One Quadratic Functional, One Kernel, All Scales

N.J.Kettlewell

Abstract

A standard quadratic functional, familiar from Helmholtz screening, Ginzburg–Landau theory, and scalar field actions, is applied without modification to macroscopic motion. Its extremisation yields a Yukawa kernel that reproduces Newton's inverse-square law in the near field, produces flat galactic rotation curves in an intermediate regime, and predicts a finite-range cut-off at large radii. With a single, universal coherence length $\ell = 1/k$ in the galactic band, Solar-System deviations are suppressed to $\mathcal{O}((kr)^2)$ and therefore negligible. The framework is falsifiable: if accelerations do not fall exponentially on galactic scales, it fails.

1 Principle

Start from the quadratic functional

$$\mathcal{C}[B] = \int \left(\alpha|\nabla B|^2 + \beta B^2\right) d^3x \; - \int J(\mathbf{x})B(\mathbf{x})\, d^3x, \tag{1}$$

with source density $J(\mathbf{x})$.

This is the familiar Helmholtz / Ginzburg–Landau energy form. Extremisation gives

$$(\nabla^2 - k^2)B = -\tfrac{1}{2\alpha}J, \qquad k^2 = \beta/\alpha, \tag{2}$$

whose Green's function is Yukawa:

$$G_k(r) = \frac{e^{-kr}}{4\pi r}. \tag{3}$$

Eliminating B yields the effective interaction energy

$$U[J] = \tfrac{1}{2}\int J(\mathbf{x})\, G_k(\mathbf{x}-\mathbf{x}')\, J(\mathbf{x}')\, d^3x\, d^3x'. \tag{4}$$

2 From U to force

For two compact sources of strengths q_1, q_2 at separation r:

$$U(r) = \frac{q_1 q_2}{4\pi}\,\frac{e^{-kr}}{r}. \tag{5}$$

The force is

$$F(r) = -\frac{dU}{dr} = -\frac{q_1 q_2}{4\pi}\left(\frac{1}{r^2} + \frac{k}{r}\right) e^{-kr}. \tag{6}$$

With the identification $q_i \propto m_i$, the Newtonian limit is recovered as $kr \to 0$:

$$F(r) \;\longrightarrow\; -\frac{Gm_1m_2}{r^2}. \tag{7}$$

3 Extended sources

For a continuous density $\rho(\mathbf{x})$:

$$\Phi(\mathbf{r}) \propto \int \rho(\mathbf{r}')\, G_k(|\mathbf{r}-\mathbf{r}'|)\, d^3r', \qquad \mathbf{a}(\mathbf{r}) = -\nabla\Phi(\mathbf{r}). \tag{8}$$

The corresponding force kernel is the gradient of the Yukawa potential,

$$\mathbf{K}_F(s) = \hat{\mathbf{s}} \left(\frac{1}{s^2} + \frac{k}{s}\right) e^{-ks}/(4\pi), \quad s = |\mathbf{r}-\mathbf{r}'|. \tag{9}$$

Convolving this kernel with an exponential disc produces: (i) an inner Newtonian rise, (ii) a broad intermediate region with $v_c(R) \approx$ const (flat rotation), (iii) and an outer exponential decline for $R \gg \ell = 1/k$.

Thus flat rotation curves and their eventual fall-off follow directly, with no additional halo component.

4 Solar-System consistency

With a universal galactic coherence length $\ell = 1/k \sim$ 5–50 kpc, the Solar System lies extremely deep in $kr \ll 1$.

The exact Newton/Yukawa ratio is

$$\frac{F}{F_N} = (1+kr)\, e^{-kr} = 1 - \tfrac{1}{2}(kr)^2 + \mathcal{O}((kr)^3). \tag{10}$$

Thus the deviation is second order, not first order. At 1 AU with $\ell = 10$ kpc,

$$kr \approx 5 \times 10^{-10}, \qquad \Delta F/F \sim 10^{-19}. \tag{11}$$

So a single, universal k produces negligible deviations locally, while predicting decisive suppression on galactic scales. No screening mechanism or scenario-dependence of k is required.

5 Discussion: coherence and least cost

The integrand

$$\alpha|\nabla B|^2 + \beta B^2 \tag{12}$$

is the standard field energy density. It also admits a natural reading as a coherence cost: the gradient term penalises rapid phase variation, the mass term penalises departure from uniform density. Minimising $\mathcal{C}$ selects standing solutions B that preserve order at least cost.

On this view, B is not merely a potential but a kernel of coherence overlap: its quadratic density contributes locally to the cost, and its bilinear interaction yields the effective energy $U[J]$ that drives dynamics. The same quadratic form long used to describe quantum correlation fields is here extended without a cut-off in scale, applying equally to astronomical systems.

An obvious extension adds a third overlap term of the form $\gamma\rho_c B^2$, coupling the kernel directly to source distributions. This preserves the quadratic structure while enabling the same least-cost principle to account for light propagation alongside orbital dynamics.

6 Falsifiability

- GR/Newton: $1/r^2$ forces with no intrinsic range.
- This framework: extended sources produce flat rotation in an intermediate regime, but accelerations must taper as e^{-kR} once $R \gg \ell$.

This predicts sharper orbital fall-offs for remote satellites and streams, and a limit to bound structures at large radii. Pulsar timing arrays (e.g. SKA) and the dynamics of galactic outskirts provide a clean test: if accelerations persist indefinitely as $1/r^2$, the framework is excluded; if a finite-range fall-off is observed, the coherence-kernel picture is confirmed.

Summary

A single quadratic functional yields a Yukawa kernel that (i) reproduces Newton locally, (ii) explains flat galactic rotation with an eventual cut-off, and (iii) remains consistent with Solar-System data since deviations are $\mathcal{O}((kr)^2)$ with ℓ in the galactic range. Only one universal length scale is introduced.

No additional halo component is required, and the formalism is the same one long used to describe correlation fields in quantum theory. If validated, this would suggest that phenomena normally treated as separate across scales — from microscopic coherence to galactic dynamics — are governed by the same underlying rule.

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