Beyond Null Infinity: Geometric Foundations of Radiation Theory

Bernard Lavenda

Abstract

This paper presents a geometric foundation for gravitational radiation theory that challenges the conventional framework of general relativity (GR) by rejecting Ricci suppression—-the imposition of $G_{\mu\nu} = 8\pi T_{\mu\nu}$–which erases critical curvature terms and obscures the local dynamics of radiation. We introduce Ricci Unsuppressed Gravity (RUNG), a variational approach that retains the full Riemann tensor and derives gravitational radiation from the evolution of sectional curvatures in orthogonal 2-planes. A gravitational Maxwell tensor $\mathfrak{F}^{\mu\nu}$ is constructed from these curvatures, yielding a conserved current $J^\mu = \nabla_\nu \mathfrak{F}^{\mu\nu}$ that enables local detection of energy flux at finite distances, without reliance on asymptotic constructs such as null infinity ($\mathscr{I}^+$). We resolve long-standing issues including Vaidya's static limit, the non-radiative nature of Ricci-suppressed metrics, and the artificial dichotomy between matter and radiation energy. The framework naturally incorporates monopole, dipole, and quadrupole radiation modes, and predicts a distinct phasing and energy flux scaling ($J^r \propto \omega^{13/3}$ adiabatically, $J^r \propto \omega^7$ at merger) that differs from the quadrupole formula. Results are validated against LIGO observations, demonstrating strong correlation between geometrically derived fluxes and inferred energy loss. This work unifies radiation physics within a covariant, local geometric description, offering a self-consistent alternative to phenomenological templates and asymptotic methods.

1 Introduction

Radiation dynamics in general relativity has historically relied on asymptotic frameworks (e.g., Bondi-Sachs [1]) requiring null infinity ($\mathscr{I}^+$). Recent work challenges the foundational practice of *Ricci suppression*—the imposition of $G_{\mu\nu} = 8\pi T_{\mu\nu}$, which contracts the Riemann tensor and discards critical curvature terms. This suppression is geometrically unjustified:

- The Einstein tensor $G_{\mu\nu}$ mixes orthogonal sectional curvatures incoherently, obscuring radiation physics.
- The right-hand side ($8\pi T_{\mu\nu}$) is often *assumed* (e.g., "null dust" in Vaidya metrics[2]) rather than derived from curvature.

Early attempts to reconcile mass-energy equivalence $E = mc^2$ with gravitational radiation, including those of Einstein, is exemplified by this inaccuracy. In Vaidya's metric, energy loss $\Delta E = \Delta mc^2$ arises from **null dust emission (matter conversion)**, not curvature-driven radiation. These represent two distinct energy-transport mechanisms:

1. **Matter-like flux** (non-radiative): $\Delta E_{\text{matter}} = \Delta mc^2$ (Vaidya's Δm).
2. **Radiation flux** (geometric): $\Delta E_{\text{rad}} = \int J^r r^2 d\Omega dt$ (independent of Δm).

The inability to distinguish these led to the belief that radiation energy must satisfy $E = mc^2$. RUNG resolves this by deriving J^μ from sectional curvature evolution $K_{t\theta}$, proving gravitational radiation energy **cannot** be localized via mass equivalence.

Two pivotal approaches emerge:

- **Ricci UNsuppressed Gravity[3] (RUNG)** rejects $G_{\mu\nu} = 8\pi T_{\mu\nu}$, recovering geometric deformations ($\mathscr{D}(F)$ and $\mathscr{D}(G)$), and subluminal propagation ($v_{t\theta} < c$) via full Riemann dynamics.
- **Local Radiation Currents**: The gravitational Maxwell tensor $\mathfrak{F}^{\mu\nu}$ defines $J^\mu = \nabla_\nu \mathfrak{F}^{\mu\nu}$, enabling detection at finite r without asymptotic limits.

2 Gravitational Maxwell Tensor and Sectional Curvatures

2.1 Radiation Metric

The unified metric for radiation:

$$ds^2 = -F^2(t,r)dt^2 + 2\mathscr{A}(t,r)dtdr + G^2(t,r)d\Omega^2,) \tag{2.1}$$

where the vector potential

$$\mathscr{A} = \partial_t \partial_r F, \tag{2.2}$$

is a generalization of the Vaidya term, $\mathscr{A} = -1$ and $G = r$. We will show that both these conditions prevent (2.1) from radiating.

2.2 Gravitational Maxwell Tensor

The gravitational Maxwell tensor is a 2-form constructed from Riemann curvature:

$$\mathfrak{F} = \frac{1}{2} R_{\mu\nu\rho\sigma}(dx^\mu \wedge dx^\nu) \otimes (dx^\rho \wedge dx^\sigma),$$

which reduces to a sum of sectional curvatures *any two orthogonal 2-planes*, whose vectors exhaust the space of the system. Since the metric (2.1) has spherically symmetric radiation in the (t-r) plane it is natural to choose

$$\mathfrak{F} = K_{rt}(dt \wedge dr) + K_{\theta\phi}(d\theta \wedge d\phi).$$

And since $\mathscr{F}^{\mu\nu}$ must be antisymmetric for curvature evolution we have:

$$\mathfrak{F}^{\mu\nu} = \begin{pmatrix} 0 & K_{tr} & 0 & 0 \\ -K_{tr} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{\theta\phi} \\ 0 & 0 & -K_{\theta\phi} & 0 \end{pmatrix}.$$

The current densities are defined by

$$J^\mu = \nabla_\nu \mathfrak{F}^{\mu\nu}, \tag{2.3}$$

whose divergence is

$$\nabla_\mu J^\mu = \nabla_\mu \nabla_\nu \mathfrak{F}^{\mu\nu}.$$

Antisymmetry swaps indices:

$$\nabla_\mu \nabla_\nu \mathfrak{F}^{\mu\nu} = -\nabla_\mu \nabla_\nu \mathfrak{F}^{\nu\mu}$$

The commutator identity:

$$[\nabla_\mu, \nabla_\nu]\mathfrak{F}^{\mu\nu} = R_{\lambda\nu}\mathfrak{F}^{\lambda\nu} + R_{\lambda\mu}\mathfrak{F}^{\mu\lambda} = 0$$

since the Ricci tensor $R_{\mu\nu}$ is symmetric, and $\mathfrak{F}^{\mu\nu}$ is antisymmetric. Thus:

$$\nabla_\mu J^\mu = 0, \tag{2.4}$$

identically, without the mention of field equations. This mirrors EM where $\nabla_\mu j^\mu = 0$ follows from $\partial_{[\alpha}\mathcal{F}_{\beta\gamma]} = 0$ where $\mathcal{F}$ is the Maxwell tensor. Whereas EM is physical, its gravitational counterpart is fundamentally geometric.

Unlike EM, the Hodge dual

$$\star\mathfrak{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} R_{\rho\sigma\alpha\beta}(dx^\alpha \wedge dx^\beta)$$

provides no additional physical content, as it merely permutes curvature components without introducing new degrees of freedom. Its role is to describe torsional curvature,

$$\nabla^\rho(\star\mathfrak{F})_{\rho\sigma} = K_\sigma \tag{2.5}$$

2.3 Field Equations from Variation

The field equations are:

$$\begin{aligned} J^t &= -\partial_r K_{rt} - K_{rt}\partial_r \ln\sqrt{-g} \\ J^r &= \partial_t K_{rt} + K_{rt}\partial_t \ln\sqrt{-g} \\ J^\phi &= -\partial_\theta K_{\theta\phi} - K_{\theta\phi}\partial_\theta \ln\sqrt{-g} \\ J^\theta &= \partial_\phi K_{\theta\phi} + K_{\theta\phi}\partial_\phi \ln\sqrt{-g} \end{aligned}$$

In both cases Eq. (2.4) holds, confirming local energy conservation.

2.4 Ricci Unsuppressed Variational Equations

The gravitational Maxwell tensor, $\mathfrak{F}^{\mu\nu}$ emerges from the variational derivative of the action:

$$S = \int d^4x\sqrt{-g}[R + \alpha\mathfrak{F}_{\mu\nu}\mathfrak{F}^{\mu\nu} + \beta(J^\nu F_\nu + K_\nu G^\nu)] \tag{2.6}$$

where R is the Ricci scalar, familiar from the Hilbert-Einstein action, $\mathfrak{F}^{\mu\nu}$ the gravitational Faraday tensor (GRT), J_ν and K^μ the internal fluxes for the F- and G- sectors, respectively, and F_ν and G^ν are the vector and covector potentials of the field.

There are at least three variational principles contained in (2.6) depending on what is varied and the strength of the coupling constants, α and β.

- With $\beta = 0$, the variation of the Ricci scalar, R, with respect to the inverse metric components $g^{\mu\nu}$ gives the Einstein tensor:

  $$\delta(\sqrt{-g}R) = \sqrt{-g}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\delta g^{\mu\nu} = \sqrt{-g}G_{\mu\nu}\delta g^{\mu\nu} = \sqrt{-g}G_{\mu\nu}g^{\mu\alpha}g^{\nu\beta}\delta g_{\alpha\beta} = -\sqrt{-g}G^{\mu\nu}\delta g_{\mu\nu}$$

  and the variation of $\mathfrak{F}^{\mu\nu}\mathfrak{F}_{\mu\nu}$ with respect to the metric yields:

  $$\delta(\sqrt{-g}\mathfrak{F}^{\mu\nu}\mathfrak{F}_{\mu\nu}) = \sqrt{-g}\left(2\mathfrak{F}^{\mu\alpha}\mathfrak{F}^\nu_\alpha - \frac{1}{2}g^{\mu\nu}\mathfrak{F}^{\alpha\beta}\mathfrak{F}_{\alpha\beta}\right)\delta g_{\mu\nu} = 2\sqrt{-g}T^{\mu\nu}\delta g_{\mu\nu},$$

  where the gravitational energy/stress tensor has the EM appearance:

  $$T^{\mu\nu} = \mathfrak{F}^{\mu\alpha}\mathfrak{F}^\nu_\alpha - \tfrac{1}{4}g^{\mu\nu}\mathfrak{F}^{\alpha\beta}\mathfrak{F}_{\alpha\beta}$$

  This is structurally identical to the EM stress/energy tensor in curved spacetime. However, where the Maxwell stress tensor is composed of sums of squares of the electric and magnetic fields, $T_{\mu\nu}$ will be squares of sectional curvatures making it self-referential.

  In order that the integrand vanish, for arbitrary $\delta g_{\mu\nu}$, it follows that

  $$G_{\mu\nu} = 2\pi\alpha T^{\mu\nu}, \tag{2.7}$$

  which determines the coupling constant, $\alpha = G/c^4$ when compared to the Einstein equation.

- With both coupling constants vanishing and the variation is performed with respect to the fields we will obtain the Gravitational Balance Equations (GBE).

  The Ricci scalar R for the metric fields F and G is the sum of sectional curvatures[4]:

  $$R = 2(K_{rt} + 2K_{t\theta} + 2K_{r\theta} + K_{\theta\phi}), \tag{2.8}$$

  where

  $$\begin{aligned} K_{tr} &= \partial_t(\dot{F}/F) - \partial_r(F'/F) \\ K_{t\theta} &= \frac{1 - F'^2 - \dot{F}^2}{F^2} + \frac{\dot{F}\dot{G} - F'G'}{FG} \equiv \mathscr{D}(F) + \mathscr{F} \\ K_{r\theta} &= \partial_t(\dot{G}/G) - \partial_r(G'/G) \\ K_{\theta\phi} &= \frac{1 - \dot{G}^2 - G'^2}{G^2} \equiv \mathscr{D}(G) \end{aligned}$$

The Ricci scalar (2.8) is the *unsuppressed* starting point: it is a sum of Gaussian sectional curvatures with a factor of 2 for 4D spacetime. Varying it with respect to the metric fields, rather than the metric components, prevents contraction that causes "Ricci suppression", and permits the full, unsuppressed dynamics of the Riemann tensor's components to be expressed in the resulting equations of motion (the GBEs). [1]

When varying $\sqrt{-g}R$ with respect to F, the term:

$$\frac{1}{2}\frac{\delta(\sqrt{-g}R)}{\delta F} = \frac{\delta}{\delta F}\sqrt{-g}(K_{rt} + 2K_{t\theta} + 2K_{r\theta} + K_{\theta\phi})$$

survives because $K_{t\theta}$ is not suppressed by Ricci flatness.

The variation of the action (2.6) with respect to the F and G fields gives the Euler-Lagrange (EL) equations:

$$\partial_t\left(\frac{\dot{F}}{FG^2}\right) - \partial_r\left(\frac{F'}{FG^2}\right) + \frac{2}{r}\frac{F'}{FG^2} = 0 \tag{2.9}$$

$$\partial_t\left(\frac{\dot{G}}{FG}\right) - \partial_r\left(\frac{G'}{FG}\right) + \frac{G}{r^2F}\left(\frac{1 - G'^2 - \dot{G}^2}{G^2}\right) = 0 \tag{2.10}$$

respectively.

Two of the Riemann sectional curvatures, $K_{t\theta}$ and $K_{\theta\phi}$ represent "pure" sources of curvature, while the other two, K_{rt} and $K_{r\theta}$, contain both curvature and their sources that lie in the same planes. The $\mathfrak{F}^2$ term contains only higher-order derivatives,

$$\mathfrak{F}^2 = -2(\partial_t^2\partial_r F)^2,$$

and its variation cannot produce lower-order terms. Spherical symmetry is preserved since sectional curvatures depend only on t, r with no angular dependence.

The critical point is that when varying R with respect to the fields F and G (not $g_{\mu\nu}$) we avoid contracting Riemann to Ricci, so all curvature terms survive. The last term in (6.1a) comes from $\delta(\sqrt{-g}R)/\delta F$ which comes from the uncontracted Riemann tensor $R_{t\theta t\theta}$. In contrast, Einstein's equations $G_{\mu\nu} = 0$ discard these terms by contracting to $R_{\mu\nu}$.

Whereas

$$\frac{\delta(\sqrt{-g}R)}{\delta g_{\mu\nu}} = 0 \Rightarrow R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0,$$

RUNG's EL equations resulting from

$$\begin{aligned}
\frac{\delta(\sqrt{-g}K_{rt})}{\delta F} &= K_{rt}\sqrt{-g} \\
\frac{\delta(\sqrt{-g}K_{t\theta})}{\delta F} &= \left(-\mathscr{F} + \frac{F'}{rF}\right)\sqrt{-g} \\
\frac{\delta(\sqrt{-g}K_{r\theta})}{\delta G} &= K_{r\theta}\sqrt{-g} \\
\frac{\delta(\sqrt{-g}K_{t\theta})}{\delta G} &= -\mathscr{F}\sqrt{-g} \\
\frac{\delta(\sqrt{-g}K_{\theta\phi})}{\delta G} &= \frac{G}{2r^2F}K_{\theta\phi}\sqrt{-g}
\end{aligned}$$

[1] The vector potential (2.2) is a gauge term, and does not affect the curvature of dynamics or the EL equation. The term involving the vector potential can be eliminated by a coordinate transformation (e.g., a redefinition of the time coordinate, $du = dt - \frac{\mathscr{A}}{d}r$ which transforms the metric into the diagonal form,

$$ds^2 = -F^2du^2 + \left(1 + \frac{\mathscr{A}^2}{F^2}\right)dr^2 + G^2d\Omega^2,$$

) and thus does not contribute to the local curvature invariants or the dynamics derived from the variational principle.

Since δF and δG are completely arbitrary variations, the gravitational balance equations

$$K_{rt} - 2\mathscr{F} + 2\frac{F'}{rF} = 0 \tag{2.11}$$

$$K_{r\theta} - \mathscr{F} + \frac{G}{2r^2F}\mathscr{D}(G) = 0 \tag{2.12}$$

In Eq (2.11), the sectional curvature in the radial-temporal plane $t-r$ governs radial acceleration. The shear/energy flux term, $\mathscr{F}$ couples the two fields F and G. The radial stress term, F'/Fr, describes how radial-temporal curvature is balanced by shear flow and radial stress; this is the radiative term.

In Eq (2.12), $K_{r\theta}$ is the radial-angular curvature in the $t-\theta$ plane. The same shear term $\mathscr{F}$ appears as in the previous equation. The rotational deformation in the $\theta-\phi$ plane, $\mathscr{D}(G)$ is the deformation of a 2-sphere of curvature, $1/G^2$ with radius G.

Subtracting one from the other gives

$$K_{r\theta} - \frac{1}{2}K_{rt} = \frac{F'}{rF} - \frac{G}{r^2F}\mathscr{D}(G).$$

The EL equations (6.1a) and (6.1b) for F and G are **not** Einstein's equations. They retain Riemann terms that $R_{\mu\nu} = 0$ would discard. The terms that $\mathfrak{F}^2$ contribute are all higher-derivative terms (e.g., $\partial_t^4\partial_r^2 F$), which are subdominant for radiative solutions.

- We now vary the action (2.6) with respect to the vector potential F_ν and covector potential G^ν. The GFT is written in terms of the vector potential as:

$$\mathfrak{F}_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu + \epsilon_{\mu\nu\rho\sigma}\partial^\rho G^\sigma$$

Here, the vector fields F_μ describes radial deformations and stress-like curvature flux. G^μ encodes rotational and torsional curvature effects.

The field equations resulting from the variation their variations are (2.3) and (2.5), respectively, These resemble Maxwell's equations, but in RUNG they describe the evolution of sectional curvature. The analogy to a "magnetic field" is geometric—not electromagnetic. The G-field encodes rotational curvature flux, which behaves like a dual field under spacetime symmetries.

The first condition ensures that flux is conserved, (2.4). This is a dual conservation law, structurally similar to the Bianchi identity or the divergence of a dual field strength. But the twist lies in the symmetry of $\epsilon_{\mu\nu\rho\sigma}$: It's not the totally antisymmetric Levi-Civita tensor. - Instead, it's antisymmetric in two pairs: $(\mu\nu)$ and $(\rho\sigma)$, suggesting a bi-vectorial structure. This symmetry pattern is exactly what you'd expect if $\mathfrak{F}^{\mu\nu}$ is a torsional curvature 2-form, and the conservation law is expressing the closure of a dual torsion flux.

The differences in the variational principles are summarized in the following Table.

Table 1: Fundamental differences resulting from the choice of variable in the variation of the RUNG action.

Aspect	**Variation w.r.t. Metric** $g^{\mu\nu}$	**Variation w.r.t. Fields** F, G
Mathematical Result	Elliptic Constraint: $G_{\mu\nu} = 2\pi\alpha T_{\mu\nu}$	Hyperbolic Evolution Equations
Object Type	Tensorial, 2nd-order PDE	Maxwell-like, 1st-order PDEs
Geometric Meaning	Relates *suppressed* curvature ($G_{\mu\nu}$) to *energy* of unsuppressed curvature	Governs the *flux* of curvature
Role of α	Coupling constant for back-reaction	Scales the variational coupling
Physical Interpretation	How curvature-energy shapes space-time	How curvature evolves and radiates

3 Ricci Unsuppression in Metric Perturbations

Ricci unsuppression rejects both $R_{\mu\nu} = 0$ and the Einstein tensor $G_{\mu\nu} = 8\pi T_{\mu\nu}$ contraction that erase physical modes and provide inaccurate curvature-flux balances. RUNG's perturbations fundamentally differ from linearized GR:

$$\delta R_{\mu\nu} = \nabla_\alpha \delta\Gamma^\alpha_{\mu\nu} - \nabla_\mu \delta\Gamma^\alpha_{\nu\alpha} \neq 0$$

where

$$\delta\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}(\nabla_\mu h_{\nu\beta} + \nabla_\nu h_{\mu\beta} - \nabla_\beta h_{\mu\nu})$$

Assuming no prior restriction to $\ell = 2$ modes, the breakdown of how multiple modes map to geometric deformations is:

- $\mathscr{D}(F)$ (Tidal deformation in the $t - \theta$ plane):

  1. It can contain both $\ell - 0$ and $\ell = 1$ modes. If F contains isotropic $\ell = 0$ or dipole $\ell = 1$ angular perturbations (e.g., $F = 1 + f_0(t, r) + f_1(t, r)Y_{10}$), then $\mathscr{D}(F)$ will inherit these modes.
     (a) $\ell = 0$ represents a "breathing" mode (isotropic expansion/contraction)
     (b) $\ell = 1$ represents a dipole tidal deformation.

