Osmotic Pressure as Momentum Counteraction: A Kinetic Theory and its Application in Energy Conversion

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Abstract

The van't Hoff formula for osmotic pressure ($\Pi = icRT$) bears a striking mathematical resemblance to the ideal gas law ($P = nkT$). Conventionally, its microscopic origin is attributed to entropy or chemical potential gradients. This paper proposes a novel, purely kinetic theory based on momentum transfer: osmotic pressure arises from the instantaneous reversal of the perpendicular momentum component of impermeable solute particles during elastic collisions with a semi-permeable membrane while undergoing Brownian motion. This reversal directly counteracts the momentum flux of water molecules impinging on the membrane from the solution side. To compensate for this momentum flux deficit, a macroscopic static pressure must be applied to the solution side, which is the osmotic pressure. We thereby rigorously derive the van't Hoff law from first principles.

Based on this theory, we further conceptualized and experimentally validated an energy conversion system. This system utilizes an electrostatic field to create asymmetric local ion concentrations near two semi-permeable membranes, generating differential solute momentum counteraction that ultimately induces a sustained water level difference between pure water columns, allowing for the extraction of gravitational work. Improved experiments observed a water level difference of up to 2.8 meters, indicating the system's ability to absorb ambient heat and convert it into mechanical work, posing a challenge to the universal applicability of the second law of thermodynamics under specific conditions. However, the consistent contradiction between the observed flow direction and theoretical predictions remains a central puzzle to be solved.

Keywords: Osmotic Pressure, Brownian Motion, Kinetic Theory, Momentum Transfer, Momentum Counteraction, Perpetual Motion Machine of the Second Kind, Semi-Permeable Membrane, Experimental Verification

1. Introduction

Since van't Hoff's first quantitative description of osmotic pressure in 1886, the mathematical isomorphism between his formula $\Pi = icRT$ and the ideal gas law $P = nkT$ has prompted deep reflection among physicists. However, standard textbook derivations predominantly rely on thermodynamic arguments, such as entropy maximization or chemical potential balancing, rather than more fundamental mechanical principles. This obscures an intuitive microscopic picture of osmotic pressure behind abstract thermodynamic concepts.

This paper aims to revisit and construct a microscopic theory of osmotic pressure based entirely on mechanical collisions and momentum exchange. We propose that the essence of osmotic pressure lies in the momentum counteraction effect produced by solute particles colliding with the semi-permeable membrane. Based on this core picture, we not only successfully re-derive the van't Hoff law but also design a unique experimental system. Observations from this system strongly suggest the possibility of continuously extracting work from ambient heat, thereby questioning the boundaries of classical thermodynamics. Although the effect itself is robust and reproducible, the systematic discrepancy between theoretical predictions and experimental observations regarding the flow direction reveals gaps in our understanding of microscopic interfacial processes.

2. Theory

2.1. Kinetic Derivation of Osmotic Pressure as Solute Momentum Recoil

Consider a system separated by a rigid semi-permeable membrane, one side containing a dilute solution with impermeable solute, the other side pure solvent. Solute particles (mass m, number density n) undergo random Brownian motion, their velocities following the Maxwell-Boltzmann distribution. When a solute particle collides elastically with the semi-permeable membrane with a perpendicular velocity component v_x, its perpendicular momentum component $p_x = mv_x$ is instantly reversed to $-mv_x$. Consequently, the momentum change imparted to the membrane-solution system per collision is $\Delta p_x = 2m \mid v_x \mid$.

Osmotic pressure Π is physically defined as the time-averaged force per unit area contributed by this momentum transfer. Let N particles strike an area A of the membrane in time interval Δt. The time-averaged force F is:

$$F = \frac{N \cdot \Delta p_x}{\Delta t}$$

According to kinetic theory, the number of particles striking unit area per unit time (collision rate) is $\Gamma = \frac{n\langle | v_x | \rangle}{2}$, where $\langle | v_x | \rangle$ is the average speed of v_x. To relate $\langle | v_x | \rangle$ to an observable statistic, we note that for a Maxwellian distribution $\langle | v_x | \rangle = \sqrt{\frac{2kT}{\pi m}}$, but a more direct derivation uses $\langle v_x^2 \rangle$. Analyzing particle flux and combining it with momentum change yields the standard kinetic result:

$$\Pi = nm\langle v_x^2 \rangle$$

Since the system is isotropic, the equipartition theorem gives $\frac{1}{2}m\langle v_x^2 \rangle = \frac{1}{2}kT$, hence $\langle v_x^2 \rangle = \frac{kT}{m}$. Substituting:

$$\Pi = nkT$$

Converting microscopic parameters to macroscopic variables: number density $n = N_A c$ (where c is the molar concentration, N_A is Avogadro's number), Boltzmann constant $k = R/N_A$ (R is the universal gas constant), we obtain:

$$\Pi = cRT \quad \text{(van't Hoff's law for } i = 1\text{)}$$

2.2. The Microscopic Balance Mechanism: Momentum Counteraction

The derivation above provides the magnitude of Π, but elucidating its physical mechanism is crucial. The core lies in understanding how solute collisions affect the transport of water molecules across the membrane.

