Demonstrating a Possible Homeostatic Mechanism using Auxiliary Parameters to Represent the Metric Tensor

Stephen P. Smith, October 2025

Abstract. This paper develops a homeostatic model for the metric tensor by introducing an auxiliary-parameter representation that reflects the bilateral structure of CPT symmetry. Each side of the symmetry possesses a set of auxiliary parameters, $W^{(1)}$ and $W^{(2)}$, whose differences vanish at equilibrium, giving rise to the emergent metric tensor. Based on facts pertaining to the factorability of symmetric matrices, the metric is parameterized as $G = WDW^T$, where W is a lower-triangular matrix of ten independent parameters and $D = diag(-1, 1, 1, 1)$ encodes the Lorentzian signature. This factorable form interprets spacetime geometry as an adaptive interface maintained by homeostatic balance between conjugate sectors, rather than as a primitive geometric field. The framework unifies algebraic, geometric, and informational descriptions by linking the Gram-type form WW^T to the Fisher information matrix and to Frieden's ten amplitude functions. The result is a dynamically generative account of the metric tensor consistent with both Riemannian and Lorentzian regimes, providing a foundation for extending homeostatic principles to curvature and field dynamics.

Keywords: Auxiliary Parameters, CPT Symmetry, Fisher Information, Homeostatic Balance, Lorentzian Geometry, Metric Tensor, Mid-Parent Equation, Riemannian Geometry, Spacetime.

1. Introduction

A metric tensor need not be viewed as a primitive geometric object but may instead arise from a deeper set of auxiliary parameters whose coordination maintains internal balance. Because CPT symmetry implies that every physical process has a complementary counterpart, it is natural to assume that these auxiliary parameters occur in dual sets, one associated with each side of the symmetry. Each set may contain twice the number of degrees of freedom normally assigned to the tensor it supports, and this excess requires a regulating principle—a homeostatic mechanism—that restores equilibrium between the conjugate sectors.

Although the auxiliary parameters themselves need not transform as tensors, the *differences* between the dual sets can acquire tensorial character or can be constrained to do so. The homeostatic condition that drives these differences toward zero then gives rise to the proper tensor field. The metric tensor, therefore, can be interpreted as the emergent equilibrium state of a bilaterally constrained system of auxiliary variables.

An instructive precedent appears in the appendix of *Smith (2025)*, where the Christoffel symbols of the second kind serve as auxiliary parameters whose differences reconstruct the Riemann curvature tensor. The present paper applies the same logic to the metric tensor itself. By introducing paired metric fields $G^{(1)}$ and $G^{(2)}$ corresponding to CPT-conjugate manifolds, a single physical metric G is obtained through homeostatic interpolation. The analysis then proceeds in matrix form to expose the algebraic anatomy of this balancing process.

The central tool is the strong factorization of symmetric matrices (e.g., *Smith 2001)*, which ensures that any real, non-singular metric tensor admits a unique decomposition $G = WDW^T$ once the signature matrix D is fixed. This result allows the metric to be parameterized by a lower-triangular matrix W containing ten independent entries—the same number as the components of $g_{\mu\nu}$. The evolution of these parameters encapsulates the geometry's internal degrees of freedom and supplies the algebraic substrate for homeostatic balance between dual manifolds.

The ensuing sections demonstrate how this auxiliary-parameter approach recovers known geometric and informational structures. Section 2.1 establishes the dual-manifold interpolation identities; Section 2.2 introduces the dynamically factorable W-field that supports the metric tensor; Section 2.3 connects the framework to Fisher information and Roy Frieden's ten amplitude functions, showing that the same Gram form underlies both; and Section 2.4 points to analogies with the mid-parent equation in quantitative genetics, where dual contributions achieve statistical equilibrium. Together, these constructions reveal that homeostatic balance may be a universal mechanism linking geometry, information, and inheritance across physical and biological systems

