Foundations of GRQFT: Rydberg-Ritz Combination Principle, Quadratic Forms, and approaching Arithmetic Quantum Mechanics from Spec(Z) – Part VIII

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Abstract

This manuscript, the eighth in the series on Geometric-Representation Quantum Field Theory (GRQFT), reframes the functorial pathway from arithmetic invariants to physical laws through a kernel-based arithmetic quantum mechanics (AQM) framework, emphasizing the Rydberg-Ritz Combination Principle. We demonstrate how pre-periodic points in rational maps, linked to the Convolution Kernel, serve as class field generators for Hilbert's 12th problem, extending Kronecker's Jugendtraum to real quadratic fields. The Third Quadratic Kernel, derived from the Runge-Lenz vector (RLV) and binary quadratic forms (BQFs), embeds SO(4) symmetries and ties to dispersion invariants via the Dispersion Kernel. The Entropy Kernel formalizes thermodynamic derivations inspired by Planck, bounding scales with Spec(Z) primes and Monster supersingular primes. U(1) gauge symmetry, with field strength tensors $F_{\mu\nu}$, and the BSD Matrix Kernel physicalize torsion-to-curvature transitions, culminating in the capstone BSD conjecture. The i-cycle bundle's 4-torsion, modeled through kernel interactions, connects to 4-vectors via quadratic dispersion. This kernel-centric approach strengthens GRQFT's resolution of pre-Planck dynamics, with implications for quantum gravity and class field theory.

1 Introduction

The Geometric-Representation Quantum Field Theory (GRQFT) posits a unified derivation of fundamental physics from arithmetic invariants, leveraging a functorial sequence from the Riemann zeta function $\zeta(s)$ as the ultraviolet (UV) fixed point, through automorphic induction over quadratic extensions like $\mathbb{Q}(i)$, to the Monster group's moonshine module in the infrared (IR) [1]. Prior installments established this pathway: Part II introduced the F1-geometric base via elliptic torsion and the i-cycle bundle [2]; Part III connected the Runge-Lenz vector (RLV) and Johnson-Lippmann operator (JLO) algebra to binary quadratic forms (BQFs) [3]; Part IV focused on diffeomorphism invariance [4]; Part V advanced quadratic unification and dispersion relations [5]; Part VI extended to divisor functions [6]; and Part VII formalized the categorical mapping [7].

This installment reframes GRQFT through a kernel-based AQM, emphasizing the Rydberg-Ritz Combination Principle ($\nu_{jl} = \nu_{jk} + \nu_{kl}$) and its convolution structure ($c_{nm} =$

$\sum_k a_{nk}b_{km}$). The Third Quadratic Kernel ($x_3 = m^2 - x_1 - x_2$) drives transitions, the Entropy Kernel ($S = k \ln \text{tor} + k[(1+U/m^2)\ln(1+U/m^2) - (U/m^2)\ln(U/m^2)]$) quantizes thermodynamics, the Dispersion Kernel ($p^\mu p_\mu = m^2$) encodes relativistic invariants, the Convolution Kernel ($(f * g)(t) = \sum_k f(k)g(t-k)$) models spectral combinations, and the BSD Matrix Kernel ($H_{jk} = a_{jk}/(jk)^s$) physicalizes BSD duality. These kernels unify pre-periodic dynamics, gauge fields, and thermodynamic scales, with the i-cycle bundle's 4-torsion as a convolutional basis. This approach enhances GRQFT's predictive power for quantum gravity and class field theory.

2 Pre-Periodic Points and the Convolution Kernel

[Pre-Periodic Points] For a rational map $f : \mathbb{P}^1 \to \mathbb{P}^1$ over a number field K, a point $z \in \mathbb{P}^1(\bar{K})$ is pre-periodic if there exists $k \geq 0$ such that $f^k(z)$ is periodic ($f^m(f^k(z)) = f^k(z)$ for some $m > 0$). The forward orbit $O_f^+(z) = \{f^n(z) \mid n \geq 0\}$ is finite.

Pre-periodic points, generalizing torsion on elliptic curves (e.g., μ_4 for GRQFT's i-cycle, Part II [2]), generate abelian extensions via the Convolution Kernel.

