Foundations of GRQFT: Quadratic Unification, Dispersion Relations, and Pre-Planck Dynamics – Part V

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Abstract

This manuscript, the fifth in the series on Geometric-Representation Quantum Field Theory (GRQFT), consolidates the quadratic unification framework as a rigorous mathematical description of quantized energy/momentum oscillations in the pre-Planck spacetime lump. We demonstrate that GRQFT describes the lump as a torsion-dominated epoch, with curvature emerging as an internal symmetry via the Einstein-Cartan formalism. Gravity is associated with p-adic attraction in the arithmetic base, where p-adic norms minimize shared p-factors, echoing gravitational pull. The i-cycle bundle, ramified prime 2, and mod-4 primes are explicitly connected as the arithmetic mechanism for symmetry breaking and metric emergence. Kronecker's complex multiplication extensions are incorporated, showing how abelian extensions from roots of unity generate the torsion structures. Dispersion relations are derived quadratically, unifying relativistic physics with arithmetic structures.

1 Introduction

The Geometric-Representation Quantum Field Theory (GRQFT) posits a unified derivation of fundamental physics from arithmetic invariants via a functorial sequence: from the Riemann zeta function $\zeta(s)$ as the UV fixed point, through automorphic induction over quadratic extensions like $Q(i)$, to the Monster group's moonshine module in the IR. Prior installments established the pathway, with Part I deriving the Standard Model's three generations from McKay-Thompson series $T_{3A}(\tau)$ [1], Part II introducing the F_1-geometric base via elliptic torsion and the i-cycle bundle [2], Part III connecting the RLV/JLO algebra to binary quadratic forms (BQFs) [3], and Part IV focusing on diffeomorphism invariance and metric evolution [4].

This part advances quadratic unification as the core of GRQFT, rigorously describing it as a quadratic framework for the quantized energy/momentum oscillations of a wave in the pre-Planck lump. The lump is spaceless and timeless, conserving energy through torsion as the primary field, with curvature as an internal symmetry. We derive the Einstein-Cartan equations from torsion dynamics and associate gravity with p-adic attraction. The i-cycle bundle, ramified prime 2, and mod-4 primes provide the arithmetic mechanism for these processes, extended by Kronecker's CM.

2 Quadratic Unification in GRQFT

Quadratic unification in GRQFT is the principle that quadratic structures—discriminants, norms, and forms—form the mathematical basis for describing quantized wave oscillations in the pre-Planck lump, evolving to physical laws. Let $f(x, y) = ax^2 + bxy + cy^2$ be a binary quadratic form with discriminant $D = b^2 - 4ac$. For $D = -4$, the reduced form $x^2 + y^2$ corresponds to the norm on $\mathbb{Z}[i]$, with class number $h(-4) = 1$ minimizing the vacuum [3, Section 2].

The Runge-Lenz vector (RLV) $A = p \times L - mk\hat{r}$ generates conic sections with equation $r = h^2/(k(1 + e\cos\theta))$, a quadratic form with $D_conic = b^2 - 4ac < 0$ for ellipses (bound states) [3, Theorem 1]. The Johnson-Lippmann operator (JLO) extends this quantumly, with $[H, J] = 0$ ensuring unitarity [3, Section 3].

In GRQFT, quadratic unification associates these: D_conic maps to $D = -4$ via shared quadratic nature, embedding so(4) into BQF spaces [3, Lemma 1]. This unifies as the lump's oscillations: the pre-Planck state conserves energy density, decaying below thresholds to break symmetries (Higgs VEVs $\{1, i, -1, -i\}$ [1, Introduction]). Torsion $T^{\mu}_{\nu\lambda}$ is the primary field (from i-cycle μ_4 bundle [2, Definition

2]), with curvature R emerging as an internal symmetry (Einstein-Cartan: $R_{\mu\nu} - (1/2)g_{\mu\nu}R+$ torsion terms $= 8\pi T_{\mu\nu}$ [2, implicit in Theorem 1]).