1. **Baseline Case (Pure Solvent):** When only pure solvent is separated by a semi-permeable membrane, water molecules constantly and randomly cross the membrane due to thermal motion. The momentum flux of water molecules perpendicularly striking the membrane is statistically symmetric in both directions, resulting in zero net momentum transfer and thus no macroscopic pressure difference.
2. **Introduction of Solute (Solution Side):** When impermeable solute is present on the solution side, these solute particles continuously collide with the semi-permeable membrane. Each collision results in the solute particle gaining a perpendicular momentum recoil directed into the solution bulk. From the membrane's perspective, it continually exerts a microscopic force on the solution directed inward.
3. **Momentum Flux Imbalance:** The physical effect of this sustained inward microscopic force is to **hinder and reduce** the net momentum flux of water molecules striking the semi-permeable membrane *from the solution side*. However, the momentum flux of water molecules striking the membrane from the pure solvent side remains unchanged.

Consequently, the momentum fluxes on the two sides of the membrane become unbalanced, generating a net momentum transfer directed towards the solution side, macroscopically manifesting as a net force pushing pure solvent into the solution side.

4. **Establishment of Mechanical Equilibrium:** To counteract this net flow and re-establish equilibrium, an additional hydrostatic pressure must be applied to the solution side. When this pressure equals $\Pi = nkT$, it precisely compensates for the "water molecule momentum flux deficit" caused by the solute momentum recoil, by increasing the number density and average kinetic energy of water molecules on the solution side. At this point, the net momentum flux of water molecules striking the membrane from both sides reaches dynamic balance again, and macroscopic net flow ceases.

Therefore, the physical essence of osmotic pressure is: **the equilibrium pressure that must be applied on the solution side to compensate for the deficit in water molecule momentum flux caused by solute momentum recoil.**

2.3. Energy Conversion System Based on Momentum Counteraction

Based on the above theory, we conceived a U-tube energy conversion system. The system is divided by two semi-permeable membranes into three chambers: a central chamber filled with a dilute electrolyte solution (e.g., sodium sulfate or citric acid), flanked by two side chambers filled with pure water.

The core operation involves using an applied electrostatic field (or using inherently charged nanofiltration membranes) to create asymmetric local ion concentrations in the regions adjacent to the two membranes in the central chamber. According to kinetic theory, this results in differential solute momentum counteraction strength at the two membranes:

- **High Local Concentration Side:** The higher concentration of impermeable ions near this membrane means more solute particles collide with the membrane per unit time. The strong solute momentum recoil significantly weakens the outward momentum flux of water molecules from the solution side at this membrane, resulting in water **inflow** into the central chamber (osmosis).
- **Low Local Concentration Side:** The lower concentration of impermeable ions near this membrane results in weaker solute momentum recoil. Consequently, the outward momentum flux of water molecules from the solution side is relatively dominant, resulting in water **outflow** from the central chamber (reverse osmosis).

This process creates a stable water level difference Δh between the pure water columns in the U-tube side arms. By connecting a drainage channel between the tops of the two side water columns, gravitational work can be continuously extracted via a water turbine. The system as a whole absorbs thermal energy from its surroundings to maintain the water level difference generated by the osmotic effect, thus theoretically achieving near 100% conversion of ambient heat into useful work within this specific framework.

3. Experimental Progress and Validation

3.1. Initial Experiment (2017)

The author first proposed this theory online and reported preliminary experimental results on a ScienceNet.cn blog in 2017. The experimental setup used an external power supply and insulated electrodes to generate an electrostatic field. Upon application of the field, a sustained water level difference between the two pure water columns and continuous droplet flow through a connecting channel were observed, providing initial empirical support for the theoretical hypothesis.

3.2. Independent Verification and Improvement (2024)

In 2024, a team led by Mr. Demin Huang conducted crucial independent verification and improvement experiments. The most significant improvement was the replacement of the external electrodes with a commercially available nanofiltration membrane possessing inherent fixed charge, allowing the required field to be generated internally within the system. This modification offered two major advantages:

1. It achieved a significantly larger stable water level difference, reaching **2.8 meters**, far exceeding the level in the initial experiment.
2. It **completely eliminated** potential artifacts from leakage currents present in the original electrode setup, greatly enhancing the reliability of the experimental results. Subsequently, Dr. Jing Fan independently reviewed and confirmed the repeatability and authenticity of the improved experiment conducted by Huang's team.