2.1 Partitioning the Metric Tensor using Auxiliary Parameters

Define dual metric tensors, $g_{ij}^{(1)}$ and $g_{ij}^{(2)}$, that share identical coordinates. These represent dual CPT-conjugate manifolds, interpreted as sheeted structures. The goal is to derive one metric tensor from these as a result of homeostatic balance, given initially as an approximation: $g_{ij} \approx ½ g_{ij}^{(1)} + ½ g_{ij}^{(2)}$. It is more expedient to shift into matrix algebra notation, rather than following the standard for tensors. Therefore, define the metric tensors with these square matrices:

$$g_{ij} = (i,j)th\ element\ of\ G$$

$$g_{ij}^{(1)} = (i,j)th\ element\ of\ G^{(1)}$$

$$g_{ij}^{(2)} = (i,j)th\ element\ of\ G^{(2)}$$

Next define the square matrices $W^{(1)}$ and $W^{(2)}$. If these represent spacetime geometry then it is normally the case that these square matrices are of order 4, which will be assumed henceforth. The functional relation between $G^{(k)}$ and $W^{(k)}$ $(k = 1 \; or \; 2)$ is given by the following matrix equations.

$$G^{(1)} = W^{(1)} D \, W^{(1)T}$$

$$G^{(2)} = W^{(2)} D \, W^{(2)T}$$

Where D is a diagonal matrix which specifies the signature of the metric. If $D = diag(1,1,1,1)$, the metric is positive definite (the Gram form) and characterizes Riemannian geometry. If $D = diag(-1,1,1,1)$, the metric is indefinite and characterizes Lorentzian geometry and finds an application with general relativity.

Next define the interpolated matrix, $W = ½W^{(1)} + ½W^{(2)}$. A remarkable set of identities follow that relate the matrix G to the mechanism of homeostatic balance:

$$WDW^T = [½W^{(1)} + ½W^{(2)}]D[½W^{(1)} + ½W^{(2)}]^T$$

$$WDW^T = ¼W^{(1)}DW^{(1)T} + ¼W^{(2)}DW^{(2)T} + ¼W^{(1)}DW^{(2)T} + ¼W^{(2)}DW^{(1)T}$$

$$WDW^T = ½G^{(1)} + ½G^{(2)} - ¼W^{(1)}DW^{(1)T} - ¼W^{(2)}DW^{(2)T}$$

$$+¼W^{(1)}DW^{(2)T} + ¼W^{(2)}DW^{(1)T}$$

$$WDW^T = ½G^{(1)} + ½G^{(2)} - ¼\Delta D \Delta^T$$

where $\Delta = W^{(1)} - W^{(2)}$. At homeostasis we expect $\Delta \to 0$, $WDW^T = ½G^{(1)} + ½G^{(2)}$ and $G \equiv G^{(1)} \equiv G^{(2)}$. What is remarkable is that the algebra here is very similar to the partitioning found with the Riemann curvature tensor when it was populated with interpolated Christoffel symbols (Smith 2025). Moreover, there is this vague but uncanny resemblance to the mid-parent equation that describes additive genetic inheritance (see Section 2.4), and a more obvious connection with the Fisher information matrix and Frieden's (2004) 10 amplitude functions that are intended to underlie the metric tensor (see Section 2.3). But before looking at those a closer look at W is needed, if only because it is not uniquely defined. Resolution is found by factorizing the metric. To relate the auxiliary matrices $W^{(k)}$ to observable geometry, we also assume the metric on each sheet admits a dynamic factorization consistent with the signature specified by D.

2.2 The W-field that Supports the Metric Tensor

Because all empirically accessible measurements of G occur in time-like frames, the Lorentzian signature $D = diag(-1,1,1,1)$ is taken as fundamental. All such metric tensors can be partitioned into a strictly negative component g_{00}, and a positive definite part indicated by a 3×3 submatrix of G corresponding to the spatial components. Because of this special structure, the matrix G is strongly factorable (any row and column permutations that set the pivot order suffice) and can be put into the form $G = LDL^T$ where L is a lower-triangular matrix (Smith 2001). Once a pivot order is selected and fixed, L is uniquely determined and composed of real numbers. We need only set $W = L$, as an a priori parameterization of G and not as an algorithmic step that computes the factorization from G .