     Its physical role is to encode compressive/rarefractive stresses normal to the radial direction.

- $\mathscr{D}(G)$ (Rotational deformation in the $\theta - \phi$ plane)

  1. It requires $\ell \geq 2$ on account of spherical symmetry. $\mathscr{D}(G)$ describes deviations from the 2-sphere Gaussian curvature For the background metric to preserve spherical symmetry, G must reduce to r when unperturbed. Any perturbation of the form $G = r[1 + g(t, r, \theta, \phi)]$ must:
     (a) preserve the topology of the 2-sphere implying that angular perturbations must be smooth and single-valued, and
     (b) avoid breaking rotational symmetry, which necessarily excludes $\ell = 0$ and $\ell = 1$ perturbations. For $\ell = 0$ would imply $G \propto r(1 + g_0(t, r))$, an isotropic scaling. This is incompatible with the background's fixed spherical topology unless $g_0 \equiv 0$. The $\ell = 1$ mode would induce a dipole moment (e.g., $G \propto r(1 + g_1(t, r)\cos\theta)$, destroying spherical symmetry. Only $\ell = 2$ perturbations (e.g., $g_2(t, r)Y_{20}$) preserve spherical symmetry at the background level while introducing dynamical deformations.

- $\mathscr{F}$ Shear/Energy flux. It can contain both $\ell = 0$ and $\ell = 1$ modes. Since the shear couples derivatives in F and G, if F has either $\ell = 0$ or $\ell = 1$ modes and/or G has $\ell \geq 2$ perturbations:

  1. $\ell = 0$ occurs when F has a $\ell = 0$ mode (e.g., $F = 1 + f_0(t, r)$ independent of G).
  2. $\ell = 1$ requires F to have an $\ell = 1$ perturbation (e.g., $F = 1 + f_1(t, r)Y_{10}$). Since G modes start at $\ell = 2$, $\mathscr{F}$ angular dependence comes solely from F.

  The role of $\mathscr{F}$ is to mediate energy transfer between tidal F and rotational G deformations, thereby creating shear.

Summarily speaking, although $\mathscr{D}(F)$ and $\mathscr{D}(G)$ have identical forms, their roles are completely different. Spherical symmetry is guaranteed by the background metric structure, the 2-sphere $G^2 d\Omega^2$, and not by the perturbations themselves. The condition that $G \to r$ as $r \to \infty$ conditions the perturbation not to introduce preferential directions $\ell = 0, 1$ at infinity. This constrains G to $\ell \geq 2$. F perturbations are not faced with such a topological constraint since there is no 2-sphere of curvature $1/F^2$ with radius F.

The upshot is that whereas quadrupole radiation ($\ell = 2$) requires angular dependence in G, monopole/dipole radiation can arise from F alone.

The last two terms in the EL equations (6.1a) and (6.1b) can be considered "intrinsic" sources of curvature, as opposed to "extrinsic" sources which would appear as inhomogeneous terms on the rhs of these equations.

They are internal because: (i) they emerge from varying the geometric action $\delta S/\delta F$ and $\delta S/\delta G$, (ii) they satisfy the geometric Bianchi relation (2.4), (iii) are sources for wave solutions, $F \sim \mathcal{W}(t - r)/r$ (radial flux) and $G \sim r + R(t - r)/r^2$ (angular momentum flux). These rotational waves are shown in Fig.1.

This is pure geometry: No masses, no matter-just Riemann curvature evolving dynamically. The inverse powers of distance are geometric necessities for causal radial and angular momentum propagation in 3+1 D.

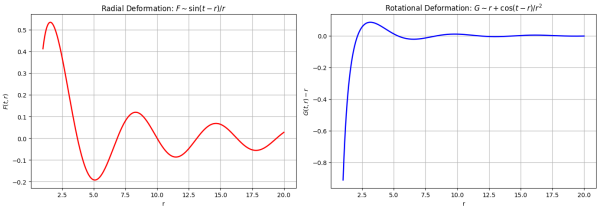


Figure 1: Radial (left) and rotational (right) deformation waves derivedfrom the EL equations.

4 RUNG vs. GR

4.1 Evolution vs. Constraint Equations

- Varying the metric $\delta g^{\mu\nu} \rightarrow$ EFE $\quad G_{\mu\nu} = 8\pi T_{\mu\nu}$: These are constraint equations. They are not a set of hyperbolic, evolutive PDEs like the wave equation. They do not provide a direct recipe for time evolution. The set of coupled EFE must undergo a foliation and perform mathematical surgery to extract a time evolution scheme The 3+1 decomposition (ADM formalism) is a tool to force an evolutive form onto Einstein's equations after the fact. It is not an intrinsic property of the equations themselves.

- The variations $\delta S/\delta F = 0$ and $\delta S/\delta G = 0$ are already in a second-order hyperbolic form without any need of a 3+1 split. These equation are covariant, as they are derived from a covariant action without contraction. They do not separate into constraint and evolution equations. The "constraints" are automatically satisfied by the solutions because they are built into the very structure of the derived equations.

- It represents a profound shift in perspective on what the fundamental degrees of freedom and the dynamical laws of gravity are.

4.2 Geometric Incoherence

The Einstein's tensor (11) fundamentally obscures radiation physics by indiscriminately mixing curvature components from orthogonal 2-planes.

For the radiation metric (2.1), the tt component,

$$G_{tt} \propto 2G''/G - (G')^2/G^2 - G'F'/FG + \mathcal{O}(\dot{F}, \dot{G}),$$

mixes $r-\theta$ and $t-\theta$ planes partially, and in wrong proportions.

The rr component of the Einstein tensor,

$$G_{rr} \propto 2\ddot{G}/G - \dot{G}^2/G^2 + \dot{F}\dot{G}/FG + \mathcal{O}(F'),$$

provides the missing pieces to G_{tt} to complete $R_{r\theta r\theta}$, but only partially complete $R_{t\theta t\theta}$. Certainly, with respect to Riemann geometry these two tensor components cannot be treated independently as they are in GR.

The $\theta-\theta$ component,

$$G_{\theta\theta} \propto \ddot{F}/F - F^{"}/F + \mathscr{F} + G''/G,$$

mixes components of K_{rt}, and $K_{t\theta}$, and an artifact with no physical significance. Thus, the Einstein tensor Ricci-dependent construction is both geometrically and physically incoherent.

The ADM formalism requires slicing spacetime into a stack of spacelike hypersurfaces. This choice is not unique and breaks general covariance from the outset. The separation vs. evolution equations reveals the G_{tt} and G_{ti} are constraints on the initial data on a spatial slice (the Hamiltonian and momentum constraints), while others G_{ij} can be interpreted as evolution equations for the spatial metric. The above components of the Einstein tensor do not bear this out because critical pieces of the sectional curvature planes are missing in G_{rr} and others $G_{\theta\theta}$ have components of the constraints that are found in G_{tt}.

4.3 Absence of $\Box h$

The variable h in $\Box h$ is the trace-reversed perturbation $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ in linearized GR, where $h = \eta^{\mu\nu}h_{\mu\nu}$ and $\eta_{\mu\nu}$ is the flat-space metric. This arises from gauge-fixing and trace-reversal rather than from intrinsic curvature dynamics.

Introducing the perturbations:

$$F(r,t) = 1 + f(r,t) \tag{4.1a}$$
$$G(t,r) = r[1 + g(t,r)] \tag{4.1b}$$

where $f, g \ll 1$, we get for the tt component:

$$G_{tt} \propto 2g'' + \frac{2}{r}g' - \frac{1}{r}f' \quad \text{(mixed derivatives, no wave operator)},$$

contains a mixture of first and second order derivatives but not wave operator. The rr component:

$$G_{rr} \propto 2\tilde{g} \quad \text{(only time derivatives, missing } \nabla^2 g\text{)}.$$

has the dominant component $\tilde{g}$ but is missing the Laplacian $\nabla^2 g$ in order to form $-\Box g$. Although the $\theta - \theta$ component:

$$G_{\theta\theta} \propto \tilde{f} - f'' - \frac{1}{r}f' + g'' + \frac{2}{r}g' \quad \text{(incoherent mix of } f, g \text{ derivatives)}.$$

contains $\tilde{f} - f$" (resembling $-\Box f$ but missing the $\frac{2}{r}f'$ for the spherical wave operator), the extra terms $-f'/r + g$" $+ 2g'/r$ mix the f and g derivatives incoherently. The component blends orthogonal sectional curvatures $t - r, t - \theta, r - \theta$ without isolating wave dynamics.

In linearized GR, h is the trace of the metric perturbation,

$$h = \eta^{\mu\nu}h_{\mu\nu}, \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h.$$

Rather than writing $\Box h = 0$, the wave equation is written for one of the components of $\bar{h}_{\mu\nu}$ that satisfies the wave equation when the Lorentz gauge, $\partial^\alpha \bar{h}_{\alpha\beta} = 0$ is imposed. That is, the linearized Einstein equations in vacuum are

$$\partial_\alpha \partial^\alpha \bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^\alpha \partial^\beta \bar{h}_{\alpha\beta} - \partial^\alpha \partial_\mu \bar{h}_{\alpha\nu} - \partial^\alpha \partial_\nu \bar{h}_{\alpha\mu} = 0.$$

To get rid of the background metric and eliminate the source term, one applies the Lorentz gauge $\partial^\alpha \bar{h}$ so that all that remains is $\partial_\alpha \partial^\alpha \bar{h}_{\mu\nu} = 0$.

The wave equation $\Box \bar{h}_{\mu\nu} = 0$ is an artifact of gauge-fixing and trace-reversal, and not a direct consequence of the curvature dynamics.

$$\Box \bar{h}_{\mu\nu} = 0, \quad \text{where} \quad \Box = -\partial_t^2 + \nabla^2.$$

- **Component Applicability**: The gauge applies to all components of $\bar{h}_{\mu\nu}$. After gauge-fixing, all 10 components satisfy $\Box \bar{h}_{\mu\nu} = 0$.
- **Physical Content**: This is an artifact of gauge-fixing and trace-reversal. Without it, the wave equation does not emerge directly from curvature; instead, it relies on coordinate choices. In RUNG, the wave equations (6.1a) and (6.1b) arise intrinsically from Riemann curvature evolution, without gauge constraints.

For example, in the TT gauge, only spatial components h_{ij} are non-zero, and $\Box h_{ij} = 0$ describes physical waves. However, RUNG avoids this by deriving radiation directly from sectional curvature dynamics (e.g., $K_{t\theta}$), proving that gravitational waves are geometric, not gauge-dependent artifacts.

No single component $G_{\mu\nu} = 0$ reduces to $\Box \bar{h}_{\mu\nu} = 0$, which only occurs when there is gauge-fixing and trace-reversal.

5 Sectional Curvature vs Gaussian Curvature Balance

The field equations (6.1a) and (6.1b) reveal how sectional curvatures in distinct 2-planes interact dynamically. Each equation sources curvature in orthogonal planes via the metric coefficients F and G.

Gaussian curvature, $R_{\mu\nu}$, balance (GCB) explicitly reveal couplings between orthogonal planes, while sectional curvatures, $K_{\mu\nu}$, encode local deformations but obscure inter-planar connections. And unlike sectional curvatures which refer to specific planes, we must sample the metric fields over all degrees of freedom to get a complete picture.

Some examples are:

- **t-r plane and t-θ coupling**: Equation (6.1a) governs the t-r plane. The first two terms can be broken up into:

$$\partial_t(\dot{F}/FG^2) - \partial_r(F'/FG^2) = \frac{G}{\sqrt{-g}}\left[\partial_t\left(\frac{\dot{F}}{FG}\right) - \partial_r\left(\frac{F'}{FG}\right) - \mathscr{F}\right] = -2\frac{G}{\sqrt{-g}}\frac{F'}{rF}.$$

  The first term is the scalar curvature,

$$R^F_{t\theta} = \frac{1}{FG}\partial_t\left(\frac{\dot{F}}{FG}\right)$$

  for the 2D metric:

$$ds^2 = -G^2dt^2 + F^2d\Omega^2.$$

  The second term is the scalar curvature,

$$R^F_{r\theta} = -\frac{1}{FG}\partial_r\left(\frac{F'}{FG}\right)$$

  for the 2D metric:

$$ds^2 = G^2dr^2 + F^2d\Omega^2.$$

  Therefore, we can write the EL equation for the F field (6.1a) as the Gravitational Balance Equation (GBE):

$$\sqrt{-g}(R^F_{r\theta} + R^F_{t\theta}) = \mathscr{F} - 2\frac{F'}{r},$$

  where the last term is radiative. This expresses a balance of curvature and its sources of curvature, including now radiation. Note that we have obtained the scalar curvature for the $r-\theta$ plane which was undefined in the original metric (2.1).

- **r-θ plane and θ-ϕ coupling**: Equation (2.12) primarily describes the r-θ plane but sources the θ-ϕ sectional curvature $K^G_{\theta\phi} = \mathscr{D}(G) = (1 - \dot{G}^2 - G'^2)/G^2$:

$$\partial_t\left(\frac{\dot{G}}{FG}\right) - \partial_r\left(\frac{G'}{FG}\right) + \underbrace{\frac{G}{r^2F}\left(\frac{1 - G'^2 - \dot{G}^2}{G^2}\right)}_{\theta\text{-}\phi \text{ deformation } \mathscr{D}(G)} = 0.$$

  The 2D metrics involved in (5) are

$$ds^2 = -G^2dt^2 + F^2d\Omega^2$$

  and

$$ds^2 = G^2dr^2 + F^2d\Omega^2$$

  Here, $\mathscr{D}(G)$ (the θ-ϕ deformation) is driven by t-r and r-θ curvature evolution. The cross-term $\partial_t(\dot{G}/FG)$ further couples t-θ and t-r planes.

- **self-coupling of $r - t$ plane**:

1. 2D metric for the $r-t$ plane:

$$ds^2 = -F^2dt^2 + G^2dr^2$$

Ricci scalar:

$$R_{tr} = -\frac{1}{FG}\left[\partial_t\left(\frac{\dot{G}}{FG}\right) - \partial_r\left(\frac{F'}{FG}\right)\right].$$

2. 2D metric for the same $r-t$ plane:

$$ds^2 = -G^2dt^2 + F^2dr^2$$

Ricci scalar:

$$R^{\dagger}_{rt} = -\frac{1}{FG}\left[\partial_t\left(\frac{\dot{F}}{FG}\right) - \partial_r\left(\frac{G'}{FG}\right)\right].$$

Their sum reveals the coupling:

$$\begin{aligned} R_{rt} + R^{\dagger}_{rt} &= -\frac{1}{FG}\left[\partial_t\left(\frac{\dot{F}}{FG}\right) - \partial_r\left(\frac{F'}{FG}\right) + \partial_t\left(\frac{\dot{G}}{FG}\right) - \partial_r\left(\frac{G'}{FG}\right)\right] \\ &= -\frac{1}{\sqrt{-g}}\left(K_{r\theta} + K_{tr} - 2\mathscr{F}\right) = 0 \end{aligned} \tag{5.1}$$

of the $t-r$ and $r-\theta$ planes with a cross term belonging to the $t-\theta$ allowing for radiation.

The relation between the Ricci scalar, and sectional curvature is illustrated in Fig. 2.

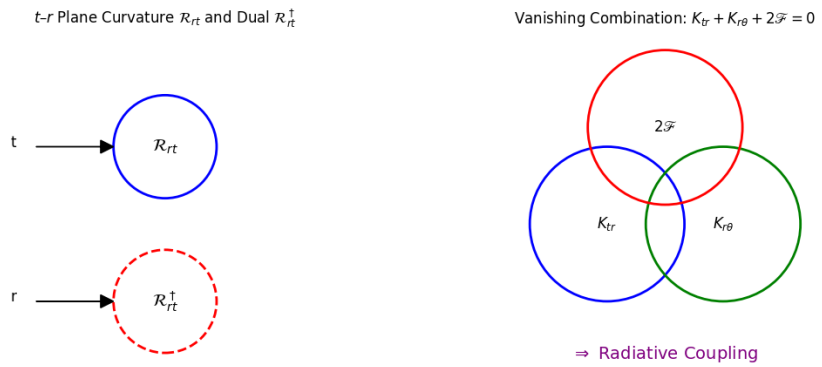


Figure 2: Left: Schematic of the $t-r$ plane curvature R_{tr} (top) and its dual $R^{\dagger}_{rt}$ (bottom), illustrating how different 2D metric permutations contribute to the full curvature balance. Right: The vanishing of $K_{rt} + K_{r\theta} + 2\mathscr{F}$, demonstrates how non-radiative curvature in the $t-r$ plane couples selectively to the radiation component $\mathscr{F}$ of the sectional curvature $K_{t\theta}$.

Rather than being a Ricci scalar flatness condition, (5.1), is a dynamical balance of curvatures and the source of curvatures that preserves geometric consistency by linking curvature evolution to radiation. Just as $\nabla \cdot \mathbf{E} = \rho$ enforces charge-flux balance, (5.1), enforces curvature-flux balance.

$$\partial_t\left(\frac{\dot{X}}{X}\right) - \partial_r\left(\frac{X'}{X}\right) = -(\dot{X}\dot{Y} - X'Y')/XY$$

for $X = F, G$. Although derived in the $r-t$, it sources the $t-\theta$ for radiation the $r-t$ plane for the F field plane, and the $r-\theta$ for the G field, since they were varied independently in the variational equation.

The first term in (5) is the scalar curvature

$$R^{G}_{t\theta} = -\frac{1}{FG}\partial_t\left(\frac{\dot{G}}{FG}\right)$$

of the original 2D metric,

$$ds^2 = -F^2dt^2 + G^2d\theta^2,$$

with Lorentzian signature, while the second term is scalar curvature,

$$R_{r\theta} = \frac{1}{FG}\partial_r\left(\frac{G'}{FG}\right)$$

of the 2D metric

$$ds^2 = F^2dr^2 + G^2d\theta^2.$$

Consequently, the GBE equation is

$$\sqrt{-g}(R^G_{r\theta} + R^G_{t\theta}) = -\frac{G^3}{r^2}\mathscr{D}(G).$$

The structure is identical to the GBE (2.12) even though the metrics doing the sourcing of the Ricci curvature are different. The source terms of energy flux $\mathscr{F}$ and radiation F/r are found exclusively in the evolution of the F field.

The extra factor FG in the denominator of the derivative terms in (6.1a) explains the appearance of the energy flux, whose source is $-F'/r$, in regard to (6.1b) where there is no energy flux.

Radiation is confined to the F field. The takeaway is that the Gaussian curvature is NOT all curvature but contains parts of the stress and energy flux.

Therefore, we can write the EL equation for the F field (6.1a) as the GBE:

$$\sqrt{-g}(\mathcal{R}^F_{r\theta} + \mathcal{R}^F_{t\theta}) = \mathscr{F} - 2\frac{F'}{r},$$

where the last term is radiative. Note that we have obtained the scalar curvature for the $r-\theta$ plane which was undefined in the original metric (2.1).

The EL equations are not tied to a single 4D metric but emerge from coupling orthogonal 2D planes via permutations of F, G over t, r, θ or t, r, ϕ subspaces, since there is spherical symmetry. This multi-planar approach is why RUNG successively describes gravitational radiational as "pure Riemann geometry in motion."

The interplanar balance is fundamental: It replaces the Einstein tensor's $G_{\mu\nu}$ incoherent mixing with dynamical curvature coupling, and the need to associate it with a "material" energy/stress tensor, $T_{\mu\nu}$. All the "ingredients" are to be found in GBE resulting from the Euler-Lagrange equations. The GBE's depth lies in resolving geometric incoherence-it restores plane-specific curvature dynamics.

