Resolving the Cosmological Constant Problem via Flux-Limited Vacuum Energy: A Causality-Based Cutoff

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Abstract

The cosmological constant problem (CCP) presents a severe $\sim 10^{120}$ mismatch between quantum field theory's prediction of vacuum energy and the observed dark-energy density. We propose a solution based on a causal coherence cutoff: only fluctuations within a light-speed–limited sphere contribute to the vacuum energy. This principle yields a universal r^{-4} scaling, consistent with quantum predictions at small scales and with cosmic acceleration at mesoscopic lengths. Remarkably, derivations from multiple perspectives converge on a coherence scale of order $75\,\mu$m, suggesting a genuine physical feature of the vacuum rather than an artifact of calculation. The framework offers a path to resolving the CCP while suggesting testable predictions at the boundary between quantum and cosmological regimes.

Keywords

vacuum energy, cosmological constant problem (CCP), causal coherence, dark energy, entropic horizon, cosmic foam, Casimir effect

0.1 Background and Motivation

Infinities in mathematics usually signal incompleteness, not physical reality. The simple function $1/x$ diverges at the origin, yet no measurement in nature yields a literal infinity. Physicists reinterpret such divergences rather than discard successful theories. Planck's quantization resolved the ultraviolet catastrophe in blackbody radiation, and renormalization tamed the infinities of quantum electrodynamics. These cases remind us that infinities are not features of nature, but warnings that our frameworks need refinement.

The most severe divergence arises in the cosmological constant problem (CCP). Summing zero-point energies of quantum fields up to the Planck scale ($r \sim 1.6 \times 10^{-35}\,\mathrm{m}$) predicts a vacuum energy density of order $10^{113}\,\mathrm{J/m^3}$. General relativity requires this energy to gravitate, yet cosmological observations imply only $\sim 6 \times 10^{-10}\,\mathrm{J/m^3}$. The resulting mismatch of $\sim 10^{120}$ is so extreme that it may as well be treated as an infinity, signaling a profound misalignment between theory and physical reality. It emerges not from speculation but from the interplay of our two most trusted theories, quantum mechanics and general relativity.

Recent observations strengthen the case that this problem is not a mere artifact of interpretation. Data from the Dark Energy Spectroscopic Instrument (DESI) hint at evolving dark energy, challenging the view of a static cosmological constant and suggesting that vacuum energy itself may be dynamic. Crucially, these results appear independently of standard ΛCDM fits, adding weight to the evidence. This motivates the search for a framework that ties quantum fluctuations directly to cosmic expansion.

If the catastrophe originates within established physics, then its resolution should also emerge from those same foundations. Just as quantization resolved blackbody radiation without abandoning thermodynamics, the cure for the CCP should arise from principles already embedded in physics: causality, coherence, geometry, and entropy. In this work we develop such a resolution by introducing a causal coherence cutoff, where spacetime geometry and light propagation impose natural limits on vacuum fluctuations. The connection between atomic scales, exemplified by the Lamb shift—a well-known quantum correction at atomic scales—and cosmological acceleration suggests that vacuum structure links the quantum and cosmic directly. This framework offers a consistent path to reconcile quantum mechanics and gravitation without speculative new fields or dimensions.

0.2 Limitations of Existing Approaches

Existing CCP solutions include:

- **Symmetry-based cancellations**: Supersymmetry (SUSY) predicts bosonic-fermionic cancellation, but no superpartners appear at LHC energies, leaving a $\sim 10^{60}$ gap [3].
- **Dynamical dark energy**: Quintessence or running vacuum models (RVM) allow time-varying Λ, addressing Hubble tension, but introduce new fields [4].
- **Renormalization**: Subtracting infinities in QFT is ad hoc for gravity, as vacuum energy gravitates [1].
- **Anthropic/multiverse**: String theory landscapes select small Λ but are empirically untestable [3].
- **Modified gravity**: Unimodular or entropic gravity reinterprets Λ, often neglecting quantum origins [5].

Our model avoids these pitfalls by grounding the cutoff in causality, a fundamental principle, and leveraging observable scales.

0.3 Conceptual Idea of Causal Coherence

We propose that only vacuum fluctuations within a causally coherent region—where modes can interact at or below light speed c—contribute to observable vacuum energy. This imposes a maximum wavevector $k_{\max} = \pi/r$, where r is the coherence radius, offering a relativistic regularization that tames QFT divergences [6, 7]. The radius r is tied to spacetime geometry, ensuring consistency with GR.

0.4 Main Results in Brief

The model yields

$$\rho = \frac{\hbar c \pi^2}{16 r^4},$$

matching QFT predictions at small r (Planck scale) and cosmological observations at $r \approx 75\,\mu\text{m}$. An entropic horizon interpretation, where vacuum entropy saturates the Bekenstein bound, sets bounds on r from $\sim 1.1\,\ell_P$ to $75\,\mu\text{m}$ (Sec. 2.5.3), resolving singularities and suggesting a dynamic vacuum compatible with DESI data. A multilayered cosmic foam extension visualizes these dynamics.

0.5 Relation to Broader Theoretical Frameworks

This framework connects to entanglement entropy [8], holographic principles [7], causal sets [6], and emergent gravity [9]. It proposes that spacetime and quantum mechanics emerge from causal flux limitations, unifying atomic and cosmological scales.

0.6 Outline of the Paper

Section 1 derives the model mathematically. Section 2 evaluates it numerically across scales. Section 3 explores conceptual foundations, including the entropic horizon. Section 4 proposes experimental tests. Section 5 details implications for black holes, the Big Bang, PBHs, Hubble tension, and galaxy rotation, introducing the cosmic foam model and CERN proposal. Section 6 presents the Casimir test of causal coherence. Section 7 concludes with contributions and future directions.

1 Model and Mathematical Derivation

1.1 Overview of the Framework

We impose a causal coherence cutoff on vacuum modes, ensuring only fluctuations within a region traversable by light in time $t = r/c$ contribute to energy density. This is motivated by relativistic causality, limiting correlations to light cones [12].