Therefore, *the* 4×4 Lorentzian metric tensor may be written in an alternative form $G = WDW^T$, where W is a nonsingular lower-triangular matrix and $D = diag(-1, 1, 1, 1)$ encodes the metric signature. This representation is unique once the sign convention is fixed and provides a complete parameterization of the ten independent degrees of freedom of $g_{\mu\nu}$. The entries of W thus constitute a dynamically sufficient set of auxiliary parameters whose evolution determines the geometry. Moreover, for paired or dual manifolds, variations in the metric are expressible directly as differences between their respective W-fields, enabling a first-order algebraic treatment of dual geometric evolution. The factorable form therefore establishes a firm foundation for viewing the metric tensor as a dynamically generative structure rather than a primitive field variable.

Having introduced dual matrices $W^{(1)}$ and $W^{(2)}$ in Section 2.1, we now specify their algebraic structure as the identical lower-triangular structure inherent in W. Although the matrices W and Δ are not tensors—owing to their lower-triangular structure and coordinate dependence—the combination,

$$G \approx WDW^T = ½G^{(1)} + ½G^{(2)} - ¼\Delta D\Delta^T \ ,$$

is tensorial because it is constructed directly from the legitimate metric tensors $G^{(1)}$ and $G^{(2)}$. The auxiliary quantity WDW^T therefore serves only as an interpolated representation of the metric, not as a tensor in its own right. At homeostasis, where $\Delta \to 0$, the non-tensorial contributions vanish and $G = WDW^T$ acquires full tensorial character. In this sense, the homeostatic condition functions analogously to the Euler–Lagrange constraint that restores covariance on-shell.

2.3 Fisher Information and Amplitude Functions

In Section 2.1 and prior to setting $W = L$, the Fisher information matrix was identified with the Gram form $G = WDW^T$ where $D = I$. However, this comparison involving $G = WW^T$ is not limited to the case where W is square. It could be rectangular and constructed from an array of auxiliary parameters $W_{\mu n}$ that encode the local sensitivities of a system's variables. This identification highlighted that the positive-definite nature of G mirrors that of the classical Fisher information matrix, which quantifies the curvature of a likelihood surface with respect to its parameters. The Gram representation thus revealed a geometric basis for information: each column of W acts as a differential vector spanning an information manifold, and the contraction WW^T defines a local metric over its parameter space.

Frieden's *Extreme Physical Information* (EPI) program generalizes this correspondence by replacing the discrete auxiliary array W with a continuous set of *amplitude functions* $q_n(x)$ defined over spacetime or configuration space. These amplitudes generate the probability densities $p_n(x) = q_n^2(x)$, and the Fisher information becomes an integrated functional,

$$\mathrm{I}_{\mu\nu} = \int d^4x\, \rho(x) \sum_n \left(\partial_\mu q_n\right)(\partial_\nu q_n) \quad ,$$

where $\rho(x)$ is a weighting measure. The integrand inside this expression is formally identical to the local Gram structure of Section 2.1 when one identifies $W_{\mu n} \propto \partial_\mu q_n$. Thus, the local quantity WW^T represents the pointwise Fisher information density whose spacetime integral yields the global Fisher matrix employed in the EPI formalism.

While the Gram form G is by construction positive-definite, physical theories such as relativity require an indefinite metric signature. Frieden achieves this not by altering the algebraic definition of the Fisher tensor but by contracting its indices with an underlying Minkowski metric, $\eta^{\mu\nu}$, within the information functional itself. Hence, Frieden uses the trace, W^TDW, rather than working directly with WW^T. In this way, the sign structure of spacetime is embedded directly in the variational principle. The dynamic-factorization framework developed in Section 2.2 permits an equivalent implementation by treating W as a 4×4 lower-triangular matrix with ten independent parameters and by fixing $D = diag(-1, 1, 1, 1)$. The dynamically factorable metric provides a constructive realization of Frieden's Fisher tensor rather than merely a formal analogy.