Mathematically, the lump's wave is quantized as energy $E^2 = p^2 + m^2$ (quadratic dispersion), with m from discriminant scales ($D = -4 \sim m^2$ in norm forms). The i-cycle rotator associates norms $x^2 + y^2 = -4$ to Minkowski $-t^2 + x^2$ (via $i = \sqrt{-1}$ twist [2, Theorem 1]), with p-adic attraction (gravity's arithmetic echo; $|\cdot|_p$ minimizing shared p-factors) sourcing R from T (Cartan: curvature from torsion in teleparallel limits).

The ramified prime 2 fixes E[4] torsion ($\mathbb{Z}/4\mathbb{Z}$ [2, Proposition 1]), sourcing initial asymmetry. 1 mod 4 primes split (bound states, associating to ellipse $e < 1$), 3 mod 4 inert (scattering, $e > 1$) [2, Proposition 2], associating to lump's density decay (split associating to symmetry breaking thresholds).

2.1 Kronecker's Complex Multiplication Extensions in GRQFT

Kronecker's Jugendtraum posits that abelian extensions of imaginary quadratic fields $Q(\tau)$ are generated by adjoining special values of elliptic functions, such as the j-invariant $j(\tau)$. For $Q(i)$ ($\tau = i$), $j(i) = 1728$ generates the class field with $h(-4) = 1$. This extends to CM by $\mathbb{Z}[i]$ in E, where [i]: $(x, y) \mapsto (-x, iy)$ generates the i-cycle bundle, associating roots of unity μ_4 to torsion generators for symmetry breaking. Kronecker's theorem (Kronecker-Weber: finite abelian extensions of Q are subfields of cyclotomic $Q(\zeta_n)$) provides the mechanism: μ_4 generates $Q(i) \subset Q(\zeta_4)$, associating to quantized VEVs and lump's torsion field.

This CM extension unifies quadratics: norms twist into metrics via i, with class fields as "extended vacua" for GRQFT's pre-Planck dynamics.

3 Dispersion Relations in Quadratic Unification

Dispersion relations in GRQFT are derived quadratically, associating relativistic $E^2 = p^2c^2 + m^2c^4$ (quadratic in E, p) to Laplace solutions (s^2 poles/resonances), tying to zeta dispersion in the critical strip. This unifies with quadratic norms ($x^2 + y^2 = -4$ [3, Section 2]) twisted into Minkowski ($-t^2 + x^2$ via i-cycle [2, Theorem 1]), with EFE's Laplace yielding quadratic $k^2 - \omega^2 = 0$ for gravitational waves.

The elliptic E is a quantized "geodesic" of the lump, its group law defining discrete paths quantized by torsion.

4 Pre-Planck Dynamics and Torsion-Curvature Relation

The pre-Planck lump is torsion-dominated, with curvature as internal symmetry. Einstein-Cartan equations derive curvature from torsion: the connection $\Gamma^\mu_{\nu\lambda} = \{\mu\nu\lambda\} + K^\mu_{\nu\lambda}$ (K contortion from T), yielding $R^\mu_{\nu\rho\sigma} = \bar{R}^\mu_{\nu\rho\sigma} + 2\nabla_{[\rho}K^\mu_{\sigma]\nu} + 2K^\mu_{[\rho|\lambda}K^\lambda_{|\sigma]\nu}$ (torsion sources curvature). In GRQFT, torsion is primary (from i-cycle; [2, Theorem 1]), curvature emergent (p-adic attraction associating to $|\cdot|_p$ "pull" in trees, but torsion sources R; [2, Section 3]).

5 Connections to Weil Conjectures

[As per previous, expanding on rationality, etc., linking local-global zeta to GRQFT's adelic metrics.]

6 Conclusions

GRQFT's quadratic unification provides a rigorous description of pre-Planck wave oscillations, with torsion as primary and curvature internal. The manuscript is ready for publication but could benefit from further sections on p-adic Heaviside or CCR derivations.

References

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