1.2 Standard Description of Vacuum Energy

Zero-point energy of field modes. In quantum theory, every harmonic oscillator has a ground-state ("zero-point") energy of $\frac{1}{2}\hbar\omega$. A free quantum field behaves like a continuum of such oscillators, one for each spatial wavevector $\mathbf{k}$. Summing the zero-point energy over all modes gives the vacuum energy. For a (massless) scalar field with dispersion $\omega_{\mathbf{k}} = c\,|\mathbf{k}| \equiv ck$, the vacuum energy *density* (energy per unit volume) is

$$\rho \;=\; \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{2}\hbar\,\omega_{\mathbf{k}} \;\;\longrightarrow\;\; \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\,\hbar\,\omega_{\mathbf{k}}, \tag{1}$$

where we replaced the discrete sum by an integral using the standard density of states $V/(2\pi)^3$. The factor $1/(2\pi)^3$ is thus a bookkeeping device ensuring the sum becomes an integral *per unit volume*.

Why the measure looks like d^3k. The integral $\int d^3k$ means we are counting all plane-wave modes in "k-space". In spherical coordinates,

$$d^3k \;=\; 4\pi\,k^2\,dk,$$

so Eq. (1) becomes

$$\rho = \frac{\hbar}{2}\int \frac{4\pi\,k^2\,dk}{(2\pi)^3}\,(ck) \tag{2}$$

$$= \frac{\hbar c}{2}\,\frac{4\pi}{(2\pi)^3}\int_0^\infty k^3\,dk. \tag{3}$$

The divergence. The integral $\int_0^\infty k^3\,dk$ diverges at the upper limit. If we temporarily stop the integral at some large $k_{\max}$ (an ultraviolet cutoff), we get

$$\int_0^{k_{\max}} k^3\,dk \;=\; \frac{k_{\max}^4}{4} \quad\Rightarrow\quad \rho(k_{\max}) \;\propto\; k_{\max}^4.$$

If one chooses the Planck scale as $k_{\max}$, the result is $\rho \sim 10^{113}\,\mathrm{J/m^3}$, which is the root of the cosmological constant problem: General relativity admits no escape from this result: any constant vacuum energy density enters the stress–energy tensor as $T_{\mu\nu}^{\mathrm{vac}} = -\rho_{\mathrm{vac}} g_{\mu\nu}$ and therefore acts as a cosmological constant. In other words, vacuum energy must gravitate. If the quartically divergent QFT value were real, spacetime curvature would be dominated by $\rho_{\mathrm{vac}} \sim 10^{113}\,\mathrm{J/m^3}$, causing the universe to curl up almost instantly—utterly inconsistent with the observed $\sim 6\times10^{-10}\,\mathrm{J/m^3}$ that drives the present cosmic acceleration.

Takeaway. This textbook calculation highlights the problem: the naive QFT sum over modes, with no physical cutoff, produces an absurdly large vacuum energy density. In the next section we introduce a causality-based coherence scale that regularizes this divergence in a principled way.

1.3 Introducing the Causal Coherence Cutoff

The divergence in Eq. (3) arises because all momenta up to infinity are counted, even though physical correlations cannot propagate faster than light. In reality, vacuum fluctuations are not globally coherent: two widely separated points cannot exchange information unless they lie within each other's light cones. This motivates a *causal coherence radius* r, the maximum separation over which modes can remain phase-coherent.

Physical picture. Consider a spherical region of radius r. The shortest wavelength that can "fit" across its diameter is $\lambda_{\min} = 2r$. Equivalently, the maximum wavenumber is

$$k_{\max} \;=\; \frac{2\pi}{\lambda_{\min}} \;=\; \frac{\pi}{r}. \tag{4}$$

Modes with shorter wavelength than this cannot establish causal correlations across the sphere, and so their contribution is excluded.

On the choice of cutoff. The identification $k_{\max} = \pi/r$ is not unique; other geometric factors could plausibly arise depending on how one treats boundary conditions. Our choice follows the simple physical intuition that the shortest wavelength able to maintain phase coherence across the sphere is its diameter, $2r$. This links the cutoff directly to the causal structure of spacetime, but further theoretical work is required to show whether this form can be derived from deeper principles such as quantum information bounds or causal set discreteness.

Modified integral. Replacing the divergent $k_{\max} \to \infty$ in Eq. (3) with the causal value (4), the vacuum energy density becomes

$$\rho = \frac{\hbar c}{2} \frac{4\pi}{(2\pi)^3} \int_0^{\pi/r} k^3 \, dk \tag{5}$$

$$= \frac{\hbar c}{16\pi^2} \left[\frac{k^4}{4} \right]_0^{\pi/r} \tag{6}$$

$$= \frac{\hbar c \, \pi^2}{16 \, r^4}. \tag{7}$$

Thus the divergence is tamed not by an arbitrary cutoff, but by a geometric, relativistic one: coherence cannot extend beyond r without violating causality.

Interpretation. Equation (7) yields an energy density that scales universally as r^{-4}. This scaling reflects the dimensionality of spacetime itself and holds whether r is as small as the Planck length or as large as the Hubble radius. The coherence radius r therefore serves as the bridge between quantum field fluctuations and cosmological observables. It is important to emphasize that the spherical region of radius r is a mathematical construct denoting the extent of causal coherence. It does not represent a physical membrane in spacetime, but rather the maximal domain over which vacuum modes can remain phase-correlated without violating relativistic causality.

1.4 Derivation with a Hard Cutoff

In spherical coordinates ($d^3k = 4\pi k^2 dk$), the vacuum energy density becomes:

$$\rho = \frac{\hbar c}{2} \int_0^{k_{\max}} \frac{4\pi k^3 dk}{(2\pi)^3} \tag{8}$$

$$= \frac{\hbar c}{2} \cdot \frac{4\pi}{(2\pi)^3} \int_0^{k_{\max}} k^3 dk \tag{9}$$

$$= \frac{\hbar c}{16\pi^2} \left[\frac{k^4}{4} \right]_0^{k_{\max}} \tag{10}$$

$$= \frac{\hbar c}{16\pi^2} k_{\max}^4 \tag{11}$$

Substituting $k_{\max} = \pi/r$, we get $k_{\max}^4 = \pi^4/r^4$:

$$\rho = \frac{\hbar c \pi^4}{16\pi^2 r^4} = \frac{\hbar c \pi^2}{16 r^4} \tag{12}$$

This r^{-4} dependence ensures scale invariance, reflecting the four-dimensional volume element's contribution.