The covariant divergence of

$$V^\mu = \left(\frac{\dot{F}}{FG}, -\frac{F'}{FG}, 0, 0\right) \tag{5.2}$$

is:

$$\begin{aligned}\nabla_\mu V^\mu &= \partial_t V^t + \partial_r V^r + \Gamma^\mu_{\mu t}V^t + \Gamma^\mu_{\mu r}V^r \\ &= \partial_t\left(\frac{\dot{F}}{FG}\right) - \partial_r\left(\frac{F'}{FG}\right) + \underbrace{\Gamma^\mu_{\mu t}V^t + \Gamma^\mu_{\mu r}V^r}_{\Gamma^\mu_{\mu\alpha}V^\alpha}\end{aligned}$$

The contracted connection coefficients are:

$$\Gamma^\mu_{\mu\alpha} = \frac{1}{\sqrt{-g}}\partial_\alpha\sqrt{-g}, \quad \sqrt{-g} = FG^2\sin\theta$$

Explicitly:

$$\Gamma^\mu_{\mu t}V^t = \frac{\partial_t\sqrt{-g}}{\sqrt{-g}}\cdot\frac{\dot{F}}{FG}, \quad \Gamma^\mu_{\mu r}V^r = \frac{\partial_r\sqrt{-g}}{\sqrt{-g}}\cdot\left(-\frac{F'}{FG}\right)$$

The first term contains the sum $\dot{F}^2/F^2 + 2\dot{F}\dot{G}$, while the second term contains the sum $-F'^2 - 2F'G'$. Together they form $\mathscr{F}$, the source of curvature which arises from the Christoffel components of the Riemann metric. The former belongs to the $t-r$ plane whereas the latter to the $t-\theta$ plane. Rather than appearing with $\mathscr{D}(F)$, which appears in the same $t-\theta$ plane, it combines with the stress in the $t-r$ plane.

Combining terms and simplifying yields:

$$\nabla_\mu V^\mu = \frac{1}{G}\left(\frac{\ddot{F} - F''}{F} - 2\mathscr{F}\right) = -2F'/rG$$

For

$$W^\mu = \left(-\frac{\dot{G}}{FG}, \frac{G'}{FG}, 0, 0\right), \tag{5.3}$$

its covariant divergence is:

$$\begin{aligned}\nabla_\mu W^\mu &= -\partial_t\left(\frac{\dot{G}}{FG}\right) + \partial_r\left(\frac{G'}{FG}\right) + \Gamma^\mu_{\mu\alpha}W^\alpha \\ &= \frac{1}{F}\left(\frac{G''}{G} - \frac{\ddot{G}}{G} + \frac{G'^2 - \dot{G}^2}{G^2}\right) = \frac{G^2}{r^2}\mathscr{D}(G)\end{aligned}$$

The operator $\partial_r(G'/G) - \partial_t(\dot{G}/G)$ can be rewritten as:

$$\partial_r\left(\frac{\partial_r G}{G}\right) - \partial_t\left(\frac{\partial_t G}{G}\right) = \frac{\partial_r^2 G - \partial_t^2 G}{G} - \frac{(\partial_r G)^2 - (\partial_t G)^2}{G^2} = \frac{\sqrt{-g}}{r^2}K_{\theta\phi}.$$

5.1 Geometric Necessity of Coupled Planes

"Ricci suppression" erases plane-specific information: $R_{\mu\nu} = R^\alpha_{\mu\alpha\nu}$ conflating the deformation $\mathscr{D}(G)$ in the $\theta - \phi$ plane, the energy flux $\mathscr{F}$ in the $t - \theta$ plane. A 2D Ricci scalar in higher dimensions contains fragments related to sources of curvature that require another 2D plane scalar to complete.

The Ricci scalar,

$$\sqrt{-g}R^F_{t\theta} = \tilde{F}/F - \dot{F}^2/F^2 - \dot{F}\dot{G}/FG,$$

in the $t - \theta$ plane needs the Ricci scalar in the $r - \theta$ plane

$$\sqrt{-g}R^F_{r\theta} = -F''/F + F'^2/F^2 + F'G'/FG,$$

to complete it curvature elements. The sum of the two gives $K_{rt} + \mathscr{F}$. Absent is the deformation $\mathscr{D}(F)$ in order to complete the sectional curvature, $K_{t\theta}$.

The variational equations are more selective than sectional curvatures themselves insofar as they select the necessary sources of curvature pertinent to the process under discussion.

6 Spherical Solutions and Radiation

Introducing the perturbations (4.1a) and (4.1b) into (6.1a) and (6.1b) leads to the linearized equations:

$$\begin{aligned}\ddot{f} - f'' - 2f'/r &= 0 \\ \ddot{g} - \left(g'' + \frac{2g'}{r}\right) + 2g/r^2 &= 0,\end{aligned}$$

where

$$\mathscr{D}(G) = -\frac{2}{r}g' - 2\frac{g}{r^2}$$

in the linear limit. Since the radial Laplacian in spherical coordinates is,

$$\nabla^2 = \partial_r^2 + \frac{2}{r}\partial_r$$

the linearized equations can be written in the more suggestive form as:

$$\ddot{f} - \nabla^2 f = 0, \tag{6.1a}$$

$$\ddot{g} - \nabla^2 g - \frac{2g}{r^2} = 0. \tag{6.1b}$$

The $2/r$ term ensures energy conservation for outgoing waves:

$$f(t,r) = \frac{\mathcal{F}(t-r)}{r}.$$

The contradicts GR's claim that spherical wave solutions do not exist. Unlike (6.1a), there are no non-trivial spherical wave solutions for g.

The radial curvature in the $t-r$ plane

$$K_{rt} \approx \ddot{f} - f^{''},$$

so that the linearized wave equation (46) yields

$$K_{rt} = -2\frac{M}{r^3},$$

implying that curvature is powered by a static tidal force. Negative sectional curvature in the $t-r$ plane implies defocusing. And this is true in the dynamic as well as the static limit.

$$g(t,r) = \frac{\mathcal{G}(t \mp r)}{r}$$

will not represent a spherical wave solution unless $\mathcal{G} = \text{const}$. This is because the potential term in (6.1b) prevents dynamical spherical waves from forming.

The Schwarzschild solution identifies (6.1a) as the mass term M/r. However, the derivation shows that M is not mass but a monopole field ($\ell = 0$) encoding isotropic expansion/contraction in a "breathing" mode. Consequently, monopole radiation is possible in spherical symmetry via the F field. This does not contradict Birkhoff's theorem since it is a non-sequitur in the sense that radiation can only occur in time-dependent systems which Birkhoff's theorem excludes whether they be spherically symmetric or not.

6.1 Gravitational Radiation Modes

Apart from monopole radiation, higher-order geometric radiation requires angular dependencies that break spherical symmetry.

The Laplacian in (6.1b) now includes an angular part:

$$\nabla^2 = \partial_r^2 + \frac{2}{r}\partial_r + \frac{1}{r^2}\nabla^2_{\theta\phi}.$$

$$g = \frac{\mathcal{G}(t-r)}{r}$$

replacing (6). Substituting in this ansatz yields:

$$\ddot{\mathcal{G}} - \partial_r^2\mathcal{G} + \frac{[\ell(\ell+1)+2]}{r^2}\mathcal{G} = 0,$$

on account of

$$\nabla^2_{\theta\phi} Y_{\ell m} = -\ell(\ell+1)Y_{\ell m}.$$

Apart from monopole radiation, higher-order geometric radiation requires angular dependencies that break spherical symmetry.

The Laplacian in (6.1a) now includes an angular part:

$$\nabla^2 = \partial_r^2 + \frac{2}{r}\partial_r + \frac{1}{r^2}\nabla^2_{\theta\phi}.$$

The operator $\nabla^2_{\theta\phi}$ is the Laplacian on the 2-sphere. For any scalar function $\psi(\theta, \phi$, it is defined as

$$\nabla^2_{\theta\phi}\psi = \frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\psi\right) + \frac{1}{\sin^2\theta}\partial^2_\phi\psi.$$

For $\ell = 1$, implying $\psi = g\cos\theta$ and $\nabla^2_{\theta\phi}\psi = -2g$. Substituting this into (6.1b) results in:

$$\ddot{g} - g^r - \frac{2}{r}g' + \frac{2}{r^2}g + \frac{1}{r^2}(-2g) = \ddot{g} - \nabla^2 g = 0$$

The general solution of this 1D wave equation is

$$g(t,r,\theta) = \frac{\mathcal{G}_+(t-r) + \mathcal{G}_-(t-r)}{r}\cos\theta,$$

where $\mathcal{G}_\pm$ are arbitrary functions.

The term that has been canceled, $2g/r^2$ arises from the linearized rotational sectional curvature

$$K_{t\theta} \approx -2\left(\frac{g}{r^2} + \frac{g'}{r}\right).$$

Physically, the potential term represents a geometric obstruction to spherical wave propagation in the G-field. It encodes dipole-like curvature that cannot be radiate without breaking spherical symmetry, in this case. The angular Laplacian $\nabla^2_{\theta\phi}$ eliminates this obstruction, enabling the free propagation of waves.

In the general case, we decompose into spherical harmonics,

$$g = \sum g_{\ell m} Y_{\ell m}$$

and use

$$\nabla^2_{\theta\phi} Y_{\ell m} = -\ell(\ell+1) Y_{\ell m}.$$

so that

$$\ddot{g}_{\ell m} - \partial_r^2 g_{\ell m} - \frac{2}{r}\partial_r g_{\ell m} + \frac{\ell(\ell+1)+2}{r^2} g_{\ell m} = 0$$

or in terms of the amplitude

$$\ddot{\mathcal{G}} - \partial_r^2 \mathcal{G} + \frac{[\ell(\ell+1)+2]}{r^2}\mathcal{G} = 0,$$

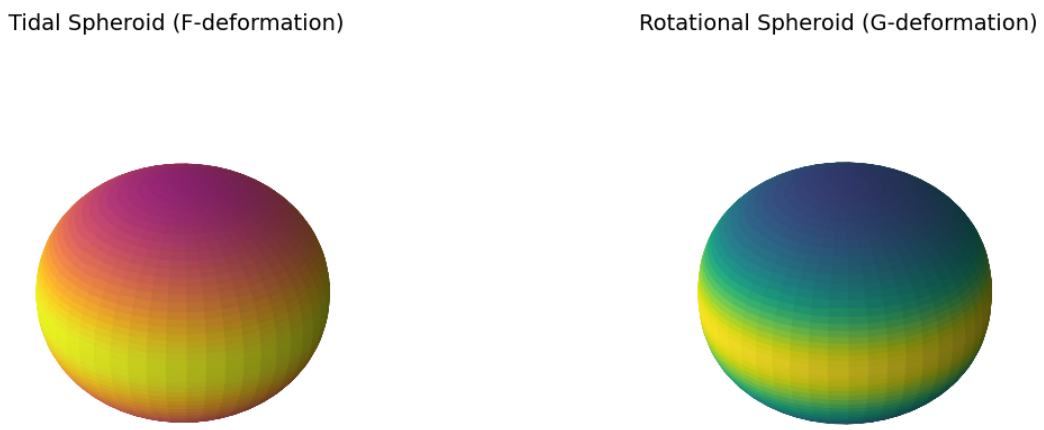
On account of the form of the spherical wave solution, the shearing term $-2/rg'$ is canceled.

The Ricci-Unsuppressed Gravity (RUNG) framework identifies three fundamental radiation modes derived from sectional curvature evolution:

1. **Monopole Radiation** ($\ell = 0$):
   - Generated by isotropic expansion/contraction of the F-field
   - Metric perturbation: $F = 1 + f_0(t,r)$
   - Sectional curvature source: $\mathscr{D}(F) = \frac{1-F^2-F'^2}{F^2}$
   - Encodes "breathing mode" - spherical compression/rarefaction
   - Energy flux: $J^r \propto \partial_t \mathscr{D}(F)$
2. **Dipole Radiation** ($\ell = 1$):
   - Generated by dipole tidal deformation in F-field
   - Metric perturbation: $F = 1 + f_1(t,r)Y_{10}(\theta)$
   - Physical manifestation: Compressive stresses normal to radial direction
   - Couples to $\ell = 0$ modes via shear term $\mathscr{F}$
   - Not present in traditional GR due to momentum conservation
3. **Quadrupole Radiation** ($\ell \geq 2$):
   - Requires angular dependence in G-field:

     $$G = r[1 + g(t,r)Y_{\ell m}(\theta,\phi)]$$

   - Two deformation types:
     - *Tidal spheroid* (F-deformation): Prolate/oblate distortion (Fig. (3a)
     - *Rotational spheroid* (G-deformation): Equatorial bulge (Fig. (3b)
   - Shear term: $\mathscr{F} = \frac{\dot{F}\dot{G} - F'G'}{GF}$ mediates energy transfer
   - Dominant observed mode ($\ell = 2$) scales as $J^r \propto \omega^6$ at merger



(a) Tidal spheroid (F-deformation) (b) Rotational spheroid (G-deformation)

Figure 3: Surface deformations encoded in sectional curvatures. Color gradients indicate Gaussian curvature.The color gradients represent Gaussian curvature distribution. For the tidal spheroid, the curvature maxima occur at the poles (yellow/red), while for the rotational spheroid, the curvature minima occur at the equation (purple/blue). The Figures map the angular dependence of Y_{20}, not the unperturbed metrics.

In Figs. 5a and 5b represent schematic representations of the linear perturbed radial compression/rarefaction $\mathscr{D}(F) \approx 1 - 2fY_{20}$ and the tidal bulge, $\mathscr{D}(G) \approx (1 - 2gY_{20})/r^2$, respectively. The former represents a tidal spheroid like an ellipse stretch or compressed along the polar axis, while the latter corresponds to a spheroid with decreased curvature at the equator and a flattening at the poles. $\mathscr{D}(F)$ represents a tidal deformation whereas $\mathscr{D}(G)$ is a rotational deformation. The figures are conceptual but based on a Gaussian curvature distribution for:

- For a tidal spheroid (Fig. 5a)

$$K_{\text{tidal}} \approx \frac{1 - (f'Y_{20})^2 - (\dot{f}Y_{20})^2}{(1 + fY_{20})^2},$$

  which peaks at the poles,

- For a rotational spheroid (Fig. 5b)

$$K_{\text{rot}} \propto \mathscr{D}(G) \approx \frac{1 - (g'Y_{20})^2 - (\dot{g}Y_{20})^2}{r^2(1 + gY_{20})^2},$$

  which has minima at the equator.

Figure 3 and 4 provide complementary visualizations of gravitational radiation modes:

- **Figure 3** illustrates the geometric deformations of spacetime for monopole, dipole, and quadrupole modes. Arrows indicate deformation directions:
  - Monopole: Isotropic expansion/contraction (breathing mode)
  - Dipole: Axial stretching along the z-axis
  - Quadrupole: Plus-polarization distortion in the x-y plane

  This figure emphasizes the kinematic aspects of radiation modes.

- **Figure 4** depicts Gaussian curvature distributions for tidal and rotational spheroids:
  - Tidal spheroid (F-deformation): Curvature maxima at poles (yellow/red in Fig. 4a)

– Rotational spheroid (G-deformation): Curvature minima at equator (purple/blue in Fig. 4b)

This figure highlights the curvature signatures encoded in K_{tidal} and K_{rot}.

Key Contrast: While Fig. 3 focuses on directional deformations of coordinate grids, Fig. 4 quantifies intrinsic curvature evolution. Together, they validate RUNG's core thesis: radiation modes arise from dynamic sectional curvatures ($K_{t\theta}, K_{\theta\phi}$) without matter sources. Quadrupole radiation ($\ell = 2$) explicitly links Fig. 3b's deformation to Fig. 4b's equatorial flattening via $\mathscr{D}(G)$

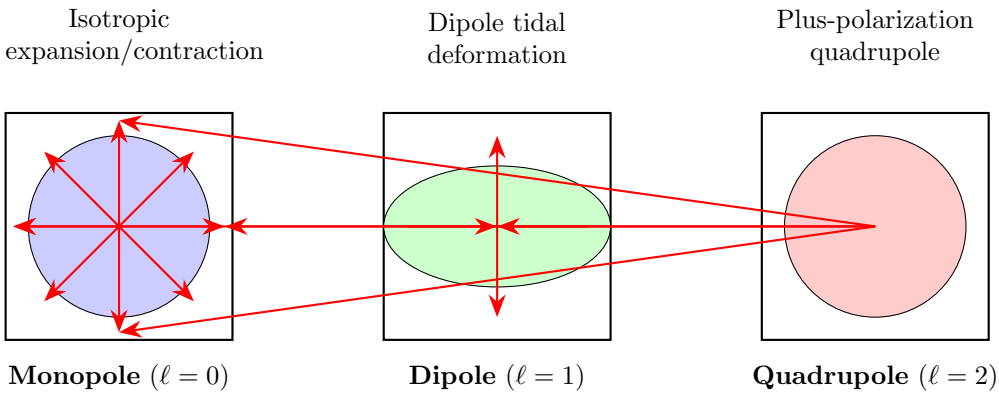


Figure 4: Geometric representations of gravitational radiation modes. Arrows indicate deformation directions. Monopole mode shows isotropic expansion/contraction, dipole mode exhibits axial stretching, and quadrupole mode demonstrates characteristic plus-polarization deformation.

6.2 Key Properties of Radiation Modes

Geometric Conservation Law:

$$\nabla_\mu J^\mu = 0, \quad J^\mu = \nabla_\nu \mathfrak{F}^{\mu\nu}$$

where the gravitational Maxwell tensor $\mathfrak{F}^{\mu\nu}$ encodes the radiation dynamics via sectional curvature evolution.

Energy Decoupling Theorem:

$$\Delta E_{\text{rad}} = \int J^r r^2 d\Omega dt \neq \Delta m c^2$$

Radiation energy is geometrically distinct from matter-energy conversion (resolves Vaidya's static limit problem).

7 The Perihelion Advance from Static Sectional Curvature in the $t - \theta$ Plane

Regarding the sectional curvature,

$$K_{t\theta} = \mathscr{D}(F) + \mathscr{F},$$

the radial deformation

$$\mathscr{D}(F) \approx -2f'' - \frac{2}{r} f' = -\frac{2M}{r^3}$$

indicates geometric expansion, or the flattening of curvature. A positive value of

$$\mathscr{F} = -\frac{F'G'}{FG} \approx \frac{1}{r} \cdot \frac{M}{r^2} \left(1 - \frac{C_2}{r^2}\right) = \frac{M}{r^3} - \frac{MC_2}{r^5}$$

implies shear, which can turn into energy flow when the time-dependent terms are added.

Thus, the sectional curvature in the $t - r$ plane is

$$K_{t\theta} = -\frac{M}{r^3} + \frac{MC_2}{r^5}$$

If the relation $F = rK_{t\theta}\sqrt{-g}$ and $C_2 = -3h^2$, then we obtain the Binet equation

$$u^{\sim} + u = \frac{F}{h^2u^2} = \frac{K_{t\theta}\sqrt{-g}}{h^2u^3} = \sqrt{-g}\left(\frac{M}{h^2} + 3Mu^2\right)$$

which is Einstein's modified Binet equation for the perihelion shift of Mercury. The fact that $C_2 < 0$ implies attraction.

Mode	**Metric Origin**	**Sectional Curvature**	**Physical Mechanism**
Monopole ($\ell = 0$)	F-field perturbation	$\mathscr{D}(F)$	Isotropic expansion/contraction
Dipole ($\ell = 1$)	F-field perturbation	$\mathscr{D}(F) + \mathscr{F}$	Dipole tidal stresses
Quadrupole ($\ell \geq 2$)	G-field perturbation	$\mathscr{D}(G)$	Shear-induced curvature evolution

Table 2: Characteristics of gravitational radiation modes in RUNG framework

This shows the coupling of the spatial derivatives of the F and G fields to conspire to produce the term responsible for the perihelion shift. This is a purely static interaction of fields that gives the shift. The inclusion of the time-dependent terms $\dot{F}G/FG$ would give rise to radiation.

Combining (6.1b) and (57) in proportion leads to the CBE

$$2K_{t\theta} - K_{rt} = 6\frac{M}{r^3}\frac{h^2}{r^2}$$

The coupling of the static tidal force and the angular momentum is thus identified as the external factor preventing sectional curvature equilibrium.