3.3. Technical Limitations and Prospects

Current experiments use commercial reverse osmosis and nanofiltration membranes, which are non-ideal semi-permeable membranes and allow some degree of solute leakage (especially the NF membrane). Nevertheless, the observed hydraulic imbalance persists

despite leakage, demonstrating the robustness of the physical effect.
The most critical technological hurdle for the practical application of this technology is the development of an "ideal semi-permeable membrane" with perfect ion selectivity. Examples include track-etched membranes with ultra-uniform pores or graphene-based atomic-level filters. Overcoming this bottleneck could unlock a potent ambient thermal energy harvesting technology.

4. Discussion

4.1. Self-Consistency and Predictive Power of the "Momentum Counteraction" Theory

The "Momentum Counteraction" model proposed here not only successfully derives the van't Hoff law from first principles but its internal logic also naturally explains why the law is **valid only for dilute solutions**, demonstrating the theory's strong self-consistency and predictive power.

- **Dilute Solution Regime:** Solute particles are widely spaced, and their collisions with the membrane are the dominant pathway for momentum exchange. Here, the momentum recoil of virtually all solute particles acts effectively on the membrane interface, precisely "counteracting" a corresponding proportion of the water molecule momentum flux. Thus, theoretical prediction and the van't Hoff formula agree perfectly.
- **Concentrated Solution Regime:** As solute concentration increases, the average inter-particle distance decreases, and particle-particle collisions become highly frequent. The path of a solute particle destined to hit the membrane and counteract water molecule momentum is likely interrupted by collisions with other solute particles. Its recoil momentum, which should have been delivered to the membrane, is thereby **dissipated and dispersed within the solution bulk**, failing to reach the interface effectively. The result is that for the same solute number density (or concentration), the actual osmotic pressure generated in concentrated solutions will be lower than predicted by the van't Hoff formula.

This picture is entirely analogous to the kinetic reason for the deviation of the ideal gas law at high pressures due to intermolecular collisions and volume effects. It compellingly suggests that the limited applicability of the van't Hoff formula itself is evidence for the mechanical nature of osmotic pressure: the formula holds provided that **the momentum**

recoil of all solute particles is effectively used to counteract the momentum of water molecules.

4.2. Challenge to Conventional Thermodynamic Explanations

The "Momentum Counteraction" model provides a purely mechanical, chemical-potential-independent microscopic explanation for the phenomenon of osmotic pressure and its application in energy conversion. In the conceived ideal system, the overall composition of the central chamber remains constant, a scenario where chemical potential theory suggests no sustained net effect should occur. However, the experiment shows clear macroscopic energy output, strongly implying that, excluding chemical energy changes, the system's only energy source is ambient heat. If this effect is ultimately confirmed with an ideal membrane system, it would indicate that the statement of the second law of thermodynamics might require modification or qualification regarding its scope of applicability in specific microscopic systems capable of "rectifying" molecular thermal motion.

4.3. An Unsolved Puzzle: The Flow Direction Discrepancy

Although the experimental effect itself is robust and verifiable, a critical, unresolved contradiction exists between the theoretically predicted and experimentally observed flow direction:

- **2017 Experiment (using Na_2SO_4 solution, external electrodes):** Theory predicted that the anode side, attracting SO_4^{2-} anions, would have a lower local ion concentration near the membrane, lower osmotic pressure, and thus experience reverse osmosis (water outflow), leading to a higher water level on that side. However, the experiment observed the precise opposite.
- **2024 Experiment (using Citric acid solution, charged NF membrane):** Theory predicted that the side with the charged nanofiltration membrane, repelling negatively charged citrate ions, would have a lower local ion concentration, lower osmotic pressure, and thus experience reverse osmosis, resulting in a higher water level. The experiment again observed the opposite phenomenon (lower water level on the NF membrane side).

This systematic reversal of the flow direction is a major puzzle that the current theoretical framework cannot explain. Its root cause may lie in unaccounted complex physicochemical interactions at the membrane-solution interface, such as specific ion-membrane

interactions, electric double layer effects, or hydrodynamic factors. Resolving this puzzle is crucial for achieving a complete theoretical understanding.

5. Conclusion

This paper has revealed the microscopic origin of osmotic pressure from the novel perspective of "Momentum Counteraction," unifying it with ideal gas pressure through rigorous kinetic derivation. This theory not only derives the van't Hoff law but also naturally explains the profound reason for its validity only in dilute solutions. The preliminary experimental results from the energy conversion system built on this principle show the potential for continuously extracting work from ambient heat, posing a challenge to the traditional boundaries of the second law of thermodynamics. However, the systematic experimental deviation in flow direction clearly indicates that our current understanding of the microscopic interfacial processes within the system remains incomplete, and the present model requires further development and refinement. This work calls for wider independent verification, theoretical deepening, and in-depth research into the rectification mechanisms of thermal motion at the nanoscale within the scholarly community.

Conflict of Interest Statement

The author declares no conflicts of interest regarding the publication of this paper.

Data Availability Statement

No new primary data were generated in this study. All existing data supporting the conclusions of this paper are cited in the reference list. Theoretical derivations were performed using publicly available software tools including DeepSeek.

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