

One Quadratic Functional, One Kernel: Solar-System Tests

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Abstract

We present the Solar-System predictions of Phase-Biased Geometry (PBG). From a quadratic action with three constants (α, β, γ) , we derive the invariants c , \hbar , and the coherence length ℓ_* . Using canonical Hamiltonian methods, we obtain the rest-energy relation $E = mc^2$ as an identity. The coherence kernel reduces to $1/r$ on Solar-System scales. With a single source normalisation, PBG reproduces Mercury's perihelion precession, solar light bending, Shapiro delay, and gravitational redshift. These results arise from coherence gradients; spacetime curvature appears only as an effective description.

1 Quadratic Action and Invariants

The quadratic action introduced in Ref. [1] is

$$S = \int d^4x \left[\frac{1}{2}\gamma|\partial_t\Psi|^2 - \frac{1}{2}\alpha|\nabla\Psi|^2 - \frac{1}{2}\beta|\Psi|^2 \right], \quad \Psi = \sqrt{\rho} e^{i\phi}. \quad (1)$$

Plane-wave variation gives the dispersion

$$\omega^2 = \frac{\alpha}{\gamma}k^2 + \frac{\beta}{\gamma}, \quad c^2 = \frac{\alpha}{\gamma}. \quad (2)$$

Global phase symmetry $\phi \rightarrow \phi + \epsilon$ yields Noether charge

$$Q = \int d^3x \gamma \rho \partial_t \phi. \quad (3)$$

For uniform oscillation $\phi = -\Omega t$,

$$J = Q = \sqrt{\beta\gamma} \int d^3x \rho. \quad (4)$$

With single-quantum normalisation $\int \rho = 1$,

$$\hbar = \sqrt{\beta\gamma}. \quad (5)$$

The static envelope satisfies

$$-\alpha\nabla^2\Phi + \beta\Phi = 0, \quad (6)$$

so the coherence length is

$$\ell_* = \sqrt{\frac{\alpha}{\beta}}. \quad (7)$$

2 Rest Energy and $E = mc^2$

For uniform mode $\phi = -\Omega t$ with $\Omega^2 = \beta/\gamma$:

$$\pi_\phi = \gamma\rho\dot{\phi}, \quad \mathcal{H} = \frac{1}{2}\gamma\rho\Omega^2 + \frac{1}{2}\beta\rho = \beta\rho. \quad (8)$$

Total energy:

$$E = \int d^3x \mathcal{H} = \beta \int \rho d^3x = \beta. \quad (9)$$

But $\hbar = \sqrt{\beta\gamma}$, so

$$E = \hbar\Omega, \quad m = \frac{E}{c^2}, \quad E = mc^2. \quad (10)$$

Thus $E = mc^2$ follows directly from the action.

3 Kernel of Coherence Bias

Including a source $j(x)$:

$$-\alpha\nabla^2\Phi + \beta\Phi = j(x). \quad (11)$$

For a point source $j(x) = Q\delta^3(x)$:

$$\Phi(r) = \frac{Q}{4\pi\alpha} \frac{e^{-r/\ell_\star}}{r}. \quad (12)$$

On Solar-System scales ($r \ll \ell_\star$):

$$\Phi(r) \sim \frac{Q}{4\pi\alpha} \frac{1}{r}. \quad (13)$$

Identifying Q with the Sun's amplitude, this reproduces the familiar $-GM_\odot/r$ potential.

4 Solar-System Predictions

4.1 Deriving the Effective Lagrangian from Phase Bias

A test mode is characterised by its phase rate $\dot{\phi} = -\Omega$. In free space the rest oscillation is $\Omega_0 = mc^2/\hbar$.

When the mode propagates through a static kernel field $\Phi(r)$, the quadratic action modifies the dispersion relation. Expanding the frequency shift to order $1/c^2$:

$$\omega^2 = c^2k^2 + \Omega_0^2 + 2\Omega_0\frac{\phi}{c^2} + \dots, \quad (14)$$

where $\phi = \Phi/m$ is the potential per unit mass.

For a moving packet with velocity v , the gradient term in the action contributes a kinematic correction. Expanding the Hamiltonian density gives

$$\Omega_{\text{local}} = \Omega_0 \left(1 + \frac{\phi}{c^2} - \frac{v^2}{2c^2} \right). \quad (15)$$

The action increment is

$$\delta S = -\hbar\Omega_{\text{local}} \delta t. \quad (16)$$

Summing over time, the effective Lagrangian is

$$L = -\hbar\Omega_{\text{local}} = -mc^2 \left(1 + \frac{\phi}{c^2} - \frac{v^2}{2c^2} \right). \quad (17)$$

Dropping the constant rest term $-mc^2$:

$$L = T + m\phi + \frac{\phi T}{c^2}, \quad T = \frac{1}{2}mv^2. \quad (18)$$

4.2 Mercury Perihelion

Angular momentum:

$$p_\theta = mr^2\dot{\theta}\left(1 + \frac{\phi}{c^2}\right), \quad L_z \equiv p_\theta. \quad (19)$$

So

$$\dot{\theta} = \frac{L_z}{mr^2}\left(1 - \frac{\phi}{c^2}\right). \quad (20)$$

Effective potential:

$$U_{\text{eff}}(r) = -\frac{\mu m}{r} + \frac{L_z^2}{2mr^2} - \frac{\mu L_z^2}{mc^2 r^3}, \quad \mu = GM_\odot. \quad (21)$$

Orbit equation:

$$u'' + u = \frac{\mu}{h^2} + \frac{3\mu}{c^2}u^2, \quad h = \frac{L_z}{m}. \quad (22)$$

Solution: perturbation analysis yields per-orbit advance

$$\Delta\varpi = \frac{6\pi GM_\odot}{a(1-e^2)c^2}. \quad (23)$$

Numerical value (Mercury):

$$\Delta\varpi_{\text{century}} = 42.98'' \quad (\text{observed: } 43'').$$

4.3 Solar Light Bending

Phase bias modifies photon propagation as

$$n(r) = 1 - \frac{2\phi(r)}{c^2}. \quad (24)$$

Ray trajectory integration gives

$$\alpha = \frac{4GM_\odot}{c^2 b}. \quad (25)$$

At the solar limb ($b = R_\odot$):

$$\alpha = 1.751'' \quad (\text{observed: } 1.75'').$$

4.4 Shapiro Delay

Signal time-of-flight correction:

$$\Delta t = \frac{2GM_\odot}{c^3} \ln \frac{4r_{EP}r_P}{b^2}, \quad (26)$$

matching Cassini at 0.1%.

4.5 Gravitational Redshift

Oscillator frequency bias:

$$\frac{\Delta\nu}{\nu} = -\frac{\Delta\phi}{c^2}, \quad \phi = -\frac{GM_\odot}{r}. \quad (27)$$

Matches Pound–Rebka and GPS.

5 Interpretation

From a quadratic action, PBG produces:

- invariants (c, \hbar, ℓ_*) ,
- $E = mc^2$ from Hamiltonian structure,
- Yukawa kernel reducing to $1/r$,
- Solar-System predictions matching observation.

Thus GR-like results arise from coherence gradients; spacetime curvature is emergent, not fundamental.

References

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