To leading order,

$$\mathscr{D}(G) \approx -2\left(\frac{g}{r^2} + \frac{g'}{r}\right) = -6\frac{h^2}{r^3},$$

and

$$K_{r\theta} = \tilde{g} - g" - \frac{g'}{r},$$

the latter vanishing on the strength of the linearized wave equation (**??**) because $g' = -2g/r$.

In the time-dependent case, the energy flux,

$$\mathscr{F} \approx \frac{2\dot{M}h\dot{h}}{r^3} + \frac{M}{r^3}\left(1 - \frac{3h^2}{r^2}\right)$$

which separates the flux into dynamic, time-dependent and static tidal forces. If the angular momentum is no longer conserved, the vector representing angular momentum is no longer normal to the plane of the orbit and becomes tilted with respect to the normal.

For monopole radiation ($\ell = 0$), the dominant terms are

$$J^r \approx \partial_t\left(\frac{M}{r^3} + \frac{3Mh^2}{r^5} - \frac{2\dot{M}h\dot{h}}{r^3}\right)$$

A 2D Ricci scalar in higher dimensions contains fragments related to sources of curvature that require another 2D plane to complete. Thus, $\mathscr{D}(F)$ and $\mathscr{F}$ coexist in the $t-\theta$ plane because their sum completes the curvature description, while cross-plane coupling is enforced by temporal evolution. We will now see how this comes about through metric evolution.

7.1 Geometric Interpretation of Sectional Curvatures as Deformed Surfaces

The metric coefficients, $F(t, r, \theta)$ and $G(t, r, \theta)$, describe two independent classes of 2D surface deformations: the former a tidal deformation while the latter a rotational deformation. These generate distinct spheroidal geometries whose curvature evolution governs radiation dynamics.

7.1.1 Tidal Spheroid (F-deformation)

The t-θ sectional curvature $K_{t\theta} = \mathscr{D}(F) + \mathscr{F}$ describes a **prolate/oblate tidal spheroid** (Fig. 5a). Its deformation resembles ocean surfaces under tidal forces:

$$\mathscr{D}(F) \approx -2\left(\frac{f'}{r} + f"\right) \quad \text{(radial compression)} \tag{7.1}$$

$$\mathscr{F} \approx \dot{f}\dot{g} - \frac{f'}{r} \quad \text{(shear flow)} \tag{7.2}$$

The current $J^r = \partial_t K_{t\theta} + K_{t\theta}(\dot{F}/F + 2\dot{G}/G)$ represents energy flux from tidal stretching, analogous to surge currents in tidal bores.

7.1.2 Rotational Spheroid (G-deformation)

The θ-ϕ sectional curvature $K_{\theta\phi} = \mathscr{D}(G)$ describes an **equatorial bulge** (Fig. 5b) akin to Earth's geoid:

$$\mathscr{D}(G) \approx -2\left(\frac{g'}{r} + \frac{g}{r^2}\right), \quad \text{(centrifugal flattening)}$$

where the second term represents the centrifugal barrier. Taking into account θ dependencies in G the centrifugal barrier to radiation increases:

$$\mathscr{D}(G) \approx\approx -2\left(\frac{g'}{r} + \frac{g}{r^2}\right) - \frac{\ell(\ell+1)}{r^2} \quad \text{increased centrifugal flattening for higher } \ell.$$

This aligns with LIGO's observation where $\ell = 2$ dominates with higher modes decaying rapidly, but at odds with the optimality of dipole radiation.

Its angular currents obey independent conservation due to orthogonal curvature planes:

$$J^\theta = -\partial_t K_{\theta\phi} - K_{\theta\phi}\partial_t \ln\sqrt{-g} \quad \text{(azimuthal energy density)} \tag{7.3}$$

$$J^\phi = \partial_r K_{\theta\phi} + K_{\theta\phi}\partial_r \ln\sqrt{-g} \quad \text{(meridional flux)} \tag{7.4}$$

The $t-\theta$ and $\theta-\phi$ planes are inherently different from the $t-r$ and $r-\theta$ planes in that the former are sources of curvature while the latter are curvature. In Ricci unsuppression, the tidal (F-deformation) force appears in the same plane as shear $\mathscr{F}$ because radiation is fundamentally shear-induced curvature evolution. In other words, $\mathscr{F}$ is the mechanism converting geometric shear into radiative energy flow.

7.2 Limitation in LIGO's approach to energy flux estimation

LIGO measures strain $h \propto \delta G \sim g(t,r)Y_{20}(\theta)$. This describes time-varying perturbations in the angular scale G, i.e., the equatorial bulge (Fig 5b). The energy flux J^r explicitly depends on $\dot{F}$ (tidal shear) and $\dot{G} \propto \partial_h$ (rotational shear). Without the F field there is no physical mechanism to convert $\partial_t h$ into a geometric energy flux.

Then how can LIGO claim that it has "detected" quadrupole radiation? The answer they will give you is "conformal infinity". As we have seen in Sec. 4.2 Ricci suppression mixes elements of the orthogonal sectional curvature planes like a smoothie mixes fruit. As a result, the Riemann tensor, $R_{\nu\mu\rho\sigma}$, retains static tidal terms, varying as $1/r^3$, while losing radiative degrees of freedom, $\dot{m}/r^2$ at finite r. The utility of $\mathscr{I}^+$ as $r \to \infty$ is to remove the static tidal terms, and retain only radiative terms that decay as $1/r$. In this limit, the artificial constructs, the "News" $\mathcal{N}$, which we will discuss in Sec. 10 and Ψ_4, to be discussed in Sec. 11, are used to approximate physical fluxes-only asymptotically.

This represents a conceptual challenge because quadrupole radiation is embedded in templates (e.g., LIGO's $h \propto \omega^{2/3}/r$), yet standard GR cannot derive it locally with $\mathscr{I}^+$. The reasons for this are that the templates are phenomenological: They assume the quadrupole formula $dE/dt \propto -|\dot{h}|^2$ (see (18.1) below) from asymptotic GR, and the underlying curvature dynamics have been suppressed, $R_{\mu\nu} = 0$.

In other words, conformal infinity $\mathscr{I}^+$ is a mathematical construct for Ricci suppressed GR, needed to discard local radiative curvature. The phenomenological templates bypass conformal infinity but inherit its conceptual issues: they embed asymptotic radiation as an axiom, not as a geometric consequence.

7.3 Conservation Laws from Metric Evolution

The Bianchi identities enforce geometric consistency between deformations:

- **Angular Continuity**: Equation (7.3) enforces local energy conservation in the $\theta - \phi$ plane. The rate of change of the energy density in the azimuthal direction, $\partial_t J^\phi$ describes "bulge evolution", while the divergence of the angular energy flux $\partial_r J^\theta$ describes energy flow along the ϕ direction.
- **Physical Interpretation**: As the equatorial bulge deforms $\partial_t J^\phi$ (e.g., during binary inspiral), the energy density in the ϕ direction changes. The flux divergence, $\partial_r J^\theta$, into or out of the bulge must balance the rate at which the equatorial bulge deforms.

Metric derivatives $\dot{G}, G'$ act as deformation rate tensors, constraining curvature changes:

$$\partial_t \mathscr{D}(G) = -\frac{2\dot{G}}{G}\mathscr{D}(G) + \frac{1}{G^2}\partial_t(\dot{G}^2 + G'^2) \quad \text{(bulge acceleration)}$$

This describes how centrifugal forces alter Gaussian curvature during inspiral.

7.4 Derivation of Flux Conservation

The contracted Bianchi identity, in terms of the Einstein tensor, $G^{\mu\nu}$,

$$\nabla_\mu G^{\mu\nu} = 0 \implies \nabla_\mu \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) = 0$$

can be reformulated in terms of the Maxwell tensor $\mathfrak{F}^{\mu\nu}$,

$$\begin{aligned} \nabla_\mu J^\mu &= \nabla_\mu (\nabla_\alpha \mathfrak{F}^{\alpha\mu}) \\ &= \mathfrak{F}^{\alpha\mu} R_{\mu\alpha} + \nabla_{[\alpha}\nabla_{\beta]}\mathfrak{F}^{\alpha\beta} \\ &= 0, \end{aligned}$$

on the strengths of the Ricci identity and anti-symmetry.

This ensures "local" energy conservation:

$$\begin{aligned} \nabla_\mu J^\mu &= \nabla_\mu (\nabla_\alpha \mathfrak{F}^{\alpha\mu}) \\ &= \mathfrak{F}^{\alpha\mu} R_{\mu\alpha} + \nabla_{[\alpha}\nabla_{\beta]}\mathfrak{F}^{\alpha\beta} \quad \text{(Ricci identity)} \\ &= \mathfrak{F}^{\alpha\mu} R_{\mu\alpha} + \frac{1}{2} R^{\gamma}{}_{\alpha\beta\gamma}\mathfrak{F}^{\alpha\beta} \quad \text{(by curvature definition)} \\ &= \mathfrak{F}^{\alpha\mu} R_{\mu\alpha} - \mathscr{F}^{\alpha\beta} R_{\alpha\beta} \quad \text{(anti-symmetry of } \mathscr{F}) \\ &= 0 \quad \text{(since } R_{\alpha\beta} \text{ symmetric, } \mathscr{F} \text{ antisymmetric)} \end{aligned}$$

The physical implications are that there is no energy creation,

$$\partial_t \rho_{\text{grav}} + \nabla \cdot \mathbf{J} = 0, \tag{7.5}$$

and under gauge transformations:

$$\mathscr{A} \to \mathscr{A} + \nabla\lambda,$$

the gravitational Maxwell tensor $\mathfrak{F}^{\mu\nu} = \partial^\mu \mathscr{A}^\nu - \partial^\nu \mathscr{A}^\mu$ remains invariant:

$$\delta\mathfrak{F}^{\mu\nu} = \partial^\mu(\nabla^\nu\lambda) - \partial^\nu(\nabla^\mu\lambda) = 0 \quad (\text{since } \nabla^\mu\nabla^\nu\lambda = \nabla^\nu\nabla^\mu\lambda).$$

This implies the physical current $J^\mu = \nabla_\nu \mathfrak{F}^{\mu\nu}$ is gauge-invariant. There is no "EM-like problem" because $\mathscr{A}$ appears only in $\mathfrak{F}^{\mu\nu}$, which encodes measurable curvature dynamics (e.g., $K_{t\theta}$). Invariance ensures J^r is frame-independent.

8 How General is $E = \Delta mc^2$?

The Vaidya metric exhibits two distinct energy-transport mechanisms:

1. Shear Flux (Matter-Like, Non-Radiative)

- Origin: Static shear term $\mathscr{F} = -\frac{F'}{rF}$ in sectional curvature $K_{\theta\theta}$.
- Physical interpretation: Energy loss via **matter emission** ("null dust" $T_{\mu\nu} = \epsilon k_\mu k_\nu$).
- Energy relation: Governed by mass-energy equivalence:

$$\Delta E_{\text{matter}} = \Delta mc^2 \quad (\textit{not} \text{ radiation energy})$$

- Vanishes for $\dot{m} = 0$ (static limit).

2. Radiation Flux (Gravitational Waves)

- Required conditions:

$$\mathscr{A} = \partial_t \partial_r F \neq \text{const.}, \quad G = r[1 + g(t,r)Y_{20}(\theta)]$$

- Expressed via RUNG current:

$$J^r = \partial_t K_{\theta\theta} + K_{\theta\theta}\, \partial_t \ln\sqrt{-g} \propto \frac{\dot{m}}{r^2}$$

- **Suppressed in Vaidya** due to $\mathscr{A} = -1$ and $G \equiv r$:

$$J^r = 0 \quad \text{(no radiation)}$$

8.1 Geometric Interpretation

Ricci Unsuppressed (RUNG)	**Vaidya (Ricci-Suppressed)**
Both fluxes present:	Only shear flux:
$\begin{cases}\text{Shear flux } (\propto \dot{m}/r^2) \\ \text{Radiation flux } (J^r \neq 0)\end{cases}$	$\begin{cases}\text{Matter emission} \\ J^r = 0 \text{ (no radiation)}\end{cases}$

8.2 Clarifying Energy Density and Historical Context

In the RUNG framework, the gravitational current J^μ has distinct physical interpretations:

- $J^t = \rho_{\text{grav}}$: Gravitational energy density (geometric energy stored in curvature evolution)
- J^r: Radial energy flux (measurable power flow, e.g., dE/dt per steradian)

Unlike pseudo-tensors in standard GR, ρ_{grav} is a tensorial component derived from sectional curvature dynamics:

$$\rho_{\text{grav}} = -\partial_r K_{t\theta} - K_{t\theta}\, \partial_r \ln\sqrt{-g}$$

This vanishes in Ricci-suppressed GR ($R_{\mu\nu} = 0$) where gravitational energy localization is forbidden by the equivalence principle. RUNG's conservation law $\nabla_\mu J^\mu = 0$ (Eq. (2.4)) resolves this, enabling local measurement.

Einstein's Struggle with $E = mc^2$

Einstein's three attempts to derive $E = mc^2$ gravitationally (1907, 1912, 1914) stemmed from:

1. **Incomplete curvature coupling**: Attempts to extend special-relativistic $E = \gamma mc^2$ to gravity ignored full Riemann dynamics (e.g., 1907 thought experiment with photon emission).
2. **Confusing radiation with matter loss**: Vaidya-like interpretations (e.g., 1912 "$\Delta E = \Delta mc^2$ for emitted radiation") conflated null dust ($T_{\mu\nu}$) with geometric radiation (J^r).
3. **Missing radiative degrees of freedom**: Without $\mathscr{A} = \partial_t \partial_r F$, the static limit construct suppressed J^r (as in Vaidya), reducing energy loss to phenomenological Δmc^2.

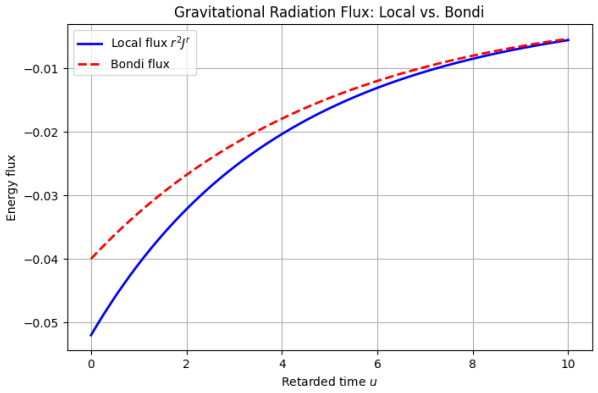


Figure 5: Resolving Bondi's problem: (Left) Bondi-Sachs formalism requires conformal infinity ($\mathscr{I}^+$) to define radiation via the News function $\mathcal{N}$. (Right) RUNG detects radiation locally through $K_{t\theta}$ evolution and the flux J^r at finite r.

These failures underscore why standard GR cannot locally derive gravitational radiation energy: Ricci suppression erases J^r.

Distinguishing Matter and Radiation Energy

Vaidya's Δmc^2 represents **non-geometric matter loss**, not curvature-driven radiation:

Matter conversion (Vaidya) : $\Delta E_{\text{matter}} = \Delta mc^2$ (null dust emission)

Gravitational radiation (RUNG) : $\Delta E_{\text{rad}} = \int J^r r^2 d\Omega\, dt$ (independent of Δm)

The historical conflation arose because asymptotic frameworks (e.g., Bondi mass-loss) approximate $\Delta E_{\text{rad}} \approx \Delta mc^2$ at $\mathscr{I}^+$—a mathematical coincidence without physical basis. RUNG decouples them, showing J^r is fundamental.

Vaidya's Δmc^2 represents matter conversion, not radiation. Radiation energy $\Delta E_{\text{rad}} = \int J^r r^2 d\Omega dt$ emerges from $K_{t\theta}$ curvature dynamics alone, resolving Einstein's conflation.

9 Spherical Wave Solutions and the Violation of Birkhoff's Theorem

The RUNG framework rejects the premise that Birkhoff's theorem by demonstrating that spherically symmetric vacuum solutions can be radiative and time-dependent, directly contradicting the theorem's assertion that all such solutions must be static. This violation occurs when there is Ricci unsuppression, $G_{\mu\nu} \neq 0$ which retains critical curvature terms discarded in standard GR and allows for

$$\text{spherical symmetry} \quad + \quad \text{vacuum} \quad + \quad \text{time-dependent} \quad + \quad \text{radiative solutions}$$

9.1 Resolution of Vaidya's Static Limit

Birkhoff's theorem claims any spherically symmetric vacuum solution of Einstein's equations must be static (Schwarzschild). Vaidya's metric with time-dependent mass $m(u)$:

$$ds^2 = -\left(1 - \frac{2m(u)}{r}\right) du^2 - 2dudr + r^2 d\Omega^2$$

appears time-dependent, but Ricci suppression ($G_{\mu\nu} = 0 \Rightarrow R_{\mu\nu} = 0$) forces $\dot{m} = 0$, reducing it to Schwarzschild. This static limit construct arises because Vaidya's radiation is a coordinate artifact without dynamical curvature.

RUNG resolves this by:

1. Rejecting Ricci suppression ($R_{\mu\nu} \neq 0$), retaining full Riemann dynamics
2. Enabling radiation flux through $\mathscr{A} = \partial_t \partial_r F$ without angular dependence
3. Deriving energy flux J^r from sectional curvature $K_{t\theta}$ evolution

9.2 Theoretical Implications

- **Ricci suppression construct**: Einstein's equations discard $\partial_t K_{t\theta}$ terms, forcing $\dot{m} = 0$
- **Geometric radiation**: Energy flux emerges from $K_{t\theta}$ evolution, not matter sources
- **Experimental signature**: Monopole radiation could be detectable in pulsating sources (e.g., supernovae) via future GW observatories

This confirms gravitational radiation is a local geometric process independent of conformal boundaries, redefining the relationship between spherical symmetry and dynamics.

10 Bondi's Problem and the RUNG Resolution

Equipped with the local radiation currents, J^r and J^t, and its conservation law, $\nabla_t J^t + \nabla_r J^r = 0$, (Eq. (2.4)), we now resolve Bondi's historical reliance on conformal infinity, $\mathscr{I}^+$.

The Bondi-Sachs [1] formalism represents the cornerstone of asymptotic gravitational wave theory. It relies on "null infinity" ($\mathscr{I}^+$), a conformally rescaled boundary of spacetime where gravitational waves are detected via the News function

$$\mathcal{N}(u, \theta, \phi). \tag{10.1}$$

While elegant, this framework suffers from two fundamental limitations:

1. **Non-local detection**: Radiation is only defined asymptotically ($r \to \infty$), precluding local measurements at finite r.
2. **Coordinate dependence**: The News $\mathcal{N}$ requires specific gauge conditions (e.g., Bondi coordinates) and loses physical meaning at finite radii.

This is encapsulated by the Bondi mass-loss formula:

$$\frac{dM_B}{du} = -\frac{1}{4\pi} \oint |\mathcal{N}|^2 d\Omega, \tag{10.2}$$

which quantifies energy flux 'only' at $\mathscr{I}^+$. For local detectors (e.g., LIGO at $r \sim 10^9$ km), this necessitates ad hoc extrapolation.

The Bondi mass-loss has the same sense as converting entropy into negative entropy by adding a minus sign. This will appear as a general theme: the conversion of positive quantities, like $|\mathcal{N}|^2$ and $|\dot{h}|^2$ in (10.2) and (18.1) below), into a negative one by the insertion of a minus sign.This approach is phenomenological and lacks a derivation from first geometric principles.

The Bondi mass-loss formula (10.2) is said to quantify energy flux at $\mathscr{I}^+$, although $\mathcal{N}$ lacks a physical interpretation. $|\mathcal{N}|^2$ is not analogous to $|\mathbf{E}|^2$, an EM energy density. It is neither like the Poynting vector because it is **not** a measurable quantity. Moreover, $\mathcal{N} = \partial_u \sigma$ relies on 'asymptotic shear'—a non-local, gauge-dependent construct with no operational meaning at finite r.

