spaced null events. If $\tau_0 > \tau_1$, add evenly spaced active events. If they are equal, then no modification is necessary. In all cases, fractional events are allowed, which brings up an interesting duality of the implied discrete sampling and (seemingly) continuous nature of comparing reference frames. A timing diagram for a single particle can be described in $\mathbb{Z}_{\geq 0}$. However, when comparing probe cadence ratios of proper time we require the rational numbers $\mathbb{Q}$ at a bare minimum. This set is dense in $\mathbb{R}$, but not continuous.

In any case, it should be clear that since we have established an invariance of proper time where $v = dx/dt$, we can show Lorentz invariance from

$$c^2 d\tau^2 = c^2 dt^2 - dx^2.$$

6.2 Ratio of Proper Time

Invariance occurs naturally and intuitively within this new framework. But we require consistency with the empirically established proper-time ratio because this represents our observations of what the capacity to interact with the vacuum must be as a function of velocity as it relates to c. We use the results of that empirical evidence to enforce the conditions on N_i that we know must be true from observations of proper time if $N_i = \tau_i$.

$$d\tau = \frac{dt}{\gamma} = dt\sqrt{1 - \frac{v^2}{c^2}} = \delta N.$$

Proper time isn't just a convenient frame of reference or an abstract view. It represents a physical measure of the actual connection that the probe has with the universe. It determines the physical experience of length. The interaction is the structure of that space. Length contraction becomes an intuitive byproduct of that notion. The need to bind spacetime as a single word almost demands that proper time is a measure of the probe's access to that structure. Probe sampling is the metric. It is the structure. One is not separate from the other.

It is helpful to make relative aging explicit with the following form: [2]

$$\frac{\Delta\tau_1}{\Delta\tau_0} = \frac{\gamma_1^{-1}}{\gamma_0^{-1}} \quad \Rightarrow \quad \Delta\tau_1 = \frac{\gamma_0}{\gamma_1}\Delta\tau_0. \tag{12}$$

We take a slight detour to show that although a square root is involved in γ implying the need for $\mathbb{R}$, this is not the case if we restrict ourselves to only representing the ratio between two τ, and this is exactly what we are doing with our timing diagrams.

$$\text{Since } \gamma_i^{-1} = \sqrt{1-\beta_i^2} \text{ with } \beta_i = v_i/c \in \mathbb{Q}, \; \frac{\Delta\tau_1}{\Delta\tau_0} = \sqrt{\frac{1-\beta_1^2}{1-\beta_0^2}} \in \overline{\mathbb{Q}} \cap \mathbb{R},$$

so all possible proper-time ratios lie within the real algebraic numbers. We do not need the full continuum of $\mathbb{R}$.

In Minkowski space this corresponds directly to:

$$\frac{\Delta\tau_1}{\Delta\tau_0} = \frac{\gamma_1^{-1}}{\gamma_0^{-1}} = \frac{\Delta N_1}{\Delta N_0} \quad \Rightarrow \quad \Delta N_1 = \frac{\gamma_0}{\gamma_1}\Delta N_0. \tag{13}$$

Our baseline N_0 is always the rest coupling where the local physical speed is 0 producing $\gamma_0 = 1$ simplifying our ratio of proper time to

$$N_i = {\gamma_i}^{-1} N_0. \tag{14}$$

The vacuum energy action budget Eq. (6) requires that the probe be at rest. However, for some arbitrary non-zero velocity the energy expressed by the probe goes up by its speed related to its mass. We do not include this in the rest coupling budget because Minkowski space provides no mechanism to account for m (see Section (8.1)). The lack of coupling implied by ${\gamma_i}^{-1}$ leaves that exact amount of energy for the increase in H_i as momentum increases due to an increase in $|v|$. This becomes the following relationship between some γ_i and the rest coupling $\gamma = 1$

$$\boxed{E_{\text{vac}}^{\max} = {\gamma_i}^{-1} E_i + \gamma_i H_i = E_i + H_i = E_0 + H_0,} \tag{15}$$

where the terms without γ represent the rest (unitary gamma) coupling. It is important to note that the field is frame independent. The adjustment through γ_i models the coupling cadence to the vacuum energy. It does not change the vacuum itself.

6.3 Accessible Energy Budget and Unit Mass

If we consider a particle in free space it has momentum $p = \gamma m v$, and with respect to lab time it has a relativistic Hamiltonian of

$$H = mc^2 \frac{dt}{d\tau}.$$

To create a fully symmetric and coordinate-free relation we can rewrite the ratio of proper times using a definition of H_i for some γ_i which leads to the lab frame canceling out as well as the gammas due to the proper time ratio

$$H_i = m_i c^2 \gamma_i = m_i c^2 \frac{dt}{d\tau_i} \quad \Longrightarrow \quad \frac{d\tau_1}{d\tau_0} = \frac{H_0/m_0c^2}{H_1/m_1c^2} = \frac{H_0/m_0}{H_1/m_1}.$$

As a result we have the fully symmetric relation

$$\frac{d\tau_1}{d\tau_0} = \frac{H_0/m_0}{H_1/m_i}.$$

[2] The reason for using $\gamma_1^{-1}/\gamma_0^{-1}$ in preference over γ_0/γ_1 is to emphasize the ratios as well as emphasize that we are interested in how N shrinks with an increase in v_1 shown by γ_1^{-1}. As we reformulate our thinking to $\tau = N$ we cleanly handle $v \to c$.