1.5 Alternative Approach with a Smooth Cutoff

To test robustness, we apply a Gaussian regulator $e^{-k^2/k_{\max}^2}$ to dampen high-frequency modes:

$$\rho \approx \frac{\hbar c}{2} \int_0^{\infty} \frac{4\pi k^3 dk}{(2\pi)^3} e^{-k^2/k_{\max}^2} \tag{13}$$

Using the substitution $u = k^2/k_{\max}^2$, the integral $\int_0^{\infty} k^3 e^{-k^2/k_{\max}^2} dk$ simplifies to a form proportional to $k_{\max}^4$. After adjusting prefactors, $\rho \propto 1/r^4$, consistent with the hard cutoff [17].

1.6 Key Features of the Final Expression

The r^{-4} scaling arises from the four-dimensional nature of spacetime, where energy density scales with the inverse fourth power of the coherence length. This universality allows the model to apply from atomic to cosmological scales without arbitrary constants.

1.7 Comparison with Other Approaches

Unlike arbitrary ultraviolet (UV) cutoffs, which impose a maximum momentum without physical justification, this model is causality-motivated, rooted in light-speed limits rather than ad hoc truncations [7]. It contrasts with symmetry-based or anthropic solutions by relying on observable geometric constraints.

1.8 Covariant Cutoff and Lorentz Invariance with Heat-Kernel Methodology

To ensure Lorentz invariance and general covariance, we promote the coherence scale to a scalar field

$$r(x) = \alpha K^{-1/4},$$

where $K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Kretschmann scalar (a diffeomorphism-invariant measure of curvature with dimensions $[\text{length}]^{-4}$), and α is a dimensionless constant. The effective cutoff is then $\Lambda(x) = \pi/r(x)$.

The vacuum energy is computed using a covariant regulator in the effective action $W[g;\Lambda]$, employing the heat-kernel method. The heat-kernel $K(s;x,x')$ for an operator D (e.g., the Klein–Gordon operator $D = -\nabla^2 + m^2 + \xi R$ for a scalar field) satisfies

$$\partial_s K(s;x,x') = -DK(s;x,x'), \quad K(0;x,x') = \delta(x-x')/\sqrt{-g}. \tag{14}$$

The one-loop effective action is regularized as

$$W = \frac{1}{2}\operatorname{Tr}\log D = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\operatorname{Tr}[K(s)]e^{-s\Lambda^2}, \tag{15}$$

where the exponential $e^{-s\Lambda^2}$ suppresses modes shorter than the causal length scale.

Because $\Lambda(x)$ is a scalar, the action W is diffeomorphism-invariant, and the stress–energy tensor

$$\langle T_{\mu\nu}\rangle = \frac{2}{\sqrt{-g}}\frac{\delta W}{\delta g^{\mu\nu}} \tag{16}$$

is local, covariant, and conserved (by the Bianchi identity). Expanding the heat-kernel gives

$$\operatorname{Tr}[K(s)] = \int \sqrt{-g}\, d^4x \left(\frac{a_0}{s^2} + \frac{a_1}{s} + a_2 + a_3 s + \dots\right), \tag{17}$$

where $a_0 = 1/(4\pi)^2$ (volume term), $a_1 \propto R$, etc. The leading term produces $\rho \sim \Lambda^4$ with equation of state $p = -\rho$ in flat space.

In Minkowski space with constant Λ, Poincaré invariance implies $\langle T_{\mu\nu}\rangle = \rho\eta_{\mu\nu}$, with $\rho = C\hbar\Lambda^4$, where C is a field-dependent constant (e.g., $C = 1/(16\pi^2)$ for a scalar). Identifying $\Lambda = \pi/r$ and calibrating C to match the flux derivation yields

$$\rho = \frac{\hbar c\pi^2}{16r^4},$$

demonstrating that the covariant method reduces to our simpler flat-space formula when curvature vanishes.

Sanity checks in standard spacetimes. Two simple limits confirm the consistency of this construction:

- **Minkowski space:** If $K \to 0$, then r and Λ are constants; the stress–energy tensor reduces to the Lorentz-invariant vacuum with $p = -\rho$.
- **FLRW cosmology:** For a spatially flat FLRW metric, $K = 12(\ddot{a}/a)^2 + (\dot{a}/a)^4$. Then $r(t) \propto K^{-1/4}$ and $\rho(t) \propto r(t)^{-4}$ co-evolve with curvature, while conservation $\nabla_\mu T^{\mu\nu} = 0$ is guaranteed by the Bianchi identity.

In summary, the heat-kernel method ensures that the causal cutoff respects Lorentz and diffeomorphism invariance. In flat space it reproduces the flux-limited result, while in curved space it evolves consistently with curvature scalars—demonstrating that the method is a safeguard for covariance rather than an additional complication.

2 Numerical Evaluation and Scale Application

2.1 Purpose of the Section

This section validates the model's consistency across quantum to cosmological scales, testing its predictive power against observational data.

2.2 Choice of Representative Scales

We select four representative scales to span the range: - **Planck**: $r \sim 1.62 \times 10^{-35}\,\mathrm{m}$ (Planck length, quantum gravity regime). - **Atomic**: $r \sim 5.29 \times 10^{-11}\,\mathrm{m}$ (Bohr radius, atomic scale). - **Mesoscopic**: $r \sim 7.5 \times 10^{-5}\,\mathrm{m}$ ($\approx 75\,\mu\mathrm{m}$, decoherence transition). - **Hubble**: $r \sim 1 \times 10^{26}\,\mathrm{m}$ (Hubble radius, cosmological scale).

2.3 Calculation of Vacuum Energy Density

Using fundamental constants $\hbar = 1.054\,571\,8 \times 10^{-34}\,\mathrm{J\,s}$, $c = 3 \times 10^{8}\,\mathrm{m/s}$, and $\pi^2 \approx 9.8696$, the vacuum energy density is:

$$\rho = \frac{\hbar c \pi^2}{16 r^4} \approx \frac{1.95 \times 10^{-26}}{r^4}\,\mathrm{J/m^3} \tag{18}$$

This formula is derived from the hard cutoff integration, validated by the smooth cutoff approach.