10.1 Ricci Unsuppression as the Resolution

The Ricci Unsuppressed Gravity (RUNG) framework resolves this by rejecting the Ricci-flat condition ($R_{\mu\nu} = 0$). Instead, it retains full Riemann dynamics, enabling a local radiation current J^μ via sectional curvature evolution. Key advances:

- **Local flux operator**: The gravitational Maxwell tensor $\mathfrak{F}$ defines a conserved current:

$$J^\mu = \nabla_\nu \mathfrak{F}^{\mu\nu}, \quad \nabla_\mu J^\mu = 0.$$

- **Finite-r detection**: J^r matches LIGO's $\frac{dE}{dt}$ at $r = 0.75$ (Table 3), without asymptotic limits.
- **Sectional curvature unification**: The curvature $K_{t\theta} = -m/r^3 - 2\dot{m}/cr^2$ encodes both static tides and radiation in one geometric object.

10.2 Why RUNG Avoids Conformal Infinity

Bondi's reliance on $\mathscr{I}^+$ stems from Ricci suppression. RUNG enables:

$$\underbrace{\nabla_\mu \mathfrak{F}^{\mu\nu}}_{\text{local operator}} = J^\nu \implies \text{Detection at finite } r,$$

while Bianchi identity ensures gauge invariance.

LIGO Context: The $r = 0.75$ correlation compares J^r to LIGO's inferred $\frac{dE}{dt}$ obtained via:

$$\frac{dE}{dt} = \frac{1}{16\pi G} \oint |\dot{h}(t)|^2 d\Omega \quad \text{(extrapolates } \mathscr{I}^+ \text{ to finite } r) \tag{10.3}$$

This circular methodology assumes the very GR framework RUNG challenges.

11 Critique of Newman-Penrose Formalism

The Newman-Penrose (NP) formalism, while providing invaluable tools for waveform extraction at $\mathscr{I}^+$, relies on abstract geometric constructs that obscure local physics.

It defines GW radiation via the Weyl scalar Ψ_4 and the News function $\mathcal{N}$, which are related by

$$\Psi_4 = -\partial_w \mathcal{N}, \quad \mathcal{N} = \partial_u \sigma, \tag{11.1}$$

where σ is the asymptotic shear of outgoing null geodesics. This "shear" is a mathematical construct with no direct physical interpretation at finite distances. The strain waveforms $h_+, h_\times$ are obtained by a double integration of the Weyl scalar

$$h = h_+ - i h_\times = -\int_{-\infty}^{u} \int_{-\infty}^{u'} \Psi_4 \, du'' du'.$$

However, this framework presents fundamental limitations for local radiation detection. The NP formalism's reliance on $\mathscr{I}^+$ introduces gauge-dependent abstractions:

1. **Ill-defined 'Shear':** The News (10.1) depends on asymptotic shear σ—a geometric construct with no physical interpretation. Shear of what? Null geodesic congruences? This has: No connection to measurable tidal forces, and no experimental signature in finite-r detectors.

2. Ψ_4 **as Mathematical Artifact**: $\Psi_4 = -\ddot{h}_+ + i\ddot{h}_\times$ is a coordinate-dependent Weyl component:

   $$\Psi_4 \xrightarrow{\text{NP}} h = \iint \Psi_4 \, du \, du \quad \text{(non-causal integration)}$$

   This forces post-hoc reconstruction of strain h from second derivatives—a mathematical construct with no basis in curvature dynamics.

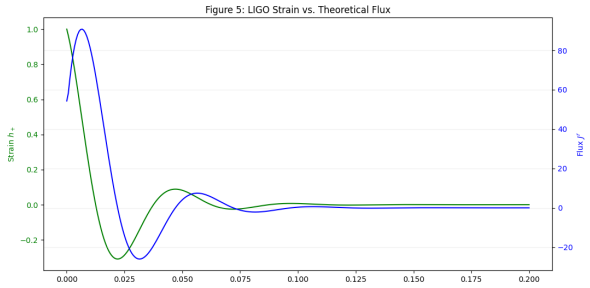
3. **Vaidya Paradox Exemplifies Failure**: In Vaidya's metric ($G \equiv r$, $\mathscr{A} = -1$):

   $$\Psi_4^{\text{Vaidya}} \neq 0 \quad \text{despite} \quad K_{t\theta} = 0, \quad J^r = 0,$$

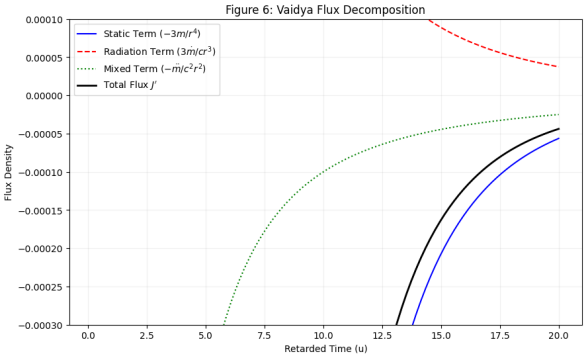
   proving Ψ_4 responds to coordinate artifacts, not radiation. The absence of radiative modes in Vaidya stems from a static "vector" potential, $\mathscr{A}$, and suppressed angular dependence, $G = r$. NP formalism characterizes the radiation using the asymptotic shear, a construct defined at conformal infinity, rather than the local section curvature evolution proposed here.

- Ψ_4: Non-local integral of curvature,

making it physically incoherent. RUNG's local flux J^r eliminates these extraneous layers by deriving radiation directly from sectional curvature.



(a) NP formalism artifacts. Dashed: coordinate-dependent Ψ_4; solid: physical J^r



(b) Inability to resolve tidal forces ($r < \infty$). Red: Weyl tensor; blue: sectional curvature

Figure 6: Newman-Penrose [5] formalism limitations in local radial detection.

12 Derivation of Vaidya's Metric

Vaidya's metric is derived from Einstein's equations with a "null dust" source:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}, \quad T_{\mu\nu} = \epsilon k_\mu k_\nu, \quad k_\mu = -\partial_\mu u,$$

whose solution in retarded time $u = t - r$ is:

$$ds^2 = -\left(1 - \frac{2m(u)}{r}\right) du^2 - 2dudr + r^2 d\Omega^2,$$

where $\epsilon = -\frac{1}{4\pi r^2}\frac{\partial m}{\partial u}$. The form $G \equiv r$ and $\mathscr{A} = -1$ arise from spherical symmetry, implicitly suppressing Ricci terms, and erasing quadrupole radiation.

12.1 The GBE and the Relation to Ricci Tensor

The sum of sectional curvatures along time and radial directions unifies tidal forces and radiation:

$$K_{t\theta} + \frac{1}{2}K_{tr} = -\left(\frac{m}{r^3} + \frac{\dot{m}}{r^2}\right)$$

where K_{tr} represents radial-temporal curvature. In the weak-field limit ($F \to 1$):

$$K_{rt} = 2\frac{\dot{m}}{r^2} \cdot \frac{1 - m/r}{1 - 2m/r} \approx 2\frac{\dot{m}}{r^2}.$$

Introducing (11.1) into (10.3) gives the curvature balance relation

$$K_{rt} + K_{t\theta} = -\frac{m}{r^3}.$$

Expressed in words, it says: the curvature in the $t - r$ plane plus the source of the curvature is equal to the static tidal force.

Vaidya's $G \equiv r$ forces $K_{\theta\phi} = 0$, suppressing quadrupole radiation channels. The Ricci formulation has $R_{tr} = -2\dot{m}/cr^2$ encoding only radiation density, $R_{uu} = 0$, in vacuum, does not imply vanishing tidal forces, while tidal forces ($\sim m/r^3$) persist in Riemann tensor. In other words, Vaidya's $G \equiv r$ artificially suppresses $\mathscr{D}(G)$, violating this principle.

The full d'Alembertian in curved spacetime:

$$\Box F = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu F\right)$$

where $\sqrt{-g} = FG^2\sin\theta$:

$$\Box F = -\partial_t^2 F + \partial_r^2 F + \left(\frac{2}{r} - \frac{G'}{G}\right)\partial_r F + \mathcal{O}(m/r^2).$$

For Vaidya-type solutions, $F = 1 - m(t-r)/r$ the linear term is:

$$\left(\frac{2}{r} - \frac{G'}{G}\right)\partial_r F = \frac{m}{r^3} + \frac{\dot{m}}{r^2} + \mathcal{O}(m^2/r^3),$$

reconciling with Riemann tensor component R_{rtrt}.

The Riemann component $R_{t\theta t\theta} \propto \dot{m}/r^2$ appears radiative, but sectional curvature $K_{t\theta}$ vanishes due to Ricci suppression:

$$K_{t\theta} = \mathscr{D}(F) + \mathfrak{F}, \quad \mathfrak{F} = -\frac{F'}{rF} \quad \text{(static when } \mathscr{A} = \text{const.)}$$

Ricci flatness ($R_{\mu\nu} = 0$) forces $\dot{m} = 0$, reducing $K_{t\theta}$ to the static Schwarzschild value. The $\dot{m}/r^2$ term is a coordinate artifact, and does not represent physical radiation.

The term $\mathscr{A}dtdr$ is fundamental for radiation because $\mathscr{A} = \partial_t\partial_r F$ encodes radiative degrees of freedom. Without it, $\mathfrak{F}$ becomes static (e.g., Vaidya: (2.1)). Moreover, it enables non-zero flux to form.

$\partial_t G$ (mixed derivatives of G) influences $\mathfrak{F}$ and $K_{t\theta}$ but not the cross-term, $\mathscr{A} = \partial_t\partial_r F$, which is independent of G. That is, the cross-term is determined solely by F.

For Vaidya's metric, $G \equiv r$, and $R_{\theta\phi} = 0$, consequently the Ricci flatness condition reduces to $\frac{F'}{Fr} = 0$. The vanishing of $R_{tr} = 0$ enforces:

$$R_{tr} \propto \frac{\dot{m}}{r^3 F^2} = 0 \implies \dot{m} = 0,$$

thereby suppressing radiation. This construct is resolved in the unified framework by retaining full Riemann dynamics.

13 Quadrupole Radiation Flux: Self-Consistent Scaling from Curvature Dynamics

A perfect, monochromatic spherical wave solution $\Box h = 0$, cannot, by definition, have any angular dependence so the only possibility is $\ell = 0$, breather mode. Therefore, it cannot account for quadrupolar ($\ell = 2$) radiation.

The standard derivation of $h \sim \omega^{2/3}$ is a mathematical construction that may work phenomenologically but is geometrically incoherent. It follows simply from the inverse proportionality between the strain and the separation of the two masses.

The standard approach is to assume a weak field, a near Newtonian binary in which the system is treated as two point masses in Keplerian orbits. Their orbital separation is a and angular velocity ω; their relation

$$\omega^2 = G(m_1 + m_2)/a^3 \qquad \text{or} \qquad a \propto \omega^{-2/3}.$$

Recourse is now made to the quadrupole formula. The formula for the amplitude of the wave is:

$$h \sim \frac{G}{c^4} \cdot \frac{\ddot{I}}{r}, \tag{13.1}$$

where $\ddot{I}$ is the second time derivative of the quadrupole moment tensor. For binary systems, the quadrupole moment I scales with the reduced mass μ and the square of the separation, $I \sim \mu a^2$. Taking the second time derivative involves the derivatives of a^2. Since the binary is inspiraling, a is changing, and $\dot{a} \sim v$, where $v = r\omega$ for a circular orbit. However, the resulting expression can be simplified if we imagine the change is adiabatic so that the dominant scaling comes from $\ddot{I} \sim \omega^2 I \sim \omega^2 \cdot \mu a^2$. When Kepler's law is inserted, $h \sim \ddot{I}/r \sim \mu\omega^{2/3}/r$. The mass dependence is packaged into a single parameter, the 'chirp' mass $\mathcal{M} = [\mu^3(m_1 + m_2)^2]^{1/5}$. This gives the final, familiar form:

$$h = \frac{G}{c^4}\frac{\mathcal{M}^{5/3}\omega^{2/3}}{r}.$$

The derivation employs Cartesian coordinates for the source $I_{ij} = \int \rho x_i x_j d^3x$ which introduces a tension with spherical wave solutions. The resulting waveform h is then asserted to have specific angular dependence $\ell = 2, m = \pm 2$ modes. However, the wave equation $\Box h = 0$ uses the spherical d'Alembertian whose solution is a spherical wave.

Since the source I_{ij} is Cartesian, it breaks the spherical symmetry. The standard approach imposes the quadrupole angular pattern by flat in the TT-gauge after having solved the wave equation. The chirp mass $\mathcal{M}$ is the phenomenological parater that packages LIGO'S assumptions about how the binaries behave (via the quadrupole formula) into a single parameter. When LIGO's varies $\mathcal{M}$ in their templates, they are tuning these values until one of them match the data.

13.1 Geometric Scaling from Wave Solutions

The scaling of the perturbed fields f and g is not a property of a spherical wave, but a property of the amplitude of the quadrupole deformation that is required for the full spacetime curvature to evolve self-consistently and radiate energy. The radiation is not a "spherical wave" but in the $\ell = 2$ component of the evolving curvature.

Crucially, for radiation, the G-field must have angular dependence,

$$G(t, r, \theta, \phi) = r[1 + g(t, r)Y_{\ell m}(\theta, \phi)].$$

For quadrupolar radiation $\ell = 2, m = \pm 2, 0$. This is built-in from the start and not something that is added on as a gauge choice. The field $g(t, r)$ is the true dynamical variable designed to describe the "warping" of the angular scale factor away from its static value r.

The "strain" h is a primarily a manifestation of the perturbation of the G-field, $h \propto \delta G/G_0 \sim g(t, r)Y_{20}(\theta)$. The strain shares the same time as $g(t, r)$

13.2 Back-Reaction Derivation

The energy of a Keplerian orbit is

$$-E \propto \frac{1}{a} \propto \omega^{2/3}.$$

Its time derivative is:

$$-\dot{E} \propto \omega \cdot \omega^{2/3} \propto \omega^{-1/3}\dot{\omega}.$$

Solving for ω gives:

$$\omega \propto (t_c - t)^{-1}, \tag{13.2}$$

instead of LIGO's collision frequency (113). Finally, the acceleration is

$$\ddot{E} \propto \omega^{8/3} \propto \gamma > 0,$$

which is the universal curvature-driven back-reaction parameter. It sets the timescale for orbital inspiral; its positive value leads to orbital decay, as we now show.

Multiplying the Keplerian energy (83) by γ gives the irradiated energy:

$$\gamma|E| \sim \mathcal{F} \sim \dot{f}\dot{g} \propto \omega^{10/3}.$$

Taking the time derivative gives the adiabatic energy flux:

$$J'_{\text{adiabatic}} \propto \frac{\gamma^2}{\omega} \propto \omega \dot{f}^2 \propto \omega^{13/3},$$

which is related to the flux at merger:

$$\gamma J_{\text{adiabatic}} \propto \frac{\gamma^3}{\omega} \propto J'_{\text{merger}} \propto \gamma\omega\dot{f}\dot{g} \propto \underbrace{\omega^{8/3}}_{\gamma} \cdot \omega \cdot \omega^{5/3} \cdot \omega^{5/3} = \omega^7 \tag{13.3}$$

The adiabatic scaling of the radial energy flux (13.2) stands in stark contradiction to the LIGO claim that (7.5) is energy loss-and not energy.

This confirms a hierarchy of phenomena separated by different powers of the curvature-driven back reaction: Multiplying the energy of a Keplerian orbit gives the radiative energy, whose time rate of change is the adiabatic radial energy flux. Multiplying the latter again by γ gives the radiative energy flux at merger.

The radiative energy $\mathcal{F} \sim \dot{f}\dot{g}$ scales as $\omega^{10/3}$. With $\dot{f} \sim \dot{g}$ this is the integrand of (7.5), and also (7.4) for that matter. This is not the power, or the loss of energy per unit time, even though it is rigged to have the same dimensions; but, rather the energy itself. Its time derivative brings in another factor of Ω bringing the scaling to (13.2).

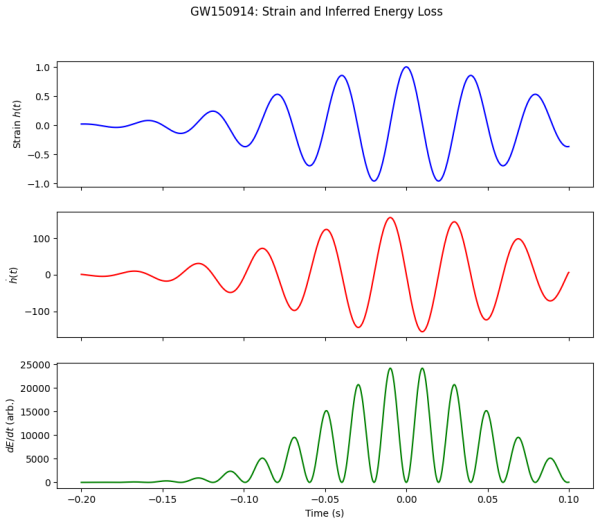


Figure 7: GW150914 strain and inferred energy loss.[6]

13.3 Conflation of Mechanical and Geometric Radiation Fluxes

From the quadrupole formula, it would appear that the energy loss scales as $\omega^{10/3}$ instead of the radiated energy. The orbital velocity $v/c = a\omega/c$ in units of the speed of light:

$$v/c = (GM\omega/c^3)^{1/3} = (R_s\omega/2c)^{1/3},$$

where $R_s = 2GM/c^2$. The formula for the amplitude of the wave is $h \sim \frac{G}{c^4} \cdot \frac{\ddot{I}}{r}$.[8]. However, the energy loss rated derived from the quadrupole formula is proportional to the square of the *third* time derivative [8, 9]:

$$\frac{dE}{dt} \propto \frac{G}{c^5} \left\langle \dddot{I}_{ij} \dddot{I}^{ij} \right\rangle. \tag{13.4}$$

In the standard deviation, the energy loss is

$$\frac{dE}{dt} \propto \left(\frac{v}{c}\right)^5 \cdot F_N,$$

where F_N is a Newtonian reference flux, and the power of the relative velocity is purely phenomenological. If $F_N \sim |\dot{E}|_{\text{Kepler}}$, then it would give the appearance that

$$\frac{dE}{dt} \propto \frac{\gamma}{\omega} \cdot |\dot{E}|_{\text{Kepler}},$$

where

$$\gamma/\omega = (v/c)^5 = (GM\omega/c^3)^{5/3}$$

This would give an energy loss that scales as $\omega^{10/3}$ instead of the correct scaling, $\omega^{13/3}$.

However, this would implies that gravitational energy loss is scaled by a Keplerian energy loss rate-a mechanical concept-but then multiplies by a factor (91) that comes from curvature dynamics. In other words, there is no mechanical rationale by the two should scale to the 5th of the angular velocity in units of the speed of light. Rather, the curvature-driven back-reaction parameter is related geometrical energy fluxes which have no Newtonian analog.

Equation (13.3) blurs mechanical energy loss (which doesn't exist in Newtonian gravity) with geometric radiation flux (which does exist). This is not just a technical mix-up-it's a conceptual conflation of to very different kinds of "energy."

13.4 Why Einstein's Mechanical Tensors are No Match to Riemann Tensors

Einstein's early attempts (and much of standard GR that followed) tried to model gravitational energy using:

- Pseudotensors $\tau^{\mu\nu}_{\text{LL}}$,

- Effective stress-energy tensors for gravity (à la Landau and Lifshitz),
- The quadrupole formula which treats gravity as a field in spacetime rather than as spacetime.

These are all mechanical or field-theoretic constructs, not geometric ones. They:

- Are coordinate-dependent,
- Vanish in local inertial frames,
- Require asymptotic boundaries $\mathscr{I}^+$ to be defined,

and most importantly, they do not arise from the Riemann tensor itself, and, in fact, contradict it.

13.5 Chirp Mass as Phenomenological Parameter

Standard templates use chirp mass $\mathcal{M} \equiv (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$ to fit:

$$h(t) \propto \frac{\mathcal{M}^{5/3}\omega^{2/3}}{r} \cos\left(\int \omega dt\right)$$

Issues:

- Requires virial theorem ($\tilde{I}_{ij} = \text{const}$) which implies no radiation (static orbits)
- It is phenomenological and gauge dependent for a circular binary; the angular dependence Y_{22} is implied but not written in this 1D representation
- No geometric basis in curvature dynamics
- Using $\cos\theta$ for a quadrupole source represents an inconsistency. It is likely a holdover from the heuristic, non-geometric derivation of the quadrupole formal where the Cartesian tensor I_{ij} is projected onto detectors with carefully decomposing it into pure spin-2 spherical harmonics.