The EPI action, $\mathcal{A}$=$\mathcal{I}$−$\mathcal{J}$, combines this Fisher term $\mathcal{I}$ with a second term $\mathcal{J}$ that represents constraints or boundary information. Extremizing $\mathcal{A}$ yields physical field equations—such as the Klein–Gordon or Einstein equations—whose metric properties derive from the chosen contraction in $\mathcal{I}$ and the structural form of $\mathcal{J}$.

The auxiliary-parameter approach presented here and the EPI framework are therefore two expressions of the same underlying geometry. The Gram form WW^T exposes the local algebraic anatomy of information, while Frieden's integrated Fisher functional provides the global variational dynamics. The homeostatic corrections that can be developed later can be interpreted as a refinement of Frieden's $\mathcal{J}$-term: they impose the regulatory balance between conjugate sectors that stabilizes the indefinite metric structure without departing from the Fisher-information foundation established in Section 2.1.

2.4 The Mid-Parent Equation in Quantitative Genetics

The form WW^T can also represent a variance-covariance matrix which is necessarily symmetric and positive definite. Homeostatic balance in this case implies two paired variance matrices that are mutually balanced, and work to constrain or renormalize the variance in a target population. A good example is the mid-parent equation that describes additive genetic inheritance and also seems to imply a variety of homeostatic renormalization. That equation is given by the following.

$$a_o = ½a_m + ½a_p + \varepsilon$$

where a_o, a_m and a_p are the additive genetic effects in the offspring, the maternal and the paternal genomes, respectively, and ε is the random effect due to Mendelian segregation. The variance of ε is taken as $½(1 - F)\sigma^2$ where F is the inbreeding coefficient in the offspring and σ^2 is the additive genetic variance.

The variance of ε is a function of F, and this creates the illusion that variance is unconstrained. This inbreeding effect arises from the diploid nature of animal genomes. Nevertheless, the total additive variances coming from gametic contributions is actually assumed constant from generation to generation.

Smith and Mäki-Tanila (1990) describe a more general mid-parent equation that is suitable for additive genetic and dominance variation under conditions of inbreeding.

Perhaps the similarities are only spurious—or perhaps the confluence of two sides naturally produces these averaged contributions, whether they be additive genetic effects, or metric or curvature tensors.

3. Conclusion

The homeostatic model developed here demonstrates that the metric tensor can be treated as a dynamically factorable structure arising from a balance between conjugate manifolds. By representing the metric in the form $G = WDW^T$, where W is a lower-triangular matrix, and D encodes the Lorentzian signature, the geometry of spacetime is recast as the product of internally regulated amplitudes rather than as a static background field. The lower-triangular structure of W provides ten independent parameters, matching the degrees of freedom of the metric itself, and allows each element to act as an adaptive variable.

The tools are general enough to treat both a Gram form $G = WW^T$ or an indefinite Lorentzian-type metric. Even when W is rectangular, we find a similar partitioning: $WW^T = ½G^{(1)} + ½G^{(2)} - ¼\Delta\Delta^T$.

At homeostasis, the auxiliary differences between paired manifolds vanish, and the residual field G acquires full tensorial status. The dynamic balance achieved at this point unites algebraic symmetry with geometric covariance, thereby linking the statistical logic of inference with the structural logic of geometry. The same Gram form that underlies Fisher information appears here as the generative source of the metric tensor, establishing a correspondence between informational and gravitational formalisms.

The resulting picture is one in which homeostasis plays a role analogous to the Euler–Lagrange constraint, restoring covariance only at equilibrium. Both the variational and informational interpretations converge on a single principle: physical laws emerge from the maintenance of balance between conjugate sectors. This insight supports a broader metaphysical view in which geometry, inference, and evolution are not separate domains but interdependent aspects of a single homeostatic order that spans both the physical and biological realms.

The present results therefore provide the metric foundation upon which such variational formulations can be constructed. When combined with the curvature-based approach of the companion work (Smith 2025), they offer a coherent route to reconcile the CPT-symmetric cosmological model with current field theories. In this view, the metric tensor is not a static geometric backdrop but a dynamically balanced interface—an adaptive structure mediating between conjugate aspects of a self-consistent universe.

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