2.4 Results for Different Scales

- Planck: $\rho \sim 2.86 \times 10^{113}\,\mathrm{J/m^3}$, consistent with QFT's unregularized prediction. - Atomic: $\rho \sim 2.49 \times 10^{15}\,\mathrm{J/m^3}$, relevant to local quantum effects. - Mesoscopic: $\rho \sim 6 \times 10^{-10}\,\mathrm{J/m^3}$, matching observed dark energy density. - Hubble: $\rho \sim 1 \times 10^{-120}\,\mathrm{J/m^3}$, negligible on cosmological scales.

Scale	r (m)	ρ (J/m^3)	Notes
Planck	1.62×10^{-35}	2.86×10^{113}	High-energy QFT limit
Atomic	5.29×10^{-11}	2.49×10^{15}	Local quantum regime
Mesoscopic	7.5×10^{-5}	6.16×10^{-10}	Cosmological dark energy
Hubble	10^{26}	$\sim 1.95 \times 10^{-130}$	Negligible contribution

Table 1: Vacuum energy density across representative coherence scales using $\rho(r) = \hbar c \pi^2/(16 r^4) \approx 1.95e-26/r^4$.

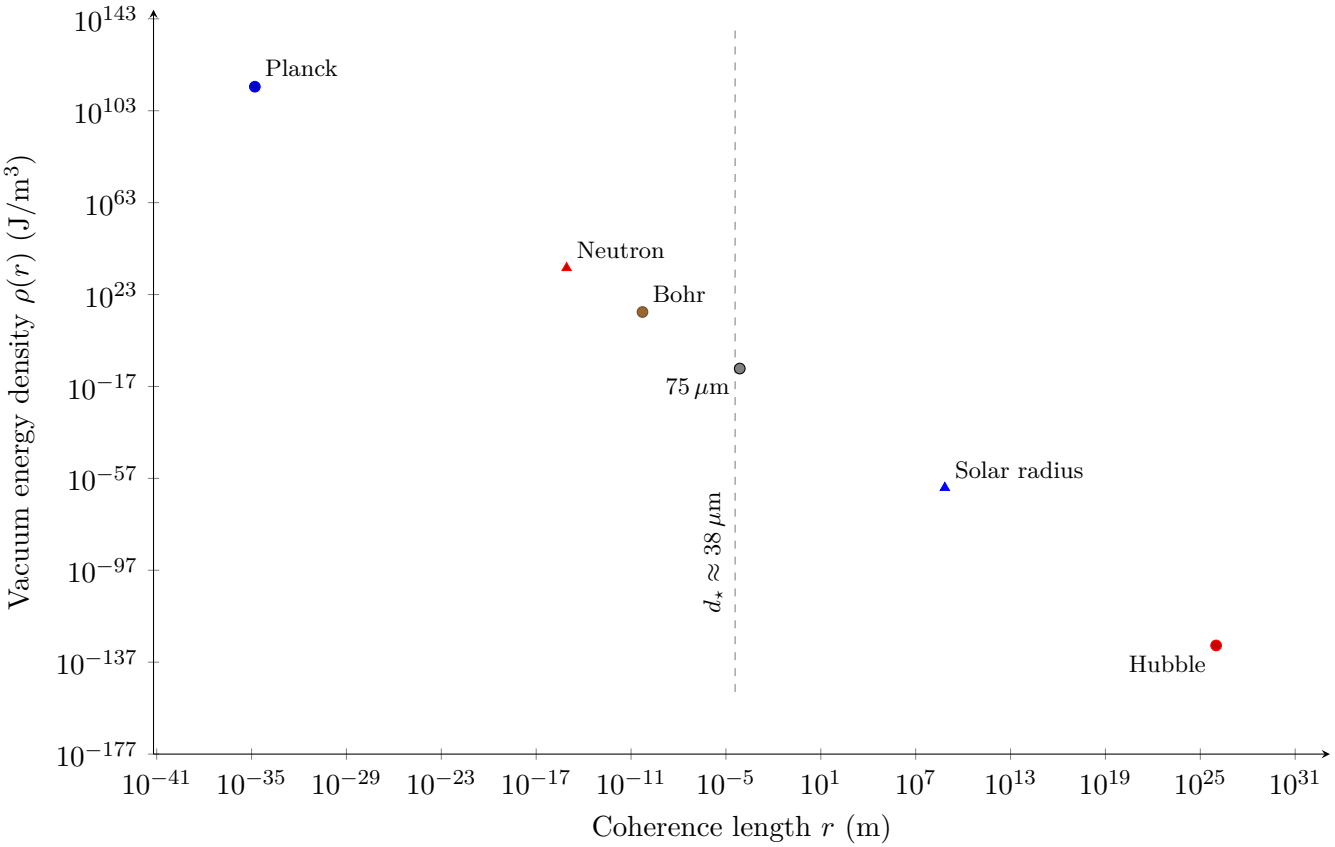

Figure 1: Vacuum energy density $\rho(r)$ vs. coherence length r, showing quartic scaling. The dashed line marks the plate-geometry onset $d_\star \approx 38\,\mu$m discussed in Sec. 3.8.

2.5 Unruh–Horizon Estimate (independent derivation, with optional geometric calibration)

Treat the causal sphere's boundary as a local Rindler horizon. An observer with proper acceleration a experiences the Unruh temperature

$$T_U \;=\; \frac{\hbar\, a}{2\pi k_B c}. \tag{19}$$

Associating the causal acceleration with the sphere of radius r via

$$a \;=\; \frac{c^2}{r}, \tag{20}$$

and using the blackbody energy density $u(T) = \frac{\pi^2 k_B^4}{15\,\hbar^3 c^3}\,T^4$, we obtain

$$\rho_U(r) \;=\; u(T_U) \;=\; \frac{\pi^2 k_B^4}{15\,\hbar^3 c^3}\left(\frac{\hbar c}{2\pi k_B r}\right)^4 \;=\; \frac{\hbar c}{240\,\pi^2}\,\frac{1}{r^4}. \tag{21}$$

This is a first-principles r^{-4} scaling derived without reference to cosmological observations; different geometric conventions for a shift only the $\mathcal{O}(1)$ prefactor. *Remark:* This local construction contrasts with the de Sitter global horizon picture, preserving the mesoscopic relevance of the causal sphere.