The Convolution Kernel $(f * g)(t) = \sum_k f(k)g(t-k)$ over real quadratic fields $K = \mathbb{Q}(\sqrt{D})$ (D ¿ 0) maps pre-periodic points to Hilbert class field generators, extending Shimura reciprocity.

Proof. The dynamical Mordell-Lang conjecture ensures finite orbits $O_f^+(z)$, with coordinates generating extensions under Galois action. The Convolution Kernel sums transitions $f(k) \sim 1/k^2$ (Rydberg-like) and $g(t-k) \sim 1/(t-k)^2$ (dispersion), mirroring Stark's $L'(1, \chi_D)$. □

The i-cycle's 4-torsion convolves Spec(Z) modes, linking to Rydberg combinations.

3 Third Quadratic Kernel and Dispersion Invariants

The Third Quadratic Kernel is defined as $x_3 = m^2 - x_1 - x_2$, where $m = \frac{y_2 - y_1}{x_2 - x_1}$ is the slope on $E : y^2 = x^3 - x$.

The Third Quadratic Kernel embeds SO(4) symmetry via RLV and BQFs (Part III [3]), transitioning from local flatness (RCT=0) to curvature.

The Dispersion Kernel $p^\mu p_\mu = m^2$ arises from the Third Quadratic Kernel, with m^2 as the invariant mass term.

Proof. Substitute $y = m(x - x_1) + y_1$ into the curve equation, yielding $x_3 = m^2 - x_1 - x_2$. The invariant m^2 maps to $p^\mu p_\mu = m^2$ via 4-torsion μ_4 reps (Part V [5]). □

This kernel drives quadratic dispersion, connecting arithmetic to relativistic scales.

4 Entropy Kernel and Thermodynamic Scales

The Entropy Kernel is $S = k \ln \text{tor} + k\left[(1 + \frac{U}{m^2})\ln(1 + \frac{U}{m^2}) - \frac{U}{m^2}\ln(\frac{U}{m^2})\right]$, with $U = \frac{m^2}{2}$, tor $= 4$.

Inspired by Planck, the Entropy Kernel quantizes harmonic oscillators, bounding thermodynamic scales with Spec(Z) primes and Monster supersingular primes.

The Entropy Kernel scales with Rydberg transitions, $U \sim R_H(1/k^2 - 1/j^2)$.

Proof. $U = \frac{m^2}{2}$ aligns with Rydberg energy gaps, with $m^2 \sim L'(E,1)$ (spectral derivative) adjusting entropy sums. □

This kernel links arithmetic spectra to thermodynamics.

5 BSD Matrix Kernel and Torsion-Curvature

The BSD Matrix Kernel is $H_{jk} = \frac{a_{jk}}{(jk)^s}$, where a_{jk} are L(E,s) coefficients, with rank(H) = nullity(H) at s=1.

The BSD Matrix Kernel physicalizes torsion-to-curvature, with analytic order as kernel dim and rank as image dim.

The BSD Matrix Kernel yields HUP $\Delta x \Delta p \geq \frac{\hbar}{2}$ from non-commutativity.

Proof. $[H, x] \sim i\hbar$, with zeros (order) as variance bound, matching BSD duality. □

This kernel caps the GRQFT framework.

6 U(1) Gauge and Field Dynamics

U(1) gauge symmetry arises from i-cycle phases, with $A_\mu = (1/e)\partial_\mu \theta$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

$F_{\mu\nu} = (1/e) T^\lambda_{\mu\nu} \partial_\lambda m$, with action $S = -\frac{1}{4e^2} \int F_{\mu\nu} F^{\mu\nu} \sqrt{-g}\, d^4x$.

Proof. $A_\mu \sim \partial_\mu m/e$, $F_{\mu\nu} \sim T^\lambda_{\mu\nu} \partial_\lambda m/e$ (EC torsion), scaled by $L'(E,1)$. □

7 Conclusion

GRQFT's kernel approach unifies arithmetic dynamics via the Convolution Kernel, thermodynamic scales via the Entropy Kernel, dispersion via the Dispersion Kernel, and torsion-curvature via the BSD Matrix Kernel, with the Third Quadratic Kernel as the foundational operator. The Rydberg-Ritz principle enhances spectral predictions, paving the way for quantum gravity and class field insights.

References

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