After substituting τ for N we can then solve for H_1 to give us any H_i with respect to H_0

$$H_i = \frac{m_i N_0}{m_0 N_i} H_0,$$

which grows in energy as the proper time N_i shrinks. As the capacity to interact with the vacuum E_i approaches $N_i \to 0^+$ the Hamiltonian H_i diverges to infinity. We can simplify this further for our baseline $i = 0$ where we assume a unit mass

$$H_i = \frac{N_0}{N_i} m_i H_0.$$

We are interested in H_i normalized to unit mass

$$\boxed{N_i H_i' = N_0 H_0 \quad \text{where} \quad H_i' = \frac{1}{m_i} H_i}$$

7 General Relativity as a Consistency Constraint

In curved spacetime, the accessible vacuum energy must vary with the local gravitational potential. The purpose of this section is not to derive the Schwarzschild metric or reproduce Einstein's equations, but to ensure that the accessible vacuum formalism remains consistent with the empirically established relationship between proper time and curvature. General Relativity provides an experimental constraint that fixes how $E_i(r)$ and $H_i(r)$ must balance to preserve the energy–action equivalence introduced by our postulates.

7.1 Empirical Constraint on Proper Time

In static spacetimes, General Relativity gives the relationship between coordinate time t and proper time τ for a clock at rest in the gravitational field:

$$\frac{d\tau}{dt} = \sqrt{g_{00}(r)}, \tag{16}$$

where $g_{00}(r)$ is the time–time component of the metric. For the Schwarzschild solution,

$$g_{00}(r) = 1 - \frac{2GM}{rc^2} = 1 - \frac{r_s}{r}, \tag{17}$$

with $r_s = 2GM/c^2$ the Schwarzschild radius. Equation (16) is an experimentally verified statement of gravitational redshift, and serves here as a constraint on the accessible vacuum energy.

7.2 Mapping Proper Time to Accessible Vacuum Energy

From Postulate 1, proper time is proportional to the capacity to interact with accessible vacuum energy:

$$d\tau = dN = f(E_i)\, dt.$$

For spatially homogeneous systems where f is locally linear, the ratio of interaction capacities equals the ratio of accessible energies:

$$\frac{N_i(r)}{N_0} = \frac{E_i(r)}{E_0}. \tag{18}$$

Consistency with the GR proper-time relation (16) therefore requires

$$\frac{E_i(r)}{E_0} = \sqrt{g_{00}(r)}. \tag{19}$$

Connection to the Lapse Function. In a static spacetime with line element $ds^2 = g_{00}(r)c^2dt^2 - g_{ij}(r)dx^i dx^j$, the metric lapse $N(r) \equiv \sqrt{g_{00}(r)}$ relates proper and coordinate time via

$$d\tau = N(r)dt.$$

Equations (19)-(17) then imply

$$\frac{d\tau}{dt} = \sqrt{g_{00}(r)} \equiv N_i(r) = f(E_i(r)), \tag{20}$$

the lapse acquires a microscopic interpretation. It encodes how accessible vacuum energy varies with position in the gravitational field. Gravitational redshift and vacuum-modified Hamiltonians are thus two limits of the same energy-action balance.

7.3 Gravitational Energy Deficit

Equation (19) implies that the presence of curvature reduces the accessible vacuum energy compared to its flat-space value E_0. We define this reduction as the gravitational vacuum energy deficit:

$$E_g(r) \equiv E_0 - E_i(r) = E_0(1 - \sqrt{g_{00}(r)}). \tag{21}$$

For Schwarzschild geometry, this becomes

$$E_g(r) = E_0(1 - \sqrt{1 - \frac{r_s}{r}}), \tag{22}$$

matching the expected dependence of gravitational redshift on potential depth. The same deficit appears in the renormalized energy density difference of QFT in curved spacetime:

$$\rho_g(r) = \langle T_{00}(r)\rangle_{\text{ren}}^{\text{curved}} - \langle T_{00}(r)\rangle_{\text{ren}}^{\text{Minkowski}}.$$

Thus, $E_g(r)$ quantifies the local reduction in vacuum energy accessibility due to curvature.

7.4 Energy–Action Balance under Curvature

The teeter–totter balance between accessible vacuum energy and Hamiltonian energy established in flat spacetime,

$$E_i + H_i = E_0 + H_0,$$

must hold in curved spacetime as well. Substituting $E_i(r) = E_0 - E_g(r)$ gives

$$\boxed{H_i(r) = H_0 + E_g(r),} \tag{23}$$

showing that as curvature suppresses accessible vacuum energy, the probe's Hamiltonian increases by an equal amount to maintain the invariant total.

7.5 Proper Time and Metric Consistency

Combining Eqs. (19) and (23), the accessible vacuum energy and Hamiltonian satisfy

$$E_0\sqrt{g_{00}(r)} + H_i(r) = E_0 + H_0.$$

The ratio of proper times follows immediately:

$$\frac{d\tau}{dt} = \frac{N_i}{N_0} = \sqrt{g_{00}(r)}, \tag{24}$$

as a direct manifestation of the vacuum energy–action balance. This demonstrates that General Relativity's observed time dilation is consistent with the modulation of accessible vacuum energy implied by our postulates.

7.6 Combining SR and GR

To unify the effects of relative motion and gravitational curvature within this framework, we adopt a covariant formulation where the accessible vacuum energy and Hamiltonian are defined as functionals over the probe's worldline $\gamma(\tau)$.

The accessible vacuum energy is given by the integral

$$E_i[\gamma] = \int_\gamma \rho_i[\Psi; g(x), \mathcal{B}(x)] ds, \tag{25}$$

where $ds = \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$ is the proper length element along the trajectory, and ρ_i is the local vacuum energy density (Eq. (2)), assuming negligible Casimir effects ($\rho_{\text{cas}} \approx 0$) for simplicity.

The interaction capacity follows as

$$N[\gamma] = \int_\gamma \mathcal{N}[\Psi; g(x), \mathcal{B}(x)] \cdot \rho_i[\Psi; g(x), \mathcal{B}(x)] ds, \tag{26}$$

with proper time emerging as $\tau = \mathbb{E}N[\gamma]$.

The energy-action balance holds invariantly over the worldline:

$$\boxed{E_{\text{vac}}^{\max} = E_i[\gamma] + H_i[\gamma] = E_0 + H_0,} \tag{27}$$

where $H_i[\gamma] = \int_\gamma \mathcal{H}_i[\Psi; g(x), \mathcal{B}(x)] ds$ is the integrated Hamiltonian density.

Locally, over an infinitesimal ds, the modulation linearizes, allowing relative motion to reduce sampling of the vacuum modes, akin to the γ^{-1} factor in flat space. Globally, the integral incorporates both motion and curvature through ds, reproducing the full relativistic time dilation:

$$\frac{d\tau}{dt} = \sqrt{g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}}. \tag{28}$$

In the SR limit (Minkowski metric), this reduces to Eq. (15). For static observers in curved space, it yields Eq. (23). For general motion in curved spacetime, the worldline integral ensures the balance without inconsistencies.

The identification $E_i/E_0 = \sqrt{g_{00}}$ gives a microscopic interpretation of gravitational redshift that parallels the relational derivations of Haggard and Rovelli [6], who obtained time dilation from differences in informational access to physical degrees of freedom. Here that relational accessibility is expressed explicitly through vacuum-energy deficits, bridging their statistical description with a field-theoretic mechanism.

8 Momentum Reinterpreted

The preceding sections examined how a probe's proper time emerges from its capacity to interact with the vacuum, considering how velocity (SR) and external gravitational fields (GR) modulate this accessibility. We now address a fundamental question deliberately postponed. How does the probe's own mass affect the vacuum structure it samples? We also address a clear inconsistency with the energy-action balance in SR for some arbitrary mass.

8.1 Self-Consistency and the Necessity of Gravitational Self-Energy

The energy-action budget derived for unit mass in Section 5 appears to encounter a fundamental inconsistency when extended to arbitrary mass in special relativity. For a probe with mass m and velocity characterized by γ, the Hamiltonian scales as $H_i = m\gamma_i c^2$, yet Minkowski space provides no mechanism to decrease the accessible vacuum energy E_i correspondingly. This would require $E_{\text{vac}}^{\max}$ to increase with mass, violating the fundamental principle that the maximum unmodulated vacuum energy is fixed.

This apparent paradox exposes an artificial constraint imposed by treating Minkowski space as physically realizable with massive objects. In nature, mass and spacetime curvature are inseparable. Every massive object, including the probe, generates a gravitational field that modifies the local vacuum structure. The inconsistency arises only when we attempt to place massive objects in perfectly flat spacetime, a configuration that cannot exist in general relativity or be physically realized.

8.1.1 Gravitational Self-Energy Contribution

For probes of arbitrary mass, the energy-action balance implies that any increase in the Hamiltonian must correspond to a proportional decrease in accessible vacuum energy. The covariant energy-action balance established in Eq. (27) provides the appropriate starting point for extending the framework to arbitrary mass:

$$E_{\text{vac}}^{\max} = E_i[\gamma] + H_i[\gamma] = E_0 + H_0. \tag{29}$$

This equality expresses the invariance of the total vacuum and Hamiltonian energy budget under both relative motion and gravitational curvature, integrated over the worldline $\gamma(\tau)$, but considers unit mass initially.

A probe contributes its own stress-energy to the vacuum field, further modifying the accessible vacuum energy in its vicinity. This contribution is the probe's gravitational self-energy, represented by an additional Hamiltonian term relative to the unit-mass baseline:

$$\Delta H_m[\gamma] = \int_\gamma (m-1)\mathcal{H}_0 \, ds, \tag{30}$$

where $ds = \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$ is the proper length element, and $\mathcal{H}_0$ is the minimal Hamiltonian density for the unit-mass case.

To preserve the invariance of Eq. (29), the accessible vacuum energy must decrease by an equal amount. This produces a local energy density of the form

$$\rho_i[\Psi; g, r, \mathcal{B}] = \rho_{\text{vac}}^{\max}[\Psi; g, r] - \rho_g[g, r] - \rho_g^{\text{self}}[g, r] - \rho_{\text{cas}}[\mathcal{B}, r], \tag{31}$$

and the corresponding integrated energy relation:

$$E_i^{(m)}[\gamma] = \int_\gamma \rho_i[\Psi; g(x), \mathcal{B}(x)] ds = E_{\text{vac}}^{\max} - E_g[\gamma] - E_g^{\text{self}}[\gamma] - E_{\text{cas}}[\gamma]. \tag{32}$$

Because the self-energy term is dynamically equivalent to the Hamiltonian increase $\Delta H_m[\gamma]$, we can express the total accessible-energy balance compactly as

$$\boxed{E_i^{(m)}[\gamma] = E_{\text{vac}}^{\max} - E_g[\gamma] - \Delta H_m[\gamma] - E_{\text{cas}}[\gamma]} \tag{33}$$

This formulation treats the probe's mass as an intrinsic modulation of the vacuum field rather than a static property. Its presence necessarily alters the local mode structure, producing a self-consistent deficit that balances the Hamiltonian energy required to sustain the probe's existence. The resulting $E_g^{\text{self}}[\gamma]$ represents the probe's stress-energy contribution in the broader energy-action framework.

Note the balance Eq. 29 defines $E_i[\gamma]$ and $H_i[\gamma]$ for a unit-mass probe. For probes of arbitrary mass m, we introduce

$$E_i^{(m)}[\gamma] \equiv E_i[\gamma] - \Delta H_m[\gamma], \qquad H_i^{(m)}[\gamma] \equiv H_i[\gamma] + \Delta H_m[\gamma],$$

where $\Delta H_m[\gamma] = \int_\gamma (m-1)\mathcal{H}_0 ds$ represents the additional Hamiltonian required to sustain the probe's self-generated vacuum deficit. This results in a two-step evaluation of solving $\{E_i[\gamma], H_i[\gamma]\}$ for unit mass, then applying $\Delta H_m[\gamma]$ to yield $E_i^{(m)}[\gamma]$, preventing circular dependence between E_i and H_i while preserving the invariant total $E_{\text{vac}}^{\max} = E_0 + H_0$.

It is essential to emphasize that neither $E_i[\gamma]$ nor $E_i^{(m)}[\gamma]$ are frame-dependent quantities. $E_i^{(m)}[\gamma]$ encodes a structural translation of the probe's own vacuum modulation through the field. This is a physical propagation of the modulation itself, not a change of reference frame. In the probe's rest frame, the modulation is static. In another frame, the same structure appears to move through the vacuum, analogous to how a stationary mass's gravitational field appears to flow past a moving observer. $E_i^{(m)}[\gamma]$ represents a single physical modulation of the vacuum field whose apparent translation depends on the observer's state of motion, while differences between observers reflect only alternate decompositions of that same evolving structure captured in the energy-action budget.

8.2 Momentum as Propagating Vacuum Modulation

Equation (33) shows that the probe's own mass introduces an additional Hamiltonian term $\Delta H_m[\gamma]$ that represents its gravitational self-energy. This energy does not merely reside within the probe. It corresponds to a local modulation of the vacuum field produced by the probe's stress–energy. As the probe moves, this modulation is continuously displaced through the vacuum, requiring energy to sustain the propagation of its own field structure. This dynamic process manifests as what we conventionally describe as momentum.

In this picture, momentum is not an intrinsic property carried by a particle through empty space but the energetic signature of a moving region of reduced vacuum accessibility. A massive probe at rest establishes a stationary self-deficit $E_g^{\text{self}}[\gamma]$. When it moves, that deficit pattern is translated modulation of local vacuum accessibility at velocity v. The Hamiltonian increase $\Delta H_m[\gamma]$ therefore represents the energy required to maintain this moving modulation coherently, ensuring that the probe's coupling to the vacuum remains self-consistent with the invariant total.

8.2.1 Local Field Modulation by Matter

Each infinitesimal displacement dx of the probe shifts the region of suppressed accessibility through the vacuum field. Maintaining continuity of this self-modulation demands a constant exchange of energy between the probe and the field. The integrated "cost" of this process is the probe's kinetic energy, expressed in its Hamiltonian as

$$H_i^{(m)}[\gamma] = \int_\gamma m\mathcal{H}_0 ds = m \int_\gamma c^2 \frac{dt}{d\tau} d\tau, \tag{34}$$

which now appears as the cumulative energy required to sustain the propagation of the mass-dependent vacuum modulation. In this framework, motion does not merely translate the probe through space. It continuously reorganizes the vacuum's accessible energy structure along its worldline.

9 Cosmological Constant as a Vacuum Energy Deficit

The accessible vacuum energy defined in this framework provides a natural interpretation of the cosmological constant. In General Relativity, a positive Λ contributes a uniform energy density that acts gravitationally as negative pressure, driving cosmic expansion. Within the present model, Λ does not represent a distinct physical substance or an unexplained residual. It arises inevitably from the global structure of the vacuum energy–action balance that governs proper time.

Quantum field theory, when regulated by a high energy cutoff, defines a maximum conceivable ground state energy of the vacuum, $E_{\text{vac}}^{\text{max}}$. However, not all of this energy is accessible for interaction. The two postulates introduced earlier demand a reciprocal relationship between a system's Hamiltonian H and its accessible vacuum energy E_i. The greater the excitation or curvature, the less vacuum energy remains available for interaction. This teeter–totter balance determines the rate at which proper time emerges for any probe $\tau = f(E_i)$.

In this view, the apparent missing vacuum energy is not lost, sequestered, or spent. It is structurally tied up in defining the very conditions that permit time to exist. Proper time arises naturally from the distribution of accessibility across the vacuum degrees of freedom. The accessible portion determines the local progression of τ, while the inaccessible portion defines the constraints on H.

On cosmological scales, the same global energy–action balance applies. The cumulative reduction in accessible vacuum energy imposed by matter, fields, and curvature manifests in Einstein's equations as the cosmological constant Λ. Thus, Λ represents the universe-wide signature of the same structural balance that produces relativistic time dilation locally. Both effects are different expressions of a single principle. The modulation of vacuum accessibility gives rise to proper time and the deficit compared to unmodulated vacuum manifests through H_i and the large-scale dynamics of cosmic expansion.

10 Vacuum-Modified Hamiltonians and the Schrödinger Foundation of Proper Time

We already define time in physics by counting oscillations of a quantum system. The SI second is set by 9, 192, 631, 770 cycles of the Cs-133 hyperfine transition, each tick marking one full round of the atom's internal state evolution with respect to the electromagnetic field.

A Rydberg atom can serve as a clock in exactly the same way. Its transition frequency tells us how fast its joint state with the field cycles between "excited" and "ground" correlations. If we place that same atom inside a cavity, we know the local vacuum mode structure changes, and its measured frequency shifts. The usual interpretation is that the cavity alters the atom's energy levels, nothing more than a Hamiltonian correction.

But we can ask a deeper question. If the clock rate of Cs-133 defines time itself, and the Rydberg system is also a clock, then what does it mean when that clock slows down or speeds up inside a cavity? Is that not a direct change in the local rate of proper time? But that view confidently assumes that the ticking of the atom is not itself what we mean by time. If the Cs-133 hyperfine oscillation defines the second, and the Rydberg system is another quantum clock, then how can the same kind of frequency shift suddenly stop measuring time when it happens in a cavity? What distinguishes one as a measure of time and the other as merely an energy adjustment? The answer is not experimental evidence or rigorous proof. The answer is that it is convention to maintain consistency with an assumption about time. If every observable tick of an atomic transition reflects the rate of a system's state evolution, then a change in that rate must represent a change in proper time.

Conventional thinking stops at the energy shift, treating it as a passive adjustment in level spacing. Here, we treat the same observation as evidence that the system's cadence of state change is a different pattern of vacuum correlation per tick. The frequency shift is not just a number in the Hamiltonian. It is a change in the physical tick rate that defines time for that subsystem, because state evolution and that correlation define the system's existence.

10.1 The Schrödinger Equation as the Universal Dynamical Law

All dynamical descriptions in quantum theory begin with the Schrödinger equation,

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = H |\Psi\rangle, \tag{35}$$

which defines how the total state of matter and field evolves with respect to an external parameter t. The equation itself does not specify what time is. It simply orders the sequence of state changes generated by the Hamiltonian. Operationally, the only quantity that changes with t is the quantum state. Therefore, the flow of time is synonymous with the progression of state evolution.

Reparametrized Schrödinger Dynamics. For nonrelativistic systems and the unitary sector of high-Q cavity experiments,we adopt a proper-time parametrization of dynamics,

$$i\hbar \frac{\partial}{\partial \tau} |\Psi\rangle = H_0 |\Psi\rangle, \qquad d\tau = f(E_i(t))dt, \tag{36}$$

which is equivalent to the t-picture with an effective Hamiltonian $H_{\text{eff}}(t) = f(E_i(t))H_0$. The observed cavity frequency shift is then a direct measurement of the local cadence $d\tau/dt = f(E_i)$, the system's proper-time rate.

10.1.1 Cadence versus Energy: The Physical Meaning of $f(E_i)$

The reparametrization $d\tau = f(E_i)dt$ does not alter the intrinsic generator of evolution H_0. In the system's own proper time, dynamics remain governed by

$$i\hbar \frac{\partial}{\partial \tau} |\Psi\rangle = H_0 |\Psi\rangle,$$

while $f(E_i)$ specifies how the subsystem's internal cadence relates to the continuous evolution of the surrounding vacuum field. When expressed in coordinate time (t), this relation appears mathematically as a rescaling of the Hamiltonian,

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = f(E_i)H_0|\Psi\rangle .$$

However, the physical content lies not in an energy-level shift of H_0 itself, rather in the change of the sampling ratio $d\tau/dt$. The system's clock is the vacuum field's own evolution at c. $f(E_i)$ measures what fraction of that universal cadence is effectively sampled as correlated state change. A reduction in accessible vacuum energy lowers this sampling density, so the subsystem accumulates fewer phase correlations per unit of coordinate time.

Thus, cadence and energy appear jointly through observation. Altering one necessarily changes the other, but the genuine physical modulation is the cadence $d\tau/dt$, not the intrinsic Hamiltonian spectrum. The dependence of f on both the field's local phase structure and its density of states makes this correspondence subtle. Vacuum accessibility governs not only how often correlations occur but also the range of modes available for those correlations to occur within.

Scope (Dirac Fields). For relativistic spinor fields the replacement $t \to \tau$ must be implemented covariantly. In static spacetimes, the Dirac equation couples to the metric via tetrads $e_a{}^{\mu}$ and a spin connection. The proper-time scaling enters through the lapse, $d\tau = \sqrt{g_{00}}dt$ (i.e. $f(E_i) = \sqrt{g_{00}}$) rather than by rescaling ∂_t by hand. A full spinor-covariant treatment follows the standard curved-spacetime Dirac formalism and is consistent with Eq. (20). Its development is deferred.

10.2 Rydberg Systems as Empirical Illustrations

High-Q cavity experiments with circular Rydberg states illustrate this point vividly. The transition frequency of a Rydberg atom depends on the density of vacuum modes near its resonance frequency. When the atom is placed in a resonant cavity, the altered local density of states modifies the effective Hamiltonian,

$$H_{\text{eff}} = H_0 + \delta H_i, \tag{37}$$

changing the rate at which the atomic superposition evolves in Hilbert space. The measured shift of roughly ± 100 kHz at $\nu_0 \approx 51$ GHz corresponds to a fractional change of order 10^{-6} in the rate of the system's internal phase evolution. What the experiment directly records is a change in the cadence of state evolution, the number of oscillations per external second, when the atom's coupling to the vacuum is modified.

Energy–Action Balance in Cavity Conditions. The sign of the vacuum modification determines how the accessible energy, proper time, and Hamiltonian respond. A resonant cavity enhances the density of accessible modes, while an off-resonant or sub–cutoff cavity suppresses them. The energy–action balance relations $E_{\text{vac}}^{\max} = E_i + H_i$ and $\tau = f(E_i)$ then imply:

Condition	E_{cas}	E_i	τ	H_i (to balance)
Cavity resonant (enhanced modes)	< 0	$\uparrow$	$\uparrow$	$\downarrow$
Cavity suppresses modes	> 0	$\downarrow$	$\downarrow$	$\uparrow$

When the cavity enhances available vacuum modes ($E_{\text{cas}} < 0$), the probe gains access to greater vacuum energy, increasing its interaction capacity N_i and therefore its proper time τ. To preserve the invariant total $E_{\text{vac}}^{\max}$, the probe's Hamiltonian contribution H_i must decrease. Conversely, a cavity that suppresses vacuum modes $E_{\text{cas}} > 0$ reduces E_i, shortening τ and requiring a compensating increase in H_i. This bidirectional behavior is consistent with both the observed sign of cavity-induced frequency shifts and the postulated energy–action balance.

10.3 Correlation Dynamics of the Atom–Field State

The Rydberg transition is not an isolated atomic process but a periodic exchange of excitation and phase coherence between the atom and the surrounding vacuum field. The relevant quantum state is the joint superposition

$$|\Psi\rangle = \alpha\,|e\rangle\,|0\rangle + \beta\,|g\rangle\,|1_{\mathbf{k}}\rangle + \cdots, \tag{38}$$

in which virtual photons are continually exchanged and the joint amplitude oscillates between "atom excited, field vacuum" and "atom ground, one photon in the field." Each full oscillation corresponds to a complete correlation cycle of the atom–vacuum composite system. Standard treatments trace over the field, reducing this process to an effective two-level Hamiltonian and describing it as a mere energy shift. In the present view, the cadence of these correlation cycles is the measure of proper time for that subsystem. The tick rate directly reflects how efficiently the system re-correlates with the accessible vacuum modes. A cavity alters that cadence by reshaping the vacuum mode structure, producing fewer or more correlation cycles per external second.

Proper-Time Interpretation. If we take the dependence of the effective Hamiltonian on the vacuum seriously, then the observed phase-evolution rate of a Rydberg atom already measures its proper time. The term δH_i in $H_{\text{eff}} = H_0 + \delta H_i$ arises solely from the local vacuum mode density, and the corresponding change in the atomic coherence frequency $\omega_{\text{eff}} = \langle H_{\text{eff}}\rangle/\hbar$ is therefore a direct measure of the cadence at which the atom re-correlates with accessible vacuum modes.

In this view, the fractional frequency shift $\delta\omega/\omega_0$ observed in cavity experiments quantifies the fractional change in the system's proper-time rate,

$$\frac{d\tau}{dt} = f(E_i(t)) = 1 + \frac{\delta\omega}{\omega_0}. \tag{39}$$

Thus, what is conventionally described as a Lamb-type energy shift is, in physical terms, a change in the local rate of proper time.

Reduced Dynamics and Rate Scaling. For an effectively Markovian environment, the reduced state ρ obeys

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_{\text{eff}}(t), \rho] + \mathcal{L}(\rho), \qquad H_{\text{eff}}(t) = f(E_i(t))H_0. \tag{40}$$

Changing variables using $d\tau = f(E_i)dt$ gives

$$\frac{d\rho}{d\tau} = -\frac{i}{\hbar}[H_0, \rho] + \frac{1}{f(E_i)}\mathcal{L}(\rho). \tag{41}$$

Thus, unitary phases advance per τ with generator H_0, while dissipative rates scale as $1/f$. If all dynamical cadences (e.g. Rabi flops, Ramsey fringes, decoherence rates) change by the same fractional factor f, the effect is operationally indistinguishable from a change in the local proper-time rate.

10.4 Functional Definition of $f(E_i)$ from Field Correlations

The preceding sections established that proper time corresponds to a system's cadence of correlation with the vacuum field. To express this covariantly, we require a formulation that does not rely on a mode basis or a global notion of time. Quantum field theory provides precisely such a description through the Wightman function $G^+(x, x')$, which encodes the two-point vacuum correlation structure. The rate at which a probe re-correlates with the field along its worldline defines its local proper-time scaling.

Let $G_i^+(x, x') = \langle 0_i|\hat{\phi}(x)\hat{\phi}(x')|0_i\rangle$ denote the Wightman function of the quantum field in the local vacuum state $|0_i\rangle$. For a probe following worldline $\gamma(\tau)$, the field correlation along that trajectory is

$$C_i(\Delta\tau) = \langle 0_i|\hat{\phi}(\gamma(\tau))\,\hat{\phi}(\gamma(\tau + \Delta\tau))|0_i\rangle. \tag{42}$$

We define the microscopic correlation rate of the probe with the vacuum as the proper-time derivative of this correlation at coincidence:

$$\chi_i \equiv \frac{d}{d(\Delta\tau)} C_i(\Delta\tau)\Big|_{\Delta\tau=0}. \tag{43}$$

This operational definition of proper-time flow through field correlations connects naturally with the modular-flow formulation of Connes and Rovelli, where the Tomita-Takesaki modular operator of a statistical state defines an intrinsic notion of time. The derivative of the Wightman function in Eq. (44) can be viewed as the local, covariant representation of that modular flow, restricted to the vacuum correlations accessible along a specific worldline.

In this definition, the ratio

$$f(E_i) = \chi_i/\chi_0$$

acts as the Jacobian between the local vacuum correlation rate and its unperturbed Minkowski value, making $f(E_i)$ a directly measurable quantity derived from the field's two-point function.

The proper-time scaling functional $f(E_i)$ is then identified as the normalized ratio

$$\boxed{f(E_i) = \frac{\chi_i}{\chi_0} = \frac{\frac{d}{d(\Delta\tau)} G_i^+(\gamma(\tau), \gamma(\tau+\Delta\tau))\Big|_{\Delta\tau=0}}{\frac{d}{d(\Delta\tau)} G_0^+(\gamma(\tau), \gamma(\tau+\Delta\tau))\Big|_{\Delta\tau=0}},} \tag{44}$$

where subscript 0 refers to the unmodulated Minkowski vacuum.

This construction is manifestly covariant and yields the correct limits:

- In flat spacetime, $G_i^+ = G_0^+ \Rightarrow f(E_i) = 1$.
- In static curved spacetimes, G_i^+ redshifts by the lapse $\sqrt{g_{00}} \Rightarrow f(E_i) = \sqrt{g_{00}}$.
- In cavity-modified vacua, boundary-condition changes in G_i^+ reproduce the observed frequency shifts of quantum probes.

In this form, $f(E_i)$ represents the normalized rate at which the vacuum re-correlates with the probe along its worldline, providing a covariant operational definition of proper time as emergent vacuum accessibility.

11 Experimental Outlook

A practical falsification test arises from the Rydberg–cavity system itself. When a Rydberg atom interacts with the quantized field inside a high-Q cavity, its transition frequency reflects the local vacuum mode structure determined by the cavity geometry and dielectric boundary conditions. Standard QED treats this energy shift as purely geometric, fixed in the cavity's rest frame and unaffected by uniform motion.

Such tests align with recent quantum-clock interferometry experiments and proposals that seek to observe relational or entanglement-induced time dilation [7, 8, 9]. Those works pursue the same underlying question of emergent proper time as a possible correlation-defined quantity, but do so within composite quantum systems. The present framework extends that philosophy to the vacuum itself, and the same cavity–atom system defines its own proper time through the cadence of its vacuum correlations. When the entire cavity system moves inertially, the measured transition frequency in an external frame is predicted to scale with the local sampling factor γ_i^{-1}, preserving the internal dynamics but altering the externally observed tick rate.

Observation of a γ_i^{-1} modulation of the cavity-induced frequency shift, measured between co-moving and stationary configurations, would directly test whether vacuum-defined energy shifts transform only as geometric quantities or as true measures of proper time.

11.1 Experimental Feasibility and Integration Time

The predicted effect scales as $\gamma^{-1} = \sqrt{1 - v^2/c^2}$ for laboratory velocities. While this fractional deviation appears small, the measurement does not require instantaneous frequency resolution but rather accumulated phase difference over integration time T.

For a Rydberg transition at $\nu_0 = 51$ GHz moving at $v = 300$ m/s relative to a stationary reference, we have $\gamma^{-1} = 0.999999999999499$ and fractional deviation $1 - \gamma^{-1} = 5.01 \times 10^{-13}$. Over an integration time of $T = 3600$ s (1 hour), this accumulates to:

$$\Delta N_{\text{cycles}} = \nu_0(1 - \gamma^{-1})T \approx 92 \text{ cycles.} \tag{45}$$

Velocity	Integration Time	ΔN (cycles)
300 m/s	1 hour	92
300 m/s	24 hours	2,206
1 km/s	1 hour	1,021
3 km/s	1 hour	9,191

Table 1: Accumulated cycle differences for various experimental configurations at $\nu_0 = 51$ GHz.

This accumulated cycle count difference is readily measurable with standard frequency counters. Integration times of order hours are routine in precision spectroscopy. The experimental challenge lies primarily in maintaining phase coherence, controlling systematic effects (Doppler shifts, vibration, thermal drift), and ensuring clean comparison between co-moving and stationary cavity configurations, rather than in frequency resolution itself.

At higher velocities achievable with rotating cryogenic platforms ($v \sim 1$–3 km/s), the effect scales quadratically, reducing required integration times proportionally while maintaining feasibility within existing cavity QED technology.

Relation to Self-Gravity and Matter-Wave Interferometry. Recent simulations of the Schrödinger–Newton equation by Sahoo *et al.* [10] provide complementary evidence for the present framework. Their results show that as the particle mass increases (approaching 10^{10} atomic units), interference visibility is progressively suppressed and fringe width is reduced, while no configuration produces an enhancement. In the language of the current model, this behavior corresponds to a strictly positive self-gravitational vacuum deficit $E_g^{\text{self}}[\gamma] > 0$, which reduces the accessible vacuum energy $E_i^{(m)}[\gamma]$ and thus the interaction capacity $N[\gamma] = f(E_i^{(m)}[\gamma])$. The corresponding slowdown of the probe's proper-time cadence agrees with the prediction that self-gravity can only induce redshifts, never blue-shifts, in τ.

By contrast, Casimir-type modulations may increase E_i through enhanced mode accessibility, producing a local blue-shift in proper time. The contrast between these two regimes constitutes a clear and falsifiable asymmetry: mass-induced vacuum deficits are intrinsically suppressive, whereas geometric or resonant boundary conditions can be either suppressive or enhancing. Future matter-wave interferometry at high masses therefore offers a direct experimental test of this principle, distinguishing between gravitational and geometric modulation of vacuum accessibility. For moving probes, the propagated self-deficit could further introduce velocity-dependent asymmetries, testable in relativistic setups.

Experimental Techniques and Platforms. A growing class of precision experiments is approaching the parameter regime where such effects become testable [11]. Bose *et al.* review recent advances in optomechanical, levitated, and hybrid quantum systems that allow coherent control of mesoscopic masses and interferometric readout of phase evolution at or near the quantum ground state. These developments provide the practical means to isolate the signatures predicted here. Shifts in phase or coherence rate arising from intrinsic self-gravity or engineered vacuum structure. Levitated nanospheres, optomechanical resonators, and high-mass interferometers in the 10^8-10^{12} atomic mass range already offer sufficient sensitivity to detect such minute redshifts in local cadence. For moving probes, the propagation of a mass-dependent vacuum deficit along the worldline could produce subtle, velocity related modulations of correlated vacuum noise.

Appendix

Symbolic Verification. All analytic relations and limits presented in this paper were verified using a SymPy-based consistency checker (`paper_check.py`), ensuring algebraic and numerical invariance across the Special- and General-Relativistic limits.

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