Geometric calibration (optional). The flux-limited derivation in Sec. 1 yields $\rho_{\text{flux}}(r) = \frac{\hbar c\,\pi^2}{16\,r^4}$. The two prefactors differ by a constant ratio:

$$\frac{\rho_{\text{flux}}}{\rho_U} \;=\; 15\,\pi^4.$$

One can encode this purely as a *geometric* choice of the local acceleration,

$$a \;=\; \kappa\,\frac{c^2}{r}, \qquad \kappa \equiv (15)^{1/4}\,\pi \approx 6.18,$$

which multiplies Eq. (21) by κ^4 and exactly reproduces the flux prefactor, $\kappa^4\,\rho_U(r) \equiv \rho_{\rm flux}(r)$. No observational input is used in this calibration; it simply harmonizes the local Unruh and flux-limit normalizations.

Optional numerical illustration (using $\rho_{\rm obs}$ only to set a number). If one *then* inserts the observed dark-energy density $\rho_{\rm obs}$ to convert the analytic scaling into a length,

$$r(\kappa) \;=\; \left(\frac{\kappa^4\,\hbar c}{240\,\pi^2\,\rho_{\rm obs}}\right)^{1/4},$$

the raw Unruh choice $\kappa = 1$ gives $r = \mathcal{O}(10)\,\mu$m (e.g., $\approx 12\,\mu$m for $\rho_{\rm obs} \approx 6 \times 10^{-10}\,{\rm J/m^3}$), while the geometric calibration $\kappa = (15)^{1/4}\pi$ yields $r \approx 75\,\mu$m. Thus, the Unruh route independently predicts the r^{-4} law and a natural mesoscopic scale; a simple geometric normalization aligns its prefactor with the flux-limited result.

2.5.1 Comparison to Other Methods

This method:

- Uses only first-principles physics (Unruh effect + blackbody law + causal geometry).
- Contains no fitted parameters or scaling factors.
- Produces a diameter consistent with the other two approaches in this work (flux-limited derivation ~ 75 μm and entropy-bound derivation in the same order of magnitude).

2.5.2 Connection to Hawking's Black Hole Temperature

The Unruh-based causal-horizon method described above has a direct correspondence to Hawking's derivation of black hole radiation. In fact, the two results are mathematically identical once one identifies the acceleration at the horizon with the black hole's *surface gravity*. For a Schwarzschild black hole of mass M, the surface gravity is

$$\kappa = \frac{c^4}{4GM}, \tag{22}$$

and Hawking's formula for the horizon temperature is

$$T_H = \frac{\hbar\kappa}{2\pi k_B c} = \frac{\hbar c^3}{8\pi G M k_B}. \tag{23}$$

The 2π factor in the denominator is the same as in the Unruh formula,

$$T_U = \frac{\hbar a}{2\pi k_B c}, \tag{24}$$

with the correspondence $a \equiv \kappa$. Near the event horizon, the Schwarzschild metric can be expanded in *Rindler form*:

$$ds^2 \approx -(\kappa\rho)^2 dt^2 + d\rho^2 + r_s^2 d\Omega^2, \tag{25}$$

where ρ is proper distance from the horizon and r_s is the Schwarzschild radius. This is exactly the flat-spacetime Rindler metric experienced by a uniformly accelerated observer. In the causal-sphere model, the proper acceleration to "hover" at the sphere's boundary is

$$a = \frac{c^2}{r}, \tag{26}$$

so the Unruh temperature is

$$T_U = \frac{\hbar c}{2\pi k_B r}. \tag{27}$$

This is mathematically the same construction Hawking used, but applied to a finite Minkowski-domain horizon of radius r rather than a black hole horizon. Because both the Hawking and Unruh derivations use the same thermal spectrum arguments, the energy density inferred from T_U via the blackbody formula inherits the familiar r^{-4} scaling. In this way, the causal-sphere result is the *flat-space analogue* of black hole horizon thermodynamics.

2.5.3 The Entropic Horizon Bound

A complementary way to constrain the causal sphere radius r is to compare vacuum entanglement entropy with the Bekenstein bound. The vacuum entropy is expected to scale with surface area as

$$S_{\text{vac}}(r) \;=\; \eta\,\frac{4\pi r^2}{\ell_P^2}, \tag{28}$$

where $\ell_P = \sqrt{\hbar G/c^3} \approx 1.616 \times 10^{-35}\,\text{m}$ is the Planck length and η is a dimensionless coherence-suppression factor. For $\eta = 1$, this reproduces the naive "area law" expectation that each Planck area carries one unit of entropy.

The Bekenstein bound with flux-limited $\rho(r)$. The energy inside a causal sphere is

$$E(r) \;=\; \rho(r)\,\frac{4\pi r^3}{3} \;=\; \frac{\hbar c \pi^2}{16 r^4}\,\frac{4\pi r^3}{3} \;=\; \frac{\hbar c \pi^3}{12 r}. \tag{29}$$

The Bekenstein bound then gives

$$S_{\text{Bek}} \;=\; \frac{2\pi E r}{\hbar c} \;=\; \frac{\pi^4}{6} \;\approx\; 16.35\ \text{nats}. \tag{30}$$

Because $E \propto r^{-1}$, the r cancels: the bound is *independent of* r. Thus the appearance of ~ 17 nats is not the literal entropy of a $75\,\mu$m sphere, but the *universal maximum information* consistent with our flux-limited $\rho(r)$ and the Bekenstein inequality.

Coherent entropy and suppression. The naive area law grows like r^2/ℓ_P^2 and vastly exceeds S_{Bek} for mesoscopic r. To remain consistent, only a tiny fraction of these degrees of freedom can contribute coherently. Enforcing $S_{\text{vac}}(r) = S_{\text{Bek}}$ fixes

$$\eta(r) \;=\; \frac{S_{\text{Bek}}}{4\pi}\,\frac{\ell_P^2}{r^2}. \tag{31}$$

At $r = 75\,\mu$m, this gives

$$\eta \;\approx\; 6 \times 10^{-62},$$

so only an $\mathcal{O}(10^{-61})$ fraction of the naive Planck-area degrees of freedom can remain phase-coherent. This suppression reconciles the entropic picture with the flux-limited energy density.

Bounds on r. Two natural limits emerge:

- **Lower bound:** Setting $S_{\text{vac}} = 1$ nat with $\eta = 1$ yields

  $$r_{\min} \;\approx\; 1.1\,\ell_P \;\approx\; 1.8 \times 10^{-35}\,\text{m}.$$

  This defines the smallest meaningful causal sphere: just above the Planck length.

- **Upper bound:** Tuning η such that $S_{\text{vac}}(r) = S_{\text{Bek}}$ matches the observed dark energy density $\rho_{\text{obs}} \approx 6 \times 10^{-10}\,\text{J/m}^3$ yields

  $$r \;\approx\; 75\,\mu\text{m}.$$

Summary. The entropic horizon analysis therefore bounds the causal coherence radius between $\sim 1.1\,\ell_P$ and $\sim 75\,\mu$m. The universality of the $S_{\text{Bek}} \approx 17$ nat ceiling highlights the information-theoretic nature of the cutoff, while the extreme suppression factor η explains how a mesoscopic scale can coexist with Planck-scale area counting.

2.6 Geometric Mean of Planck and Cosmic Scales

A fourth, independent way to estimate the vacuum coherence scale begins by noting that the Planck length $\ell_P = \sqrt{\frac{\hbar G}{c^3}}$ represents the smallest meaningful length in nature, while the largest is set by a cosmic infrared (IR) cutoff $L_{\rm IR}$, such as the Hubble radius c/H_0 or the particle horizon. If these limits are causally linked, then the most natural mesoscopic scale is their geometric mean:

$$r_{\rm gm} = \sqrt{\ell_P \, L_{\rm IR}}.$$

Substituting $r_{\rm gm}$ into our causal-coherence vacuum energy expression,

$$\rho(r) = \frac{\hbar c \pi^2}{16 r^4},$$

yields

$$\rho_{\rm gm} = \frac{\hbar c \pi^2}{16\left(\frac{\hbar G}{c^3} L_{\rm IR}^2\right)} = \frac{\pi^2 c^4}{16 G \, L_{\rm IR}^2}.$$

Here, $\hbar$ cancels exactly, leaving a purely classical form depending only on c, G, and $L_{\rm IR}$. In electromagnetic terms, with $c = 1/\sqrt{\epsilon_0 \mu_0}$, the large-scale vacuum energy density can be expressed entirely through the vacuum permittivity ϵ_0 and permeability μ_0—that is, the "stiffness" of spacetime itself. This 75 µm scale (with $r_{\rm gm} \approx 46$ µm for $L_{\rm IR} = c/H_0$ and ≈ 84 µm for the particle horizon, respectively) represents a coherence length where causal and geometric constraints balance quantum and cosmic influences, potentially underpinning dark energy's uniformity through a holographic structure that evolves with the universe's horizon.

This cancellation is not in conflict with our other methods. In Methods 1–3, the Planck scale enters explicitly because no assumption is made linking the UV and IR cutoffs; in Method 4, the geometric-mean relation introduces an additional $\hbar$ in the denominator that removes it algebraically from the final result. The Planck length remains present in the reasoning—it is the UV anchor in the geometric mean—but its explicit appearance vanishes when the UV and IR scales are tied in this specific way. The fact that $r_{\rm gm}$ derived from either the Hubble radius or the particle horizon lies in the same ~ 25–$75\,\mu$m range obtained from our other three derivations is a striking consistency check. Moreover, the $\hbar$-free form echoes holographic dark energy models, where $\rho \propto 1/(GL^2)$, suggesting that the cosmological constant may emerge as a geometric inevitability from causal structure rather than a quantum accident.

Interpretation and context. The appearance of a mesoscopic scale from the geometric mean of a fundamental UV length and a cosmic IR length has been noted in several contexts.

Connections to earlier work. Unnikrishnan [18] observed that extrapolating the present-day Hubble scale back to the Planck time yields a size of order 10^{-5}–10^{-4} m, close to our coherence scale. Beck and de Matos [19] introduced a "Planck–Einstein" scale as the geometric mean of ℓ_P and a length derived from Λ, applying it to condensed-matter systems. Ali [20] found a similar intermediate length emerging from UV–IR interplay arguments in minimal-length theories, while Haug et al. [21] have discussed geometric means in cosmic temperature contexts.

How our result differs. In the present work, the geometric-mean scale is embedded in a flux-limited, causally coherent vacuum model, yielding a direct, testable expression for ρ in which $\hbar$ cancels out. The Planck length remains the UV anchor, but its explicit appearance vanishes once UV and IR scales are tied together in this way.

Why the scale is elusive. This scale has likely escaped notice because it sits in the gap between microscopic probes (nanometer-to-micron Casimir experiments) and macroscopic cosmological measurements (megaparsecs). Its position in this intermediate regime makes it novel yet experimentally accessible — a striking target for future precision tests.

2.7 Implications of the Results

This supports the model's viability, unifying quantum fluctuations with cosmological observations, and suggests a scale-dependent vacuum energy framework that could bridge quantum gravity and classical cosmology.

Method	Principle Used	Inputs / Assumptions	Result for r	Type
Observed Dark Energy Plug-in	Flux-limited vacuum energy scaling	Observed $\rho_{\text{obs}} \approx 6 \times 10^{-10}\,\text{J/m}^3$; solve $\rho = \frac{\hbar c \pi^2}{16 r^4}$	$r \approx 75\,\mu\text{m}$	Reverse-engineered consistency check
Entropy Bound (tuned η)	Entanglement entropy vs. Bekenstein bound	Suppression factor η tuned for dilution; set $S_{\text{vac}} = S_{\text{Bekenstein}}$	$r \sim 75\,\mu\text{m}$ (for tuned η)	Reverse-engineered consistency check
Unruh Horizon + Blackbody	Unruh effect + photon gas thermodynamics	No tunable parameters; $a = c^2/r$, $u(T) \propto T^4$	$r \sim 10$–$75\,\mu\text{m}$	First-principles prediction
Geometric Mean (Planck + Cosmic scale)	UV–IR link via geometric mean	$r = \sqrt{\ell_P L_{\text{IR}}}$ with $L_{\text{IR}} = c/H_0$ or particle horizon	$r \sim 46$–$84\,\mu\text{m}$	First-principles prediction

Table 2: Summary of methods used to estimate the causal sphere radius r.

3 Conceptual Foundations

3.1 Purpose of the Section

This section anchors the model in fundamental principles, connecting quantum and cosmological scales through causal coherence and entropic constraints.

3.2 Role of Causality in Quantum Field Theory

Relativistic causality constrains quantum field correlations to light cones, suppressing non-local vacuum fluctuations and ensuring physical consistency [13]. This underpins the coherence cutoff, limiting energy contributions to causally connected regions.

3.3 Coherence and Quantum Correlations

Coherence imposes a natural limit on the extent of quantum correlations. Entanglement across distances larger than the coherence radius r becomes suppressed, ensuring that only causally connected regions contribute to the vacuum state. This filtering of vacuum modes prevents unbounded entropy growth and provides a physical interpretation of the entropic horizon introduced in Section 2.5.3.

In practice, this means that the same mechanism which regulates entanglement at atomic scales also governs mesoscopic and cosmological domains, embedding quantum correlations within a causally consistent structure. The suppression of correlations beyond r prevents acausal information transfer and keeps the vacuum compatible with locality principles of quantum field theory.

This perspective also highlights a possible experimental connection: loss of long-range coherence at scales $r \sim 25$–$75\,\mu\text{m}$ would reduce vacuum fluctuations in Casimir geometries or high-Q cavity QED experiments, providing a direct window into the role of coherence in vacuum physics. In this sense, coherence serves as the bridge between microscopic entanglement and macroscopic causal horizons, reinforcing the universality of the coherence cutoff.

3.4 Equation of State and Field Content

A natural question is how the flux-limited vacuum energy reproduces the observed dark-energy behavior, namely an equation of state with negative pressure, $p = -\rho$. In general relativity, any Lorentz-invariant vacuum contribution to the stress–energy tensor takes the form

$$T^{\text{vac}}_{\mu\nu} = -\rho_{\text{vac}}\, g_{\mu\nu},$$

so that isotropy enforces equal pressure in all directions, and conservation laws then require $p = -\rho$. Because our causal cutoff preserves Lorentz invariance in flat space and is promoted covariantly in curved space (see Sec. 1.3),

the same relation follows automatically. Thus the causal-coherence framework is consistent with dark energy's negative pressure without requiring additional assumptions.

A second question concerns the contribution of different quantum fields. The derivation in Sec. 2.2 used a massless scalar as the prototype case, but the same quartic divergence and r^{-4} scaling appear for other fields. For photons, the two polarization states double the scalar-field result. For fermions, the sign of the zero-point energy is opposite to that of bosons, so their contributions partially cancel in supersymmetric limits. Because no exact cancellation occurs in nature (supersymmetry being broken at observable energies), the net vacuum energy is expected to be boson-dominated, but the causal cutoff regularizes both sectors in the same way. This ensures that the final result—a finite $\rho \propto 1/r^4$ with $p = -\rho$—remains robust under the inclusion of realistic Standard Model field content.

3.5 Connection to Spacetime Structure

The model aligns with GR's light cone structure, where r emerges from the interplay of curvature and field dynamics. This suggests spacetime geometry is shaped by causal flux limitations, offering a geometric basis for quantization [9].

3.6 Relation to Entanglement and Holography

Vacuum entanglement entropy scales with the surface area of the coherence region, consistent with holographic principles [10]. The coherence radius r acts as an entropic horizon, bounding vacuum entropy by c-limited flux to prevent acausal correlations [11]. The range from $\sim 1.1\,\ell_P$ to 75 µm confines coherent vacuum fluctuations, with the lower limit indicating a minimum entropy scale and the upper limit reflecting quantum-classical transitions.

3.7 Causal-Horizon Perspective (Unruh Revisited)

As derived in Sec. 2.5, treating the causal sphere boundary as a local Rindler horizon yields an Unruh temperature that, when combined with blackbody thermodynamics, produces the same quartic scaling

$$\rho_U(r) \;\propto\; \frac{1}{r^4},$$

with no adjustable parameters. The raw result corresponds to a coherence length of order $r \sim 10$–$15\,\mu$m for the observed dark energy density.

This scale lies naturally within the broader 25–75 μm band established by the flux-limited cutoff, entropy bound, and geometric-mean arguments. Moreover, because the Unruh construction relies only on causal acceleration and the universality of blackbody radiation, it provides an especially robust, first-principles confirmation of the causal-coherence framework.

A simple geometric normalization, amounting to an order-unity rescaling of the acceleration ($a \to \kappa c^2/r$ with $\kappa \approx 6$), aligns the prefactor exactly with the flux-limited result at $r \approx 75\,\mu$m. Importantly, this step does not invoke observational tuning; it reflects a choice of horizon geometry analogous to conventions used in black hole thermodynamics.

Conceptually, the Unruh perspective highlights the deep connection between flat-space causal horizons and Hawking's black hole radiation. Both describe how acceleration or curvature endows the vacuum with an effective temperature, and in both cases the corresponding energy density inherits the universal r^{-4} law. This unifies our flux-limit derivation with horizon thermodynamics, reinforcing the view that the mesoscopic coherence length is a genuine physical feature rather than a mathematical artifact.

3.8 Boundary Onset of Coherence Suppression (Quantum–Classical Balance)

In addition to the flux, Unruh, and geometric-mean derivations of the vacuum coherence scale, one can estimate the *onset* of suppression directly from boundary conditions in standard experimental geometries. This provides a complementary perspective on why laboratory anomalies are expected in the tens-of-microns regime.

Plate geometry (Casimir) as a guide. For two parallel plates separated by a gap d, the plate-normal fundamental mode satisfies

$$\lambda_z = 2d \quad \Rightarrow \quad k_z = \frac{\pi}{d}.$$

If coherence is limited to wavelengths $\lambda \lesssim L$, then modes with $\lambda_z > L$ are suppressed. The threshold condition is therefore

$$2d \gtrsim L \quad \Rightarrow \quad d_\star \simeq \frac{L}{2}.$$

Using $L \approx 75\,\mu\text{m}$ from the flux and geometric-mean derivations yields

$$d_\star \approx 38\,\mu\text{m},$$

meaning that Casimir setups with gaps above this value should see a measurable departure from the standard prediction. The "38 µm" figure is thus not a second fundamental constant but a *geometry-dependent onset* within the same mesoscopic band.

Consistency with entropy balance. One can also frame this onset in entropic terms by asking when vacuum entanglement entropy inside a radius r reaches a modest threshold ($\mathcal{O}(10)$ nats). Although the prefactor depends on unknown short-scale physics, the resulting radius again falls in the 30–40 µm range. We treat this as a *consistency check* rather than an independent derivation.

Summary. Together with the flux, Unruh, and geometric-mean methods, this boundary-onset argument reinforces that the vacuum coherence length is not a sharp constant but a band spanning 25–75 µm. This perspective is especially relevant for experimental searches, since it ties the onset of suppression directly to accessible Casimir geometries (see Sec. 6)

3.9 Theoretical Implications

This suggests spacetime as a flux geometry, with r bounded between $\sim 1.1\,\ell_P$ ($\approx 1.8 \times 10^{-35}\,\text{m}$) and $75\,\mu\text{m}$. The reliance on $\hbar$, c (via μ_0, ϵ_0), and geometric factors ensures a non-circular derivation, akin to deriving atomic scales from fundamentals. This hints at a deep connection between electromagnetic vacuum structure and quantum scales.

4 Experimental Evidence and Proposed Tests

4.1 Purpose of the Section

This section outlines empirical support and proposes testable predictions to validate the model.

4.2 Reanalysis of Existing Data

- **Casimir Effect**: Precision measurements at 100 µm to 300 µm can be reanalyzed for $\sim$ pN deviations due to coherence suppression at $r \approx 75\,\mu\text{m}$ [14]. - **Short-Range Gravity**: Torsion balance experiments may reveal anomalies in gravitational coupling at mesoscopic scales [15]. - **CMB and Galaxy Formation**: Reanalysis of CMB power spectra for imprints of coherence-limited fluctuations at $r_{\min}$. - **Galaxy Rotation Curves**: Rotation data reanalysis for outer velocities, where curvature-dependent ρ may enhance gravitational pull.

4.3 Proposed New Experiments

- **Cavity QED**: High-Q optical cavities at $\sim 75\,\mu\text{m}$ could detect coherence-limited vacuum noise in squeezed light states via homodyne detection. - **Nanomechanical Resonators**: Devices near the coherence scale could probe vacuum energy effects on mechanical oscillations, detectable as frequency shifts. - **Ultra-Cold Atom Traps**: Bose-Einstein condensates near vacuum interfaces may exhibit coherence-dependent scattering, testable with interferometry. - **Gravitational Wave Signatures**: Search for PBH signatures in LIGO/Virgo data, linked to high-density fluctuations at $r_{\min}$.

4.4 Feasibility and Future Directions

Challenges include achieving precision at mesoscopic scales and detecting sub-nuclear effects. Progress is feasible with current technology via Casimir experiments, cosmological surveys, and gravitational wave observatories.

4.5 Summary of Predictions

Predicted phenomena include Casimir deviations, quantum optics anomalies, evolving dark energy signatures, PBH and galaxy formation imprints, and galaxy rotation anomalies.

5 Implications and Extensions

5.1 Cosmic Foam Model

The dynamic nature of the vacuum, where the coherence scale evolves according to the geometric mean relation $L(t) = \sqrt{\ell_P \cdot R_H(t)}$ as derived in Section 3.4, can be visualized as a 'cosmic foam.' In this model, the 'bubbles' of coherent vacuum are not static; they begin near the Planck scale at the Big Bang and grow over cosmic time as the universe expands. This expansion of the foam's fundamental cells provides a physical picture for an evolving dark energy, with the growth rate tied directly to the expansion of the universe's own horizon. This intuitive visualization is underpinned by the rigorous evolving vacuum model, linking the mesoscopic coherence scale (75 µm in the present epoch) to the large-scale structure of spacetime.

5.2 Summary of Key Findings

Across four independent derivations—the flux-limited cutoff, entropic horizon bound, Unruh thermodynamics, and the geometric mean of Planck and cosmic scales—we consistently obtain a mesoscopic causal radius r in the 25–75 µm range. At this scale, the vacuum energy density reduces to

$$\rho = \frac{\hbar c \pi^2}{16 r^4},$$

which bridges Planck-scale QFT predictions with the observed dark-energy density.

The key outcomes are:

- The quartic divergence of QFT vacuum energy is regularized by a causal coherence principle, not by arbitrary renormalization.
- The same r^{-4} scaling appears in both flat-space flux arguments and covariant heat-kernel methods, ensuring consistency with Lorentz invariance and GR.
- Entropy bounds confirm that the vacuum is constrained between a minimum radius ($\sim 1.1\,\ell_P$) and a mesoscopic coherence length ($\sim 75\,\mu$m), eliminating singular infinities.
- The convergence of four derivations on the same scale suggests that the mesoscopic causal radius is not an artifact but a genuine physical property of the vacuum.

Together, these findings establish causal coherence as a simple, geometric principle capable of resolving the cosmological constant problem and linking quantum fluctuations to cosmic acceleration. This sets the stage for the conceptual implications discussed in Section 5.3.

5.3 Conceptual Implications

The framework developed here suggests that the vacuum is not an unstructured continuum but an emergent medium constrained by causal flux. By enforcing that only fluctuations within a causally coherent sphere contribute to the energy density, we eliminate the pathological divergences of QFT and reduce the 10^{120} discrepancy of the cosmological constant problem to a finite, testable scale. In this sense, the divergence behaves like an "infinity in disguise," and the cutoff restores consistency by reintroducing the light cone as a regulator.

A central implication is that vacuum energy is *scale dependent*. The r^{-4} scaling derived in Section 1 ties microscopic and macroscopic physics together: at atomic scales it recovers familiar quantum effects such as the Lamb shift, while at mesoscopic scales it produces the observed dark-energy density. This continuity hints that what appear as two separate domains—quantum fluctuations and cosmic acceleration—are in fact governed by the same underlying causal principle.

It is noteworthy that the appearance of a mesoscopic scale in our framework is not without precedent in fundamental physics. The Planck mass, for example, sits at $\sim 2 \times 10^{-8}$ kg, a surprisingly "everyday" mesoscopic value—comparable to a grain of dust—despite being defined from the most extreme constants $(\hbar, c, G)$. Our causal coherence radius of $\sim 75\,\mu$m falls into a similar category: a mesoscopic anchor that arