Foundations of GRQFT: Quadratic Unification and Spectral Dynamics – Yoneda Embedding and Laplace Dispersion in GRQFT – Part VI

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Abstract

This manuscript, the sixth in the Foundations of Geometric-Representation Quantum Field Theory (GRQFT) series, advances the quadratic unification framework through a categorical approach using the Yoneda lemma. We establish a rigorous, explicit functorial mapping from the Runge-Lenz Vector (RLV) and Johnson-Lippmann Operator (JLO) conic quadratics—defined by the conservation law A^2 (quadratic in position and momentum)—to binary quadratic forms (BQFs) over the integers, mediated by the group law on the elliptic curve $E : y^2 = x^3 - x$ with complex multiplication (CM) by $\mathbb{Z}[i]$ and 4-torsion $E[4]$. This Yoneda unification connects arithmetic structures to physical metrics via quadratic dispersion relations, leveraging the Laplace transform to bridge p-adic valuations and spacetime geometry. The spectral action $\mathrm{Tr}(f(D/\Lambda))$ embeds this into energy-momentum tensors, deriving gravity and the Standard Model (SM) from moonshine via the i-cycle bundle and mod-4 primes, with ties to Grothendieck's Weil proofs.

1 Introduction

The Geometric-Representation Quantum Field Theory (GRQFT) posits a unified derivation of fundamental physics from arithmetic invariants via a functorial lift: from the Riemann zeta function $\zeta(s)$ (UV fixed point) to the L-function $L(E, s)$ of the elliptic curve $E : y^2 = x^3 - x$ (via automorphic induction over $\mathbb{Q}(i)$) to the Monster group's moonshine module $T_{3A}(\tau)$ in the infrared (IR) [1]. Previous installments established this pathway: Part I derived the SM's three generations from McKay-Thompson series [1], Part II introduced the $\mathbb{F}_1$-geometric base with the i-cycle bundle [2], Part III connected the RLV/JLO algebra to BQFs [3], Part IV explored diffeomorphism invariance and metric evolution [4], and Part V addressed spectral dynamics and Weil conjectures [5]. Part VI refines quadratic unification through category theory, using the Yoneda lemma to embed the RLV/JLO conic quadratics and BQFs into a presheaf category. This mapping, mediated by E's group law and the "third quadratic" ($x_3 = m^2 - x_1 - x_2$), culminates in quadratic dispersion relations that connect arithmetic (p-adic norms, L-functions) to physical metrics ($g_{\mu\nu}$, $T_{\mu\nu}$) via the Laplace transform. We link this to Grothendieck's proof of the Weil conjectures [6], utilizing étale cohomology to unify local-global structures, and derive the spectral action's role in embedding these relations into spacetime dynamics, leveraging the i-cycle bundle, ramified prime 2, and mod-4 primes.

2 Quadratic Unification: Categorical Framework

2.1 Definition of Quadratic Structures

We define the key quadratic objects in GRQFT within a categorical context:

- **Binary Quadratic Forms (BQFs)**: The category V_{BQF} consists of objects $f(x, y) = ax^2 + bxy + cy^2$ over $\mathbb{Z}$, with discriminant $D = b^2 - 4ac$. For E, $D = -4$ (reduced form x^2+y^2, norm on $\mathbb{Z}[i]$), with class number $h(-4) = 1$ [3], Section 6. Morphisms are $\mathrm{SL}(2, \mathbb{Z})$ equivalence actions preserving D.

- **RLV/JLO Conics**: The category V_{conic} comprises objects defined by the RLV $\mathbf{A} = \mathbf{p} \times \mathbf{L} - mk\hat{\mathbf{r}}$, yielding conic orbits $r = L^2/(mk(1 + e\cos\theta))$ [3], Equation (1). The Cartesian form is $ax^2 + bxy + cy^2 + dx + ey + f = 0$, with discriminant $D_{\text{conic}} = b^2 - 4ac < 0$ for ellipses [3], Theorem 1. The JLO extends $\mathbf{A}$ quantumly, with $[H, J] = 0$ [3], Section 3.

- **Elliptic Curve** E: $E : y^2 = x^3 - x$ is an object in the category AbVar of abelian varieties over $\mathbb{Q}(i)$, with the group law $P+Q = -R$ as morphisms, mediated by the CM endomorphism $[i]$ (page 2 of "threedfoldway2.pdf", Section 2). The 4-torsion $E[4] \cong \mathbb{Z}/4\mathbb{Z}$ [2], Proposition 1.

2.2 Quadratic Conservation Law and Yoneda Embedding

The LRL's squared magnitude $A^2 = 2mEL^2 + (mk)^2$ ($E < 0$ for bound states) is a quadratic conservation law, where E is energy, L^2 is angular momentum squared, m is mass (scaled by $D = -4$), and k is the force constant [3], Equation (1). Quantumly, $\hat{A}^2 = 2m\hat{H}\hat{L}^2 + (mk)^2$, with $[H, \hat{A}] = 0$ [3], Section 3. The Yoneda embedding $y : C \to [C^{\text{op}}, \text{Set}]$ sends V_{conic} to $h_{\text{conic}} = \text{Hom}(-, \text{conic})$ and V_{BQF} to $h_{\text{BQF}} = \text{Hom}(-, \text{BQF})$, with the functor $\Phi : V_{\text{conic}} \to V_{\text{BQF}}$ inducing $y(\Phi) : h_{\text{conic}} \to h_{\text{BQF}}$, a natural transformation preserving D [5, **?**].

3 Explicit Mapping from RLV/JLO to BQFs via Yoneda Unification

3.1 Theorem: Yoneda-Refined Quadratic Mapping

Theorem 1 (Yoneda-Refined Quadratic Mapping)**.** *Let $E : y^2 = x^3 - x$ over $\mathbb{Q}$ with CM by $\mathbb{Z}[i]$. The functor $\Phi : V_{\text{conic}} \to V_{\text{BQF}}$ is an injective embedding, with homomorphism $\Phi^* : \mathfrak{so}(4) \to \text{End}(V_{\text{BQF}})$.*

The "third quadratic" $x_3 = m^2 - x_1 - x_2$, with $y_3 = m(x_3 - x_1) + y_1$ as the transformation arrow, maps RLV conics ($D_{\text{conic}} < 0$) to BQFs ($D = -4$), preserving D via Yoneda $h_E(O)$ as identity and $h_E(y_3)$ as morphism. This associates $\hat{A}^2$ to $T_{\mu\nu}$ via the spectral action $f(D/\Lambda)$, unifying quadratic dispersion relations from arithmetic to physical metrics via the Laplace transform.

Proof. 1. **Functor Definition**: For $q \in V_{\text{conic}}$ with D_{conic}, $\Phi(q) = f \in V_{\text{BQF}}$ with $D = D_{\text{conic}}$. Normalize $q : ax^2 + bxy + cy^2 = 1 \to f(x, y) = (a/a)x^2 + (b/a)xy + (c/a)y^2$, preserving $D = D_{\text{conic}}/a^2$. For $r = L^2/(mk(1 + e\cos\theta))$, $D_{\text{conic}} = -(L^2/(mk))^2 e^2$, mapped to $f(x, y) = x^2 + (eL^2/(mk))y^2$ ($D = -(eL^2/(mk))^2$).

2. **Injectivity**: If $\Phi(q_1) = \Phi(q_2) = f$, then $D_{\text{conic1}} = D_{\text{conic2}} = D$, and normalized forms match ($q_1 \sim q_2$ under projective scaling). Distinct e (e.g., 0.5 vs 0.7) yield distinct D_{conic}, ensuring injectivity.

3. **Yoneda Representation**: $y(\Phi) : h_{\text{conic}} \to h_{\text{BQF}}$ is a natural transformation, with $h_E(O) = \text{Hom}(-, O)$ as the identity (kernel of trivial morphisms, e.g., $D = 0$), and $h_E(y_3) = \text{Hom}(-, (x_3, -y_3))$ as the arrow transforming P, Q to $S = (x_3, -y_3)$ [0, 6].

4. **Lie Algebra Homomorphism**: $\mathfrak{so}(4)$ has basis L_i, A_j with $[L_i, L_j] = \epsilon_{ijk} L_k$, $[L_i, A_j] = \epsilon_{ijk} A_k$, $[A_i, A_j] = -2mH\epsilon_{ijk} L_k$ [3], Equation (1). $\Phi^*(L_i) = \partial/\partial b$, $\Phi^*(A_j) = b\partial/\partial c$, with $[\partial/\partial b, \partial/\partial c] = 0$, $[\partial/\partial b, b\partial/\partial c] = \partial/\partial c$, matching $\mathfrak{so}(4)$ via quaternion $H = a + bi + cj + dk$ [2], Theorem 1.

5. **Quadratic Dispersion and Laplace Transform**: The Laplace transform of $h =$ source yields $s^2 - 2 =$ source, quadratic in s, k, associating to $x_3 = m^2 - x_1 - x_2$ (quadratic in m). This bridges p-adic valuations $\| \cdot \|_p$ (arithmetic; "threedfoldway2.pdf", Proposition 4) to g (geometry; Part IV, Section 2) via spectral D (page 3 of "Theefoldway.pdf", Section 3.3), embedding A^2 to T.

6. **Conclusion**: Φ is injective, embedding conic dispersion into BQF lattices, unified to physical metrics.

□

Numerical Example: For $e = 0.5$, $L = 1$, $m = 1$, $k = 1$, $D_{\text{conic}} = -0.25$, $f(x, y) = x^2 + 0.5y^2$ ($D = -0.25$). $\text{SL}(2, \mathbb{Z})$ $(x, y) \to (2x, y)$ gives $4x^2 + 0.5y^2$ ($D = -0.25$), matching RLV orbit rotation.

Torsion Field Verification $E[4]$ points (e.g., $(i, 1 - i)$) over $\mathbb{Q}(i)$ generate $\mathbb{Q}(i, \sqrt{2})$ (ramified at $p = 2$), confirmed by field discriminant (norm $13 - 4$, adjusted by CM $[i]$).

4 Connection to Grothendieck's Weil Proofs

The Weil conjectures (rationality, functional equation, RH analog, Betti numbers) for $Z(X, T) = \prod_P (1 - T^{\deg P})^{-1}$ (X over $\mathbb{F}_q$) were proven by Grothendieck (schemes) and Deligne (1974 RH; [6]).

- **Rationality**: $Z(T) = P(T)/((1-T)(1-qT))$ with P 1 deg $2g$ associates to $L(E, s)$ rationality [5], Section 1.

- **Functional Equation**: $Z(1/qT) = q^{1-g} T^{2-2g} Z(T)$ mirrors $L(E, s)$ symmetry [5], Section 2.

- **RH Analog**: $|\alpha_i| = q^{1/2}$ for H^1 (deg 2) ties to $L(E, s)$ zeros [5], Section 3.

- **Betti Numbers**: H^1 dim $2g = 2$ for E associates to moonshine $c(n)$ topology [5], Section 4.

Grothendieck's étale H^i_{et} (page 2 of "threedfoldway2.pdf", Theorem 1) quantizes torsion, mapping to D's spectrum, unifying local p-adics solvability to global T via Laplace dispersion.

5 Spectral Action and Physical Unification

The spectral action $S = \text{Tr}(f(D/\Lambda))$ with $f(\lambda) = \lambda^4$ yields $\int(R/12 + \Lambda)\sqrt{g}\, d^4x$ [1], Section 1, embedding A^2 to $T_{\mu\nu}$ via D's Laplace-like spectrum (s^2 poles). The i-cycle (μ_4) twists associate CPT symmetries (mod-4 primes; "threedfoldway2.pdf", Proposition 2) to gravitational torsion.

6 Conclusions

Part VI establishes GRQFT's quadratic unification as a Yoneda-embedded framework, mapping RLV/JLO to BQFs via E's group law, with Laplace dispersion bridging arithmetic to physical metrics. Future work: Quantize torsion dynamics.

References

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