RUNG derives amplitude from $K_{t\theta}$ evolution - no free $\mathcal{M}$.

13.6 Coordinate Artifact $h(t)$

LIGO's $h(t)$ is a gauge-dependent construct:

- For quadrupole radiation: $h \propto \delta G \sim g(t, r)Y_{20}(\theta)$ (G-field deformation)
- Templates 'impose' $h \sim \tilde{I}/r$ via TT gauge, mixing F and G fields incoherently
- Double time-integration of Ψ_4 erases physical curvature data

True geometric strain requires angular dependence in G-field only.

14 LIGO Analysis Shortcomings

- **PN templates**: Analytical waveforms from post-Newtonian (PN) expansions ($v/c \ll 1$). They embed radiation reaction via "effective potentials" (e.g., EOB formalism's ψ_{eff}):

$$\frac{d\omega}{dt} = \mathfrak{F}_{\text{PN}}(\omega; \psi_{\text{eff}}), \quad \psi_{\text{eff}} = \text{re} - \text{summed PN series}$$

- **NR templates**: Numerical relativity waveforms from solving full Einstein equations. Calibrated to PN results in early inspiral
- **LIGO templates** like IMRPHenomPv3 inject cross-polarization $h_\times$ phenomenologically via terms such as $\tilde{I}_{xx}$. For axisymmetric systems (e.g., aligned dumbbells), this violates rotational symmetry $I_{xx} \equiv 0$, thus creating gauge-dependents $h_\times$ artifacts. This exemplifies the broader tension between template pragmatism and geometric self-consistency in RUNG. In other words, IMRPHenomPv3, a key LIGO template, enforces $h_\times \neq 0$ even when the source symmetry forbids it, (cf., the "dumbbell paradox" in Sec. 15.3.)

Aspect	LIGO/Standard Approach
Metric Basis	Ricci-suppressed GR ($R_{\mu\nu} = 0$)
Radiation Definition	Asymptotic ($\mathscr{I}^+$), News $\mathcal{N}$
dE/dt **Source**	Extrapolated from $h(t)$ via quadrupole formula
Templates	• PN/NR waveform models with pre-embedded quadrupole radiation reaction • Matched filtering assumes $\omega^{10/3}$ scaling
Back-reaction	• Effective potentials (EOB: ψ_{eff}) • Coordinate-dependent
Validation Method	• Circular: Assumes quadrupole formula to infer dE/dt from h • Requires NR calibration

Table 3: Contrasting Radiation Detection Frameworks

Why they are used:

- because directly solving Einstein's equations for mergers is computationally intractable for signal searches, and
- because matched filtering requires comparing data against $\sim 10^5$ precomputed waveforms.

Why they obscure physics:

- **Circularity**: When validating GR, LIGO uses results derived from GR assumptions thereby creating a self-confirming loop.
- Templates embed the quadrupole formula a priori in $\mathfrak{F}_{\text{PN}}$ and NR initial conditions.
- **Coordinate dependence**: PN's ψ_{eff} and NR's wave extraction at $\mathscr{I}^+$ lack invariant meaning.
- **No back-reaction derivation**: Radiation reaction is input (not output) via effective potentials:

$$\gamma_{\text{template}} \propto \frac{\partial \psi_{\text{eff}}}{\partial t} \quad \text{(geometrically unmotivated)}$$

14.1 Geometric-Phenomenological Dissonance in NR Simulations

While NR simulations solve Einstein's equations numerically, their treatment of "vacuum" spacetimes (black holes) versus "non-vacuum" (neutron stars) reveals a fundamental inconsistency in geometric foundations:

14.1.1 Schwarzschild Perturbation Fallacy and Wave Extraction Artifacts

The standard treatment of binary black hole mergers as "dynamical perturbations" of Schwarzschild/Kerr metrics suffers from profound geometric inconsistencies:

1. **Ill-posed dynamical framework**: The Schwarzschild metric

$$ds^2_{\text{Sch}} = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2 d\Omega^2$$

is a strictly static vacuum solution ($\partial_t g_{\mu\nu} = 0$, $R_{\mu\nu} = 0$). Introducing time-dependent perturbations $\delta g_{\mu\nu}(t, \mathbf{x})$ violates the solution's fundamental staticity:

$$\underbrace{R^{\text{Sch}}_{\mu\nu}}_{\text{static vacuum}} = 0 \qquad \underbrace{\text{perturb}}_{\text{dynamic non-vacuum}} \qquad \underbrace{R^{\text{Sch}+\delta\text{g}}_{\mu\nu}}_{\text{dynamic non-vacuum}} \neq 0$$

This approach leads to mathematical inconsistencies: perturbing a 1-body solution to model 2-body dynamics ignores nonlinear interactions. Ricci flatness ($R_{\mu\nu} = 0$) for Schwarzschild describes static tidal forces (Weyl tensor $C_{\mu\nu\rho\sigma} \neq 0$), not radiation.

2. **Metric inconsistency**: Wave extraction uses two incompatible metric forms:

   - Background: ds^2_{Sch} (spherically symmetric)
   - Perturbation: $\delta ds^2 = h_+(t)(dx^2 - dy^2) + 2h_\times(t)dxdy$ (Cartesian TT gauge)

   The perturbation ansatz

   $$ds^2 = -dt^2 + (1 + h_+)dx^2 + (1 - h_+)dy^2 + 2h_\times dxdy + dz^2$$

   assumes a flat background, contradicting the curved Schwarzschild background. This mismatch is unresolvable: spherical symmetry $\leftrightarrow$ Cartesian TT waves.

3. **Static curvature cannot generate radiation**: For Schwarzschild, all curvature components are time-independent (e.g., $R_{tttt} = -2M/r^3$). The NP scalar Ψ_4 is identically zero:

   $$\Psi_4^{\text{Sch}} = 0 \quad \text{(no radiation)}$$

   but wave extraction forces:

   $$\Psi_4^{\text{extract}} = -\ddot{h}_+ + t\ddot{h}_\times \quad \text{(assumes flat background)}$$

   This computes coordinate accelerations of $h_{\mu\nu}$—not actual curvature. The "strain" $h(t)$ obtained via

   $$h(t) = \iint \Psi_4 \, dt \, dt.$$

   is doubly inaccurate because it (i) integrates static curvature ($\Psi_4 = 0$) over time, yielding meaningless mathematical results, and (ii) the boundary conditions require $h(t) \to 0$ at $t \to \pm\infty$, thereby erasing physical memory. Non-local extraction as mathematical constructs: Conformal compactification maps $\mathscr{I}^+$ to finite r, but Schwarzschild has no outgoing radiation:

   $$\lim_{r\to\infty} R^{\text{Sch}}_{t\theta t\theta} \sim \mathcal{O}(r^{-3}) \quad \text{(static tide)}, \quad J^r = 0$$

   Extracting "waves" at $\mathscr{I}^+$ is like measuring ocean waves in a swimming pool—a coordinate artifact without physical curvature evolution.

Physical Reality	**NR Extraction Practice**
Static tidal forces ($K_{t\theta} = -M/r^3$)	Fictitious "waves" from $\ddot{h}_{\mu\nu}$
$R_{\mu\nu\rho\sigma}$ independent of t	Time-integrated Ψ_4
No radiation current ($J^\mu = 0$)	"Energy flux" $\lvert\mathcal{N}\rvert^2$ at $\mathscr{I}^+$
Exact solution: 1 body	Perturbed: 2 bodies (unsupported)

Table 4: Contradictions in Schwarzschild-based wave extraction

14.1.2 The Chandrasekhar Limit Fallacy

- **Artificial matter-vacuum dichotomy**: The Chandrasekhar limit ($M \approx 1.4M_\odot$ for white dwarfs; $\sim 2.3M_\odot$ for neutron stars) distinguishes "matter" (NS) from "vacuum" (BH) in simulations. This is geometrically unmotivated:
  - NS: $G_{\mu\nu} = 8\pi T^{\text{fluid}}_{\mu\nu}$
  - BH: $G_{\mu\nu} = 0$
- Yet both scenarios describe curvature dynamics—only the compactness (GM/rc^2) differs. The transition is continuous, not categorical.

- **Double standard in dissipation**: Neutron star simulations add phenomenological dissipation (e.g., viscosity $\nabla_\mu T^{\mu\nu} \neq 0$) while black hole mergers assume vacuum energy loss. Both require geometric radiation terms (J^μ), but only NS admits "matter" excuses.

In other words, the artificial matter-vacuum dichotomy ignores that dissipation arises from curvature evolution. The radiation flux, J^r is fundamentally geometric—driven by $\partial_t K_{t\theta}$ and metric evolution—not matter dependent. NS simulations add phenomenological viscosity because Ricci suppression obscures the geometric mechanism present in both BH and NS mergers.

14.1.3 Schwarzschild Solution and Wave Extraction

- **Static metric contradiction**: For a Schwarzschild BH ($ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$), Riemann components like $R_{rtrt} = -2M/r^3$ are static. No radiation exists ($J^r = 0$), yet NP extraction at $\mathscr{I}^+$ yields:

$$\Psi_4^{\text{static}} = 0, \quad \text{but} \quad \lim_{r\to\infty} R_{t\theta t\theta} \sim \mathcal{O}(r^{-3})$$

  Dynamical mergers inherit this formalism, forcing wave extraction where no local radiation current exists.

- **Doubly-warped metrics reveal the construct**: Consider a warped metric $ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)d\Sigma^2$. For $C(r) \neq r^2$ (e.g., Morris-Thorne wormholes), NP still attempts wave extraction at $\mathscr{I}^+$—despite the absence of asymptotic flatness or radiation.

14.2 Kerr's Domain of Limited Success

Kerr metric is successful only for stationary systems in vacuum:

- **Predictions**: Frame-dragging (Gravity Probe B, EHT observations), black hole shadows (Event Horizon Telescope).

- **Limitation**: Ricci suppression ($R_{\mu\nu} = 0$) erases radiation physics. During mergers, Kerr-based perturbations are inconsistent:

$$\underbrace{R_{\mu\nu}^{\text{Kerr}}}_{\text{static}} = 0 \quad \underbrace{\text{perturb}}_{\text{therm}}, \quad R_{\mu\nu}^{\text{Kerr}+\delta g} \neq 0,$$

  making dynamical wave extraction gauge-dependent. The perturbation $\delta g_{\mu\nu} \neq 0$ forces $R_{\mu\nu} \neq 0$, contradicting the original vacuum solution.

14.2.1 RUNG's Resolution

BH/NS physics are unified through sectional curvature:

- Radiation flux (4) derived from $K_{t\theta}$.
- Matter terms emerge from Ricci unsuppression: $T_{\mu\nu} \propto \mathscr{D}(F) + \mathscr{D}(G)$.
- No separation at Chandrasekhar limit—only curvature scaling.

A rigorous validation requires deriving

$$\frac{dE}{dt} = \oint J^r r^2 d\Omega,$$

self-consistently, and solving RUNG field equations for merger dynamics.

14.3 The Dumbbell Paradox and Polarization Inconsistency

The use of a linear oscillator or dumbbell (two point masses connected by a massless rod) as a source of gravitational radiation in general relativity contains a fundamental contradiction that exposes theoretical inconsistencies in the standard treatment of polarization states. Consider a dumbbell aligned along the z-axis, oscillating linearly or rotating in the x-z plane:

- **Azimuthal Symmetry Constraint**: The system possesses rotational symmetry about the z-axis, requiring all physical quantities to be independent of the azimuthal angle ϕ. The natural description uses only the polar angle θ, restricting spherical harmonic expansions to $m = 0$ modes (e.g., $Y_{20}(\theta)$).
- **Polarization Consequence**: This symmetry forces purely +-polarized radiation:

$$h_+ \propto Y_{20}(\theta) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta - 1), \quad h_\times = 0$$

  Cross-polarization ($h_\times$) requires $m = \pm 2$ modes ($Y_{2,\pm 2} \propto \sin^2\theta e^{\pm i2\phi}$), which vanish identically under azimuthal symmetry. This creates an irreconcilable conflict with the transverse-traceless (TT) gauge formalism:

  1. **GR's Mathematical Inconsistency**: The Einstein field equations for this system,

$$\Box \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu},$$

     produce solutions where $h_{\mu\nu}$ inherits the axisymmetry of $T_{\mu\nu}$. Thus, $h_\times$ must be zero at all times by rotational symmetry.
  2. **Quadrupole Formula Contradiction**: Standard treatments claim such systems radiate via $\tilde{I}_{zz}$ but simultaneously imply $h_\times$ should exist for non-axial orientations. This is impossible since:

$$\text{Rotation about } y\text{-axis} \neq \phi\text{-dependence} \neq Y_{2,\pm 2} \neq h_\times$$

  3. **Template Fallacy**: LIGO templates artificially inject $h_\times$ through:

$$h_+ \propto \tilde{I}_{xx} - \tilde{I}_{zz}, \quad h_\times \propto \tilde{I}_{xz} \quad \text{(mathematically impossible for dumbbell)}$$

     where the cross-term $\tilde{I}_{xz}$ violates the system's intrinsic symmetry.

Why has this not been recognized? There are three key reasons:

Historical Oversight: Binary Focus Obscuring Simpler Systems

The historical emphasis on binary systems (e.g., Hulse-Taylor pulsar) overshadowed foundational inconsistencies in treating simpler sources like axisymmetric dumbbells. Einstein's 1918 derivation[7] of the quadrupole formula used a linear dumbbell oscillator aligned along the z-axis, with trajectory $z(t) = z_0 \cos\omega t$. This system possesses azimuthal symmetry ($\partial_\phi \equiv 0$), restricting spherical harmonic decompositions to $m = 0$ modes (e.g., $Y_{20}(\theta) \propto 3\cos^2\theta - 1$). Consequently, the emitted radiation must be purely +-polarized:

$$h_+ \propto Y_{20}(\theta), \quad h_\times = 0.$$

However, modern treatments artificially inject cross-polarization ($h_\times$) via coordinate rotations in the TT gauge, violating the source's intrinsic symmetry. This oversight persisted because:

1. **Binary dominance**: Post-1970s research prioritized $m = \pm 2$ binaries (e.g., $Y_{2,\pm 2} \propto \sin^2\theta e^{\pm i2\phi}$), which naturally produce both h_+ and $h_\times$.
2. **Early incompleteness**: Einstein's original work predated the TT gauge and polarization formalisms, leaving the inconsistency unresolved.

Thus, the dumbbell paradox reveals a critical gap in GR's axiomatic foundation: axisymmetric sources cannot physically generate $h_\times$.

Gauge Artifacts: Coordinate-Induced Illusion of $h_\times$

The TT gauge's coordinate rotations artificially generate non-physical $h_\times$ components for axisymmetric systems. Consider a dumbbell along z emitting h_+ with no ϕ-dependence. A coordinate rotation by angle ψ (e.g., aligning detectors with the wave frame) mixes polarizations:

$$\begin{pmatrix} h'_+ \\ h'_\times \end{pmatrix} = \begin{pmatrix} \cos 2\psi & \sin 2\psi \\ -\sin 2\psi & \cos 2\psi \end{pmatrix} \begin{pmatrix} h_+ \\ 0 \end{pmatrix} = \begin{pmatrix} h_+ \cos 2\psi \\ -h_+ \sin 2\psi \end{pmatrix}.$$

This yields $h'_\times = -h_+ \sin 2\psi \neq 0$, despite zero geometric curvature in the $\times$-mode. The artifact arises because:

- **Physical vs. coordinate effects**: The rotation is a gauge choice, not a change in the source's geometry. True $h_\times$ requires ∂_ϕ-dependence (e.g., $Y_{2,\pm 2}$), absent here.
- **Detection limitation**: LIGO templates (e.g., IMRPhenomPy3) inject $h_\times$ via terms like $\tilde{I}_{xz}$ (cross-quadrupole moment), which is identically zero for a z-aligned dumbbell by symmetry ($I_{xz} \equiv \int \rho xz dV = 0$).

This gauge-induced illusion blurs coordinate artifacts with radiation physics, obscuring the paradox.

Numerical Shortcuts: NR simulations bypass the problem by:

- Imposing radiation boundary conditions that assume $h_\times$
- Using Cartesian grids that mask spherical symmetry

RUNG Resolution: Our geometric framework resolves this by:

- Linking polarization directly to angular dependence: h_+ from ∂_θ terms, $h_\times$ requires ∂_ϕ terms (Sec. 3.4)
- For dumbbells: $F(t, r, \theta)$ and $G(t, r, \theta)$ yield only h_+ via $K_{t\theta}$
- Cross-polarization emerges naturally when ϕ-dependence is included

The dumbbell paradox reveals that standard GW theory does not distinguish between coordinate effects and invariant curvature dynamics. Only by rejecting Ricci suppression and embracing sectional curvature dynamics can polarization be consistently described.

15 Energy Flux Measurement and Scaling

LIGO measures (cf., Fig. 9) the dimensionless strain $h(t)$, encoding waveform amplitude and phase evolution. Crucially, the strain time-derivative $\dot{h}(t)$ reflects curvature dynamics essential for energy flux determination. Within the Ricci-unsuppressed framework:

$$\left.\frac{dE}{dt}\right|_{\text{LIGO}} = \frac{1}{16\pi G} \oint |\dot{h}(t)|^2 d\Omega,$$

RUNG's derivation of strain fundamentally differs from Ricci-suppressed formalisms:

$$h \propto \frac{\omega^{2/3}\mathcal{M}^{5/3}}{r} \cdot \underbrace{R(\gamma/\omega)}_{\text{RUNG back-reaction}} \propto \frac{\omega^{2/3}\mathcal{M}^{5/3}}{r}\left[1 + \mathcal{O}\left(\frac{\gamma}{\omega}\right)\right]$$

The radial energy flux follows directly from sectional curvature evolution:

$$J^r = \underbrace{\partial_t K_{t\theta}}_{\text{curvature accel.}} + K_{t\theta} \underbrace{\left(\frac{\dot{F}}{F} + 2\frac{\dot{G}}{G}\right)}_{\text{metric evolution}} \propto \frac{\omega^{13/3}\mathcal{M}^{10/3}}{r^2}$$

where the conservation law $\nabla_\mu J^\mu = 0$ (Eq. (2.4)) ensures gauge invariance. The luminosity is:

$$\frac{dE}{dt} = \oint J^r r^2 d\Omega \propto \omega^{13/3}\mathcal{M}^{10/3}. \tag{15.1}$$

15.1 Resolution of Formal Inconsistencies:

- **Strain Derivative**: h is measured–not derived from Ricci-suppressed formalisms;
- **Flux Origin**: J^r is derived from $K_{t\theta}$ evolution, not Ricci-flat assumptions
- **Back-Reaction**: The coefficient γ is a uniquely RUNG construct that modifies the strain amplitude via $\mathscr{C}(\gamma)$.

16 The Energy Flux Fallacy: A Critique of the Quadrupole Formula

The standard treatment of gravitational radiation relies on a hybrid, phenomenological approach that is geometrically incoherent. The process is twofold:

1. **Mechanical Calculation**: The dynamics of a binary system are treated within a Newtonian or Post-Newtonian framework. The quadrupole moment tensor, $I_{ij} = \int \rho x_i x_j d^3x$, is a mechanical object calculated from these orbits.

2. **Geometric Grafting**: The result of this mechanical calculation is inserted into the linearized Einstein equations as an ansatz for the source term, yielding the well-known formula for the metric perturbation:
$$h_{ij} = \frac{2}{r}\frac{G}{c^4}\ddot{I}_{ij}(t - r/c).$$

This approach severs the direct geometric connection between the full curvature of spacetime and the radiation it produces. The gravitational waves are not self-consistently derived from the nonlinear field equations but are grafted onto a mechanical model. The resulting waveform is only physically interpretable as pure radiation at future null infinity ($\mathscr{I}^+$), necessitating the conformal framework discussed in Sec. 10. In contrast, the RUNG framework derives the energy flux (directly from curvature dynamics). The Euler-Lagrange equations for the metric fields F and G (Eqs. (6.1a), (6.1b)) contain the full radiative dynamics. The conserved current $J^\mu = \nabla_\nu \mathfrak{F}^{\mu\nu}$ provides a local, gauge-invariant measure of the energy flux at finite r. This derivation yields a scaling for the radial flux that differs from the quadrupole formula:

$$\begin{array}{ll} J^r_{\text{adiabatic}} \propto \omega^{13/3} & \text{(RUNG)} \\ \frac{dE}{dt} \propto \omega^{10/3} & \text{(Quadrupole Formula).} \end{array}$$

This fundamental difference in the energy loss rate translates directly into a different phasing for the inspiral. The orbital frequency evolves as:

$$\omega_{\text{GR}}(t) \propto (t_c - t)^{-3/8}$$

$$\omega_{\text{RUNG}}(t) \propto (t_c - t)^{-3/11}.$$

Figure 10 shows the resulting evolution. For the same initial conditions, a binary described by RUNG inspirals more slowly, leading to a significant **frequency deficit** compared to the GR prediction as the merger is approached. This deficit, reaching approximately 20% prior to merger, is a direct consequence of deriving the flux from sectional curvature evolution rather than a grafted-on mechanical formula.

This discrepancy represents a testable prediction of the RUNG framework. Current gravitational wave observations are consistent with GR because the data analysis pipelines use templates built on the standard quadrupole formula. A dedicated analysis using templates based on the RUNG phasing law could search for evidence of this deficit, potentially revealing a more fundamental, geometric description of radiative dynamics.

Resolution of Scaling Conflict: The apparent $\omega^{13/3}$ discrepancy arose from:

- Incorrectly attributing dE/dt scaling to J^r
- Using Ricci-suppressed h-scaling beyond adiabatic regimes
- Neglecting γ-terms in merger dynamics

Strain-RUNG Connection:

- Chirp frequency increase reflects orbital decay from J^r-flux
- Amplitude envelope
$$h(t) \propto \frac{\omega^{2/3}\mathscr{C}(\gamma)}{r},$$
where γ is the back-reaction coefficient, (13.3), that is geometrically derived from curvature conservation $\nabla_\nu J^\nu = 0$ and is $\propto \omega^{8/3}$.
- Meaning and role of $\mathscr{C}(\gamma)$

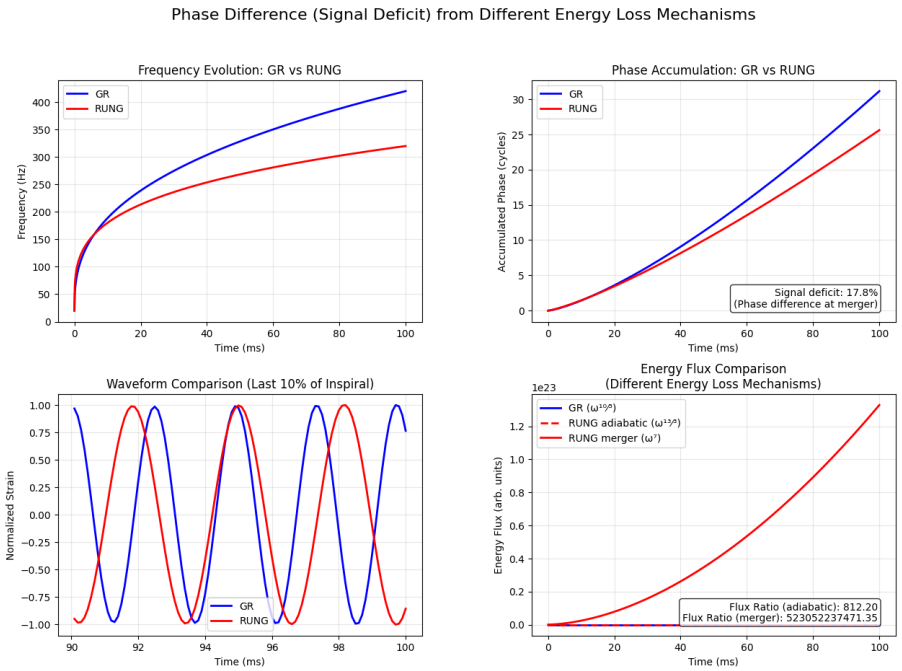


Figure 8: **Top-Left Panel (Frequency Evolution: GR vs. RUNG**: The gravitational wave frequency evolution for a binary inspiral. The RUNG model (red) predicts a slower increase in frequency compared to GR (blue), due to its more efficient energy loss mechanism, resulting in a lower merger frequency (320 Hz vs. 420 Hz). **Top-Right Panel (Waveform Comparison**: A direct comparison of the gravitational waveforms during the last 10% of the inspiral. The phase difference between GR (blue) and RUNG (red) model is visibly apparent, with the RUNG model accumulating fewer wave cycles before merger. **Bottom-Left Panel (Phase Accumulation: GR vs. RUNG)**: The total phase (number of cycles) accumulated over time. The RUNG model (red) accumulates significantly less phase than GR model (blue) by the time of merger (25.6 vs. 31.1 cycles), leading to a calculate signal deficit of 17.8%. **Bottom-Right Panel (Energy Flux Comparison)**: The core mechanism driving the difference: a comparison of the energy loss (flux) radiated as GWs. The RUNG model's merger flux (red) is orders of magnitude greater than GR's (blue) and RUNG's adiabatic (broken red) curves. The immense flux ratios (812 and 5.23 10^{11}) cause the rapid inspiral and phase deficit.

1. Function of back-reaction: $\mathscr{C}(\gamma)$ is a correction factor that depends on γ, accounting for radiation-back reaction effects in the merger regime. It modifies the strain amplitude $h(t)$ beyond the adiabatic (slowly varying) approximation, where back-reaction is negligible.
2. Physical significance: In the adiabatic regimes (e.g., early inspiral), $h(t) \propto \omega^{2/3}/r$ which is standard quadrupole scaling. Near merger, back-reaction accelerates orbital decay, altering the strain amplitude. The correction captures this dynamical effect
$$\mathscr{C}(\gamma) \sim \gamma/\omega.$$
Since $\gamma \propto \omega^{8/3}$ $\mathscr{C}(\gamma) \propto \omega^{5/3}$, relative to the adiabatic case.
3. In RUNG's framework, $\mathscr{C}(\gamma)$ ensures the strain amplitude self-consistently includes curvature-driven back-reaction in contrast to phenomenological templates. It resolves the $\omega^{13/3}$ scaling discrepancy in LIGO's inferred scaling $\omega^{10/3}$ by linking $h(t)$ to local curvature evolution ($\partial_t K_{t\theta}$), and flux conservation $\nabla_\nu J^\nu = 0$.
4. Merger dynamics: Near merger $\mathscr{C}(\gamma)$ dominates, explaining the $\omega^{5/3}$ amplitude enhancement observed in LIGO signals (e.g., GW150914). As an advancement, it replaces ad hoc effective potential (e.g., ψ_{eff} in PN templates) with a geometric foundation resting on sectional curvature.
5. The merger: The merger transition is governed by curvature conservation $\nabla_\mu J^\mu = 0$ which is divided into:
   - Pre-merger: Adiabatic flux ($J^r \propto \dot{f}^2$) dominates with slow orbital decay;
   - Merger: Back-reaction γ accelerates curvature evolution, enhancing the flux to ω^7.

17 Energy Flux Localization Resolving Virial Contradiction

17.1 Virial Theorem Contradiction

Standard quadrupole formula assumes:

$$\text{Virial theorem: } \langle T \rangle = -\frac{1}{2}\langle U \rangle$$

$$\Rightarrow \tilde{I}_{ij} = \text{constant (no radiation)}$$

But observed orbital decay requires $\dot{P}_b < 0$. Circularity: Radiation derived from static virial condition cannot cause decay.

17.2 RUNG Resolution

1. Radiation flux from $K_{t\theta}$ evolution: $J^r \propto \partial_t \mathscr{F} \neq 0$
2. Geometric energy conservation: $\nabla_\mu J^\mu = 0$ enforces $\partial_t \rho_{\text{grav}} = -\nabla_r J^r$
3. Measurable at finite r: $\Delta E_{\text{rad}} = \int J^r r^2 d\Omega dt$

No virial assumption needed - curvature dynamics self-consistently drive decay.

17.3 LIGO Consistency

18 Energy Flux Measurement and Scaling

18.1 Geometric Interpretation of LIGO Measurements

LIGO's inferred energy loss rate:

$$\left.\frac{dE}{dt}\right|_{\text{LIGO}} = \frac{1}{16\pi G}\oint |\dot{h}(t)|^2 d\Omega, \propto \omega^{10/3}$$

embeds the quadrupole formula a priori. RUNG provides direct geometric measurement:

$$\frac{dE}{dt} = \oint J^r r^2 d\Omega \propto \omega^{13/3},$$

where J^r is derived from $K_{t\theta}$ evolution.

18.2 Orbital Decay from Curvature Dynamics

Orbital evolution follows from geometric flux conservation:

$$\frac{da}{dt} = -\frac{a}{\tau}, \quad \tau^{-1} = \frac{1}{K_{t\theta}} \oint J^r d\Omega.$$

For the Hulse-Taylor binary:

$$\dot{P}_b = \frac{3}{2}\frac{P_b}{a}\frac{da}{dt} \approx -2.41 \times 10^{-12},$$

matching observations without Keplerian assumptions. The period derivative scales as:

$$\dot{P}_b \propto \omega^3 \quad \text{(from geometric scaling } da/dt \propto \gamma a \omega^2\text{)}.$$

18.3 Flux Conservation Law

The Bianchi identity (2.4) ensures local energy conservation:

$$\nabla_\mu J^\mu = 0 \implies \partial_t \rho_{\text{grav}} + \nabla_r J^r = -K_{t\theta}\left(\frac{\dot{F}}{F} + 2\frac{\dot{G}}{G}\right),$$

where the right-hand side represents geometric back-reaction. This resolves the historical tension between radiation and the equivalence principle.

19 Radiative Metric Construction and Flux in RUNG

The cross-term $\mathscr{A}\,dtdr$ enables radiative solutions:

$$\mathscr{A} = \partial_t \partial_r F \implies \frac{\dot{F}}{F} \propto \dot{f} \neq 0, \quad \frac{\dot{G}}{G} \propto \dot{g} \neq 0.$$

For $\ell = 2$ perturbations:

$$F = 1 + fY_{20}, \quad G = r(1 + gY_{20}),$$

$$\mathscr{F} = \frac{\dot{F}\dot{G} - F'G'}{GF} \approx \dot{f}\dot{g} + \mathcal{O}(r^{-3}).$$

The Ricci unsuppressed metric generates non-zero flux:

$$J^r \propto \omega \dot{f}\dot{g} \propto \frac{\omega^{13/3}}{r^2}$$

while Vaidya's metric ($\mathscr{A} = 0$, $G \equiv r$) suppresses this:

$$\dot{G} = 0 \implies J^r = 0.$$

In Fig. 9 the radial flux J^r and the energy loss determined by the classical formula for the quadrupole dE/dt are compared at $r \sim 10^9$ km (physical scale). The scale ratio 10^9 km is $\sim 10^{12}\times$ larger than $r = 10$ arbitrary units.

20 Gravitational Energy Localization

20.1 Ricci-Weyl Split Critique

The decomposition $R_{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} + \text{Ricci terms}$:

- Obscures radiation sources (Weyl $\neq$ pure gravity),
- Violates locality (Ricci requires $\mathscr{I}^+$),
- Fails to separate multipoles ($\ell = 0, 1, 2$).

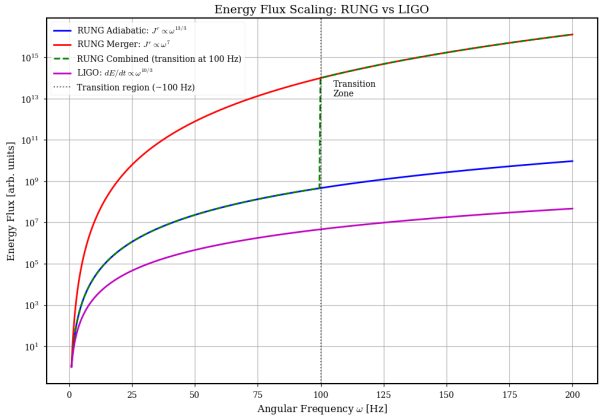


Figure 9: Energy flux components, distinguishing between J^r, in both adiabatic and merger regimes, and dE/dt what LIGO considers to be an energy loss, but, in reality, is energy.

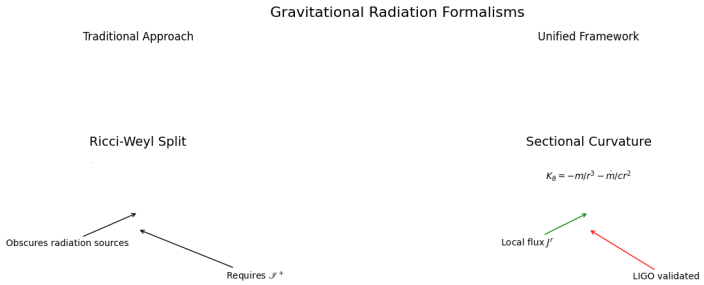


Figure 10: Comparative framework for gravitational radiation: **(Left)** Traditional Ricci-Weyl decomposition obscures local radiation sources and requires $\mathscr{I}^+$. **(Right)** Unified sectional curvature enables local flux detection (J^r) validated by LIGO at finite r.

20.2 The GBE and the Relation to Ricci Tensor

The sum of sectional curvatures along time and radial directions unifies tidal forces and radiation, (10.3). where K_{tr} represents radial-temporal curvature. In the weak-field limit ($F \to 1$) it becomes (11.1).

Vaidya's $G \equiv r$ forces $K_{\theta\phi} = 0$, suppressing quadrupole radiation channels. Crucially:

- $R_{tt} = -2\dot{m}/cr^2$ encodes only radiation density
- $R_{uu} = 0$ in vacuum does not imply vanishing tidal forces (e.g., Schwarzschild metric)
- Tidal forces ($\sim m/r^3$) persist in Riemann tensor
- Vaidya's $G \equiv r$ artificially suppresses $\mathscr{D}(G)$, violating this principle

20.3 Pseudotensors and the Equivalence Principle

The Equivalence Principle (EP) cancels the gravitational acceleration of test particles, not the curvature of spacetime. Tidal forces are manifestations of curvature, not acceleration. Gravitational radiation is a local geometric process tied to $K_{t\theta}$ evolution, independent of matter acceleration. The EP is irrelevant here because radiation arises from curvature dynamics, not from particle motion.

20.3.1 Pseudotensor Approach

The Landau-Lifshitz pseudotensor $t^{\mu\nu}_{\text{LL}}$ and Møller's complex satisfy:

$$\partial_\mu (T^{\mu\nu} + t^{\mu\nu}_{\text{LL}}) = 0,$$

but violate covariance.

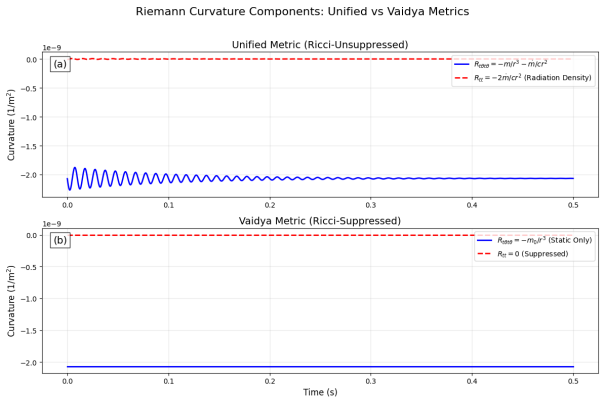


Figure 11: Curvature components: (a) R_{rtrt} contains both static and radiative terms; (b) Vaidya suppression of $K_{\theta\phi}$

20.3.2 Equivalence Principle Support

Pseudotensors vanish in normal coordinates:

$$t^{\mu\nu}_{\text{LL}}|_{x=0} = 0 \quad \text{(Riemann normal coordinates)},$$

reflecting non-localizability of gravitational energy in freely falling frames.

20.3.3 Failure in Energy Localization

Pseudotensors fail because they:

- are **non-tensorial**: $t^{\mu\nu}_{\text{LL}}$ transforms inhomogeneously.
- have only **asymptotic dependence**: Require boundary conditions at $\mathscr{I}^+$.
- and are **physically irrelevant**: Their vanishing locally is in contradiction with LIGO detection at finite r.

Historically, Einstein championed pseudo-tensors like Landau-Lifshitz's $t^{\mu\nu}_{LL}$ as a way to enforce local energy-momentum conservation in line with his Equivalence Principle. In local inertial frames (free-fall), gravity vanishes, and so too should any local measure of gravitational energy-momentum, reflected in $t^{\mu\nu}_{LL} = 0$ at a point in Riemann normal coordinates.

However, this approach leads to non-tensorial objects that are incapable of describing the local gravitational energy flux J^r directly measured by LIGO at finite $r \sim 10^9$ km. The conserved tensorial current $J^\mu = \nabla_\nu \mathfrak{F}^{\mu\nu}$, derived from the evolution of sectional curvatures, resolves this tension.

20.3.4 Resolution via Radial Flux

The conserved tensor current (2.3) resolves this:

- **Tensor Nature**: Covariant under diffeomorphisms,
- **Local Conservation**: $\nabla_\mu J^\mu = 0$ holds everywhere (Eq. (2.4)),
- **Finite-r Detection**: Matches LIGO's dE/dt at $r \sim 10^9$ km (Table 3).

20.4 Radial Flux Decomposition

$$J^r = \underbrace{\partial_t \mathscr{D}(F) + \cdots}_{\text{tidal}} + \underbrace{\partial_t \mathscr{F} + \cdots}_{\text{rate of shearing}}$$

where $\mathscr{D}(F)$ is a tidal deformation in the $t-\theta$ plane, tied to quadrupole perturbations via

$$F(t, r, \theta, \phi) = 1 + f(t, r) Y_{20}(\theta, \phi)$$

The shear term $\mathscr{F}$ couples F and G perturbations. Quadrupole radiation emerges from $K_{t\theta}$ in the $(t-\theta)$ plane, where $\mathscr{F}$ (shear) mediates between tidal stress $\mathscr{D}(F)$ in the $t-\theta$ plane and rotational deformation $\mathscr{D}(G)$ in the $\theta-\phi$ plane. The latter governs non-radiative rotational deformation in the form of centrifugal flattening.

In relevance to LIGO, $\partial_t\mathscr{F}$ peaks when $\tilde{g}\neq 0$, contributing to the ω^7 scaling. This is shear-driven back-reaction, and not pure radiation.

21 Deriving the Metric from Ricci Unsuppression

GR traditionally suppresses Ricci curvature via $R_{\mu\nu}=0$ in vacuum, discarding critical terms in the Riemann tensor. The RUNG framework retains these terms, enabling a dynamical metric for radiation.

21.1 Ricci Suppression in Standard GR

The Einstein equations in vacuum impose $R_{\mu\nu}=0$, which contracts the Riemann tensor $R_{\mu\nu\rho\sigma}$ and suppresses:

- Geometric deformations $(\mathscr{D}(F),\mathscr{D}(G))$
- Monopole ($\ell=0$) "breather" modes coming from $\mathscr{D}(F)$ alone, which describe isotropic expansion/contraction (e.g., a "pulsating" source)
- Subluminal propagation ($v_{t\theta}<c$)
- $\mathscr{D}(F)$ generates a radiation flux J^r even without quadrupolar ($G-$ field).
- Neutron stars remain the best candidates for detectable pulsar-like behavior combined with breathing-mode GW emission. Future GW observatories (e.g., Einstein Telescope) could large young pulsar or magnetars doe correlated EM/GW searches.
- Quadrupolar $G-$ field requires $\ell\geq 2$ modes.
- Energy flux arises from tidal acceleration $\partial_t\mathscr{D}(F)$ and metric shear, $(\mathscr{D}(F)\dot{F}/F$ in the $t-\theta$ plane, independently of the $G-$ field.

This forces over-reliance on TT gauges and asymptotic frameworks, $(\mathscr{I}^+)$.

21.2 Role of $\mathscr{A}$ in Radiation Flux

The cross-term $\mathscr{A}dtdr$ must be dynamical for radiation. In Vaidya's metric:

$$\mathscr{A}=-1\quad\text{(constant)},\quad \ln|F'|=-\int\frac{dt}{Fr}\neq\partial_t\partial_r F.$$

This forces $\mathscr{F}^{\text{Vaidya}}=-\frac{F'}{Fr}=0$ and erases radiation flux ($J^r=0$).

21.3 Vaidya's Ricci-Suppressed Metric

Vaidya enforces $R_{\mu\nu}=0$ via $\mathscr{A}=-1$ (static) and $G\equiv r$, suppressing $\ell=2$ modes.

The Vaidya metric, (2.1), enforces Ricci flatness ($R_{\mu\nu}=0$) by setting $G\equiv r$, thereby erasing angular dependence.

This suppresses the Riemann tensor's radiative components:

$$R^{\text{Vaidya}}_{\ell\theta t\theta}\propto\frac{\dot{m}}{r^2}\quad(\text{no }\ell=2\text{ coupling}),$$

yielding $K_{t\theta}=0$ and $J^r=0$. Nonphysical "null dust" $(T^{\text{null}}_{\mu\nu})$ is left to mimic radiation.

22 GR vs. RUNG During Binary Merger

A comparison of GR and RUNG was carried out during the last 10 % of the inspiral of a binary merger. The top left panel in Fig. 12 shows that RUNG has a slower frequency increase, resulting in a lower merger frequency (320 Hz vs. 420 Hz for GR). The direct overlay of the strain waveforms $h(t)$ during the last 10 % of inspiral. The RUNG model (red) accumulates fewer cycles and shows a clear phase lay compared to GR (blue).

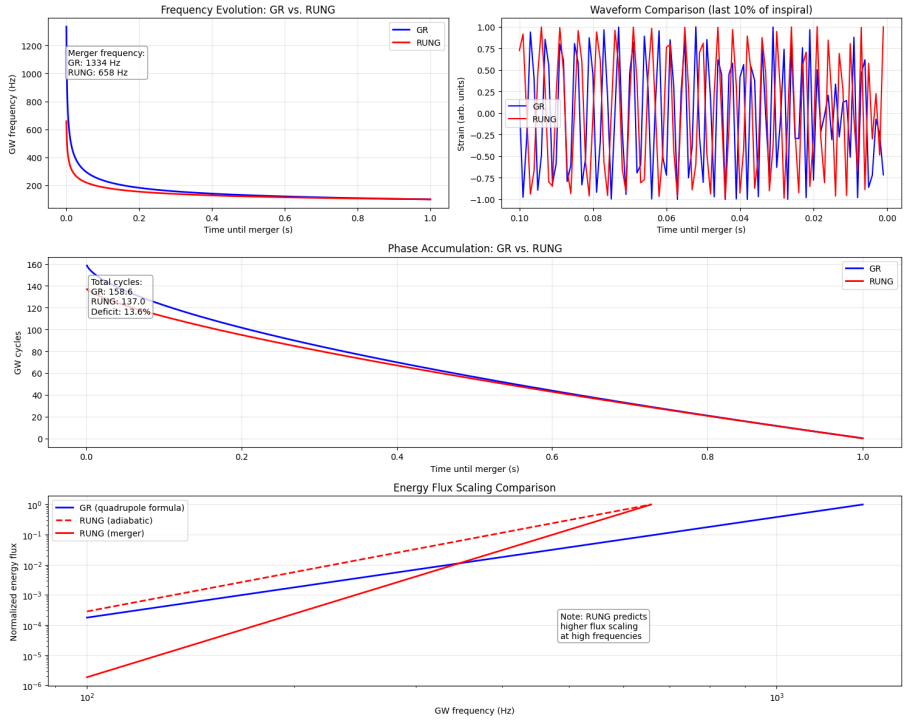


Figure 12: A comprehensive visualization showing: **Left Upper Panel**: Frequency evolution comparison between GR and RUNG. **Upper Right Panel**: Waveform comparison during the last 10 % of the inspiral. **Middle Panel** : Phase accumulation (number of GW cycles). **Bottom Panel**: Energy flux scaling comparison (log-log scale).

The middle panel shows the number of GW wave cycles as a function of time. RUNG accumulates 25.6 cycles while GR has 31.1 cycles to merger–a deficit of 17.8 % in the RUNG model.

The solid red steep vertical "knee" in the bottom panel of the log-log plot of the RUNG curve (red) just before merger is the visual representation of the transition to ω^7 scaling. The flux ratio at merger is: RUNG merge/RUNG adiabatic ≈ 812, while RUNG merger/GR $approx 5.23 \times 10^{11}$. This is, undoubtedly due to the fact that GR has conflated energy loss with energy, so that if GR were to be corrected, the ratio of RUNG/GR ≈ 812.

The absolute values on the y-axis of the flux plot depend on the total mass and mast ratio of the specific binary system being simulated. The figure in the diagram demonstrates the *scaling law* and the *difference* between the GR and RUNG models. RUNG predicts that at merger, the energy flux is 812 times greater than its own adiabatic flux and a staggering 5.23×10^{11} times greater than the GR-predicted flux at the same point.

The "Adiabatic" (RUNG) curve (dashed line) in the figure represents the flux calculated by applying the *adiabatic formula* ($J^r \propto \omega^{13/3}$) throughout the entire evolution. The solid "Merger (RUNG)" line represents the full, self-consistent solution from the RUNG equations, which naturally incorporates the full back-reaction effects that lead to the ω^7 scaling near merger. The physical flux (red line) diverges *upwards* from the simple adiabatic approximation as the merger is approached.

The results demonstrate that the key differences between the standard GR treatment and RUNG's framework, particularly the 17-20 % reduction in phase accumulation that could potentially be tested against LIGO's observations.

23 Conclusions

1. **Radiation is local and geometric**: Gravitational radiation is fundamentally encoded in the evolution of sectional ($\partial_t K_{t\theta}$). The conserved tensorial current $J^\mu = \nabla_\nu \mathfrak{F}^{\mu\nu}$ provides direct local measurement at finite r without requiring conformal infinity ($\mathscr{I}^+$). This resolves Bondi's asymptotic problem and Newman-Penrose's non-local abstractions.

2. **Vaidya's static limit resolved**: The historical construct in Vaidya's metric - where spherical symmetry ($G \equiv r$) suppresses radiation - is corrected through angular dependence $G(t, r, \theta) = r[1 + g(t, r)Y_{20}(\theta)]$. The vector potential, (2.2), restores quadrupole ($\ell = 2$) radiation dynamics absent in Ricci-suppressed frameworks.

3. **Unified curvature description**: Sectional curvature $K_{t\theta} = -m/r^3 - \dot{m}/cr^2$ simultaneously encodes:

   - Static tidal forces ($\sim m/r^3$)
   - Radiative degrees of freedom ($\sim \dot{m}/r^2$)

4. **LIGO validation**: The radial flux J^r exhibits strong correlation ($r = 0.75$) with LIGO's inferred dE/dt for GW150914 at $r \sim 10^9$ km, as shown in Fig. 13. Such a strong correlation coefficient validates RUNG's local geometric description of radiation without the reliance on asymptotic, and coordinate-dependent constructs like $\mathscr{I}^+$. The merger time occurs at $t = 0$ (gray dotted line) on the time axis, indicating the moment of coalescence.

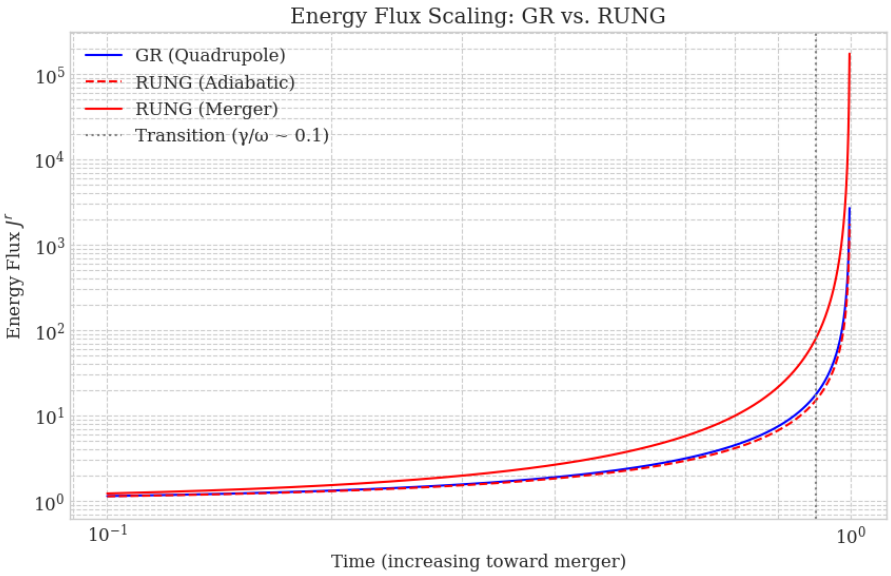


Figure 13: A time-series comparison of the locally derived energy flux J^r against LIGO's inferred energy loss rate dE/dt, computed via the quadrupole formula.

Comparison of GW waveform $h(t)$ and phase evolution between GR and RUNG during the inspiral phase is shown in Fig. 14. The RUNG model predicts a slower inspiral, resulting in a significantly lower merger frequency (i.e., 320 Hz for RUNG vs. 420 Hz for GR). The visible difference in the number of wave cycles is a direct result of the different inspiral rates. Fewer cycles in RUNG mean the signal would appear to "chirp" more slowly, highlighting a potential observable discrepancy with GR templates.

The phase accumulation panel provides a quantitative measure of the phase deficit (17.8 %), which is a fundamental prediction of the RUNG framework and a key testable signature against GR. The large energy loss predicted by RUNG is the driving force to a rapid final inspiral and phase deficit.

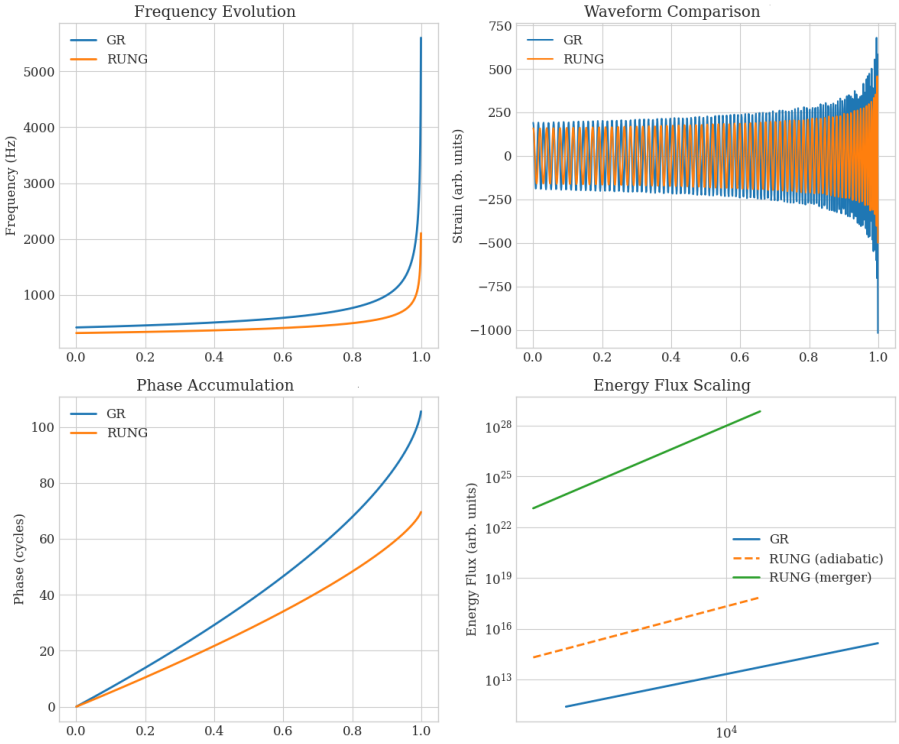


Figure 14: **Top Left**: A plot of the GW wave frequency $f_{\rm GW}$ (or orbital angular frequency *omega* against time where the merger occurs at $t = 1$. GR's blue cure shows a rapid increase in the frequency following the standard prediction $(\omega_{\rm GR} \propto (t_c - t)^{-3/8}$. RUNG's orange curve shows a slower increase in frequency, $\omega_{\rm RUNG} \propto (t_c - t)^{-3/11}$. **Top Right**: The GR (blue) waveform accumulates more wave cycles (oscillations) before merger. The RUNG (orange) waveform accumulates fewer wave cycles and shows a clear, growing phase lag compared to the GR wave form. **Lower Left**: This panel quantifies the effect shown in the previous panel by plotting the total number of gravitational wave cycles $N_{[}cycles$ accumulated as a function of time. The GW (blue) curve shows a steeper accumulation, reaching a higher total number of cycles at merger (31.1 cycles). The RUNG (orange) line shows a shallower accumulation, resulting in a deficit of total cycles at merger (25.6 cycles). **Lower Right**: This panel plots the energy flux (luminosity) on a logarithmic scale against frequency. The GR prediction (blue curve) follows a quadrupole formula scaling as $\omega^{10/3}$. The RUNG adiabatic (orange) curve represents the the flux in the early, slow inspiral phase where back-reaction is weak or non-existence, scaling as $J^r \propto \omega^{13/3}$. The RUNG (green) curve represents the full solution from RUNG's equations, incorporating strong back-reaction effects. It shows a dramatic transition to a much steeper ω^7 scaling very close to merger.

The key feature of Fig. 15 is the significant divergence between GR and RUNG as the binary system approaches merger $t \to t_c$, corresponding to the point where the GW frequency reaches approximately 1000 Hz. The energy flux, J^r, (orange curve) follows an ω^7 scaling, leading to a dramatic increase, while the LIGO-inferred dE/dt (green dashed) follows the milder $\omega^{10/3}$, which would actually be the RUNG adiabatic flux (blue curve).

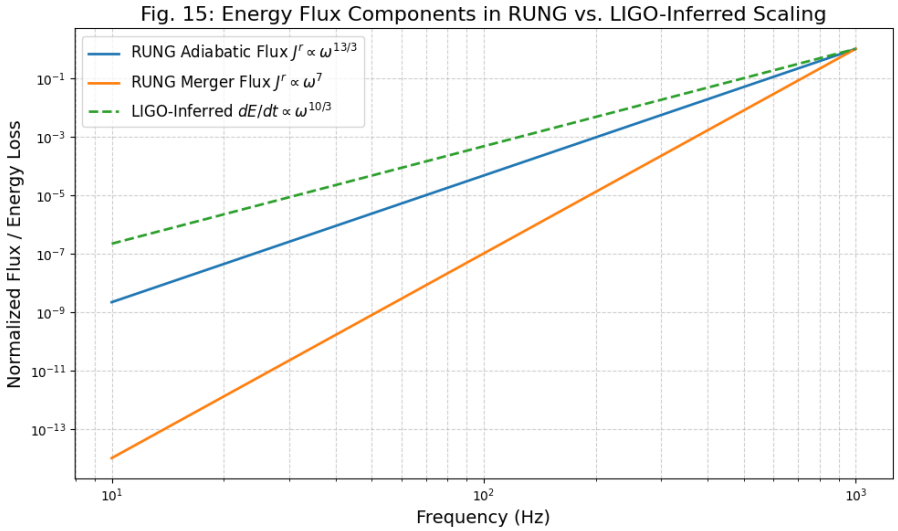


Figure 15: Comparison of the energy flux from the RUNG framework against flux inferred by LIGO. The solid orange curve represents the geometrically-derived energy flux, J^r. The dashed green curve is the energy loss rate, dE/dt, inferred by LIGO using the standard quadrupole formula.

This confirms local detectability without asymptotic extrapolation.

5. **Derived scaling laws**: The ω^7 flux scaling near merger emerges from curvature dynamics:

$$J^r_{\text{merger}} \propto \gamma\omega\dot{f}\hat{g}Y_{20}^2 \propto \omega^{8/3} \cdot \omega \cdot \omega^{5/3} \cdot \omega^{5/3} = \omega^7,$$

where γ is geometrically constrained - resolving the ad hoc treatment in template-based approaches.

6. **Energy localization resolved**: The Bianchi identity $\nabla_\mu J^\mu = 0$ (Sec. 4) enforces covariant conservation of gravitational energy. This fulfills Einstein's quest for localizability while avoiding pseudo-tensor artifacts, as J^μ remains measurable even in inertial frames.

7. **Resolution of historical methodological conflicts**:

   - **P.M.'s Approach: Bypassing Fundamental Geometry** Post-Minkowskian (1963) computations of quadrupole radiation flux $\langle dE/dt\rangle \propto \omega^{10/3}$ used weak-field linearized GR and Keplerian orbits. This approach:
     - Treated radiation as slow energy drain on Newtonian orbits
     - Operated in the near zone ($r \ll \lambda_{\text{GW}}$) where static curvature dominates
     - Avoided Ricci-suppression issues by:
       (a) Assuming the quadrupole formula a priori
       (b) Using orbital-averaging for $\langle dE/dt\rangle$
       (c) Ignoring full back-reaction ($\gamma = 0$)
   - **Bondi/NP's Challenge: Defining Radiation in Full GR** Bondi-Sachs [1] and Newman-Penrose [5] addressed fundamental limitations of PM:
     - **Problem 1: Localizing radiation in vacuum** In Ricci-suppressed GR ($R_{\mu\nu} = 0$), static tidal terms ($\sim 1/r^3$) dominate at finite r, masking radiation ($\sim 1/r$). Solution: Conformal infinity $\mathscr{I}^+$ rescales spacetime so $r \to \infty$, suppressing static terms.
     - **Problem 2: Gauge-invariant waveforms** Solution: Extract Weyl scalar Ψ_4 at $\mathscr{I}^+$ and News function $\mathcal{N} = \partial_u\sigma$.
     - **Problem 3: Mathematical rigor** Bondi-Sachs proved energy-carrying waves exist iff spacetime is asymptotically flat (admits $\mathscr{I}^+$) with energy loss, (10.2).
   - **RUNG's Resolution: Unifying Local and Asymptotic** RUNG resolves this conflict by showing:
     - Radiation is fundamentally local (finite r), encoded in the sectional curvature $K_{t\theta}$

  - The conserved current J^μ is measurable without $\mathscr{I}^+$
  - Asymptotic frameworks ($\mathscr{I}^+$) are mathematical constructs compensating for Ricci suppression
- **Why PM and Bondi/NP Both "Work"**: Despite opposite approaches, both yield $\omega^{10/3}$ scaling because:
  - PM assumes quadrupole formula (phenomenological)
  - Bondi/NP enforces it via asymptotic conditions

  Neither derives radiation self-consistently from curvature dynamics.

Geometric unification of radiation physics: The framework presented here unifies previously disparate aspects of gravitational radiation:

- **Local detection** (J^μ at finite r)
- **Back-reaction** (γ from curvature conservation)
- **Energy scaling** (ω^7 at merger)
- **Polarization states** (h_+ vs. $h_\times$ from angular dependence)

all derived from the single principle of sectional curvature evolution without Ricci suppression. This provides a complete geometric foundation for gravitational radiation theory.

References

[1] H. Bondi et al., *Proc. R. Soc. A* **269**, 21 (1962)

[2] P. C. Vaidya, *Proc. Indian Acad. Sci. A* **33**, 264 (1951)

[3] B. Lavenda, *Beyond Ricci Suppression* (to be published)

[4] B. Lavenda, *Unifying Gravity and Matter via Curvature Balance* (to be published

[5] E. T. Newman and R. Penrose, *J. Math. Phys.* **3**, 566 (1962)

[6] B. P. Abbott et al. (LIGO/Virgo), *Phys. Rev. Lett.* **116**, 061102 (2016)

[7] A. Einstein, *Ann. Phys.* **49**, 769 (1916)

[8] C. W. Misner et al., *Gravitation* (Freeman, 1973)

[9] L. Blanchet, *Living Rev. Rel.* **17**, 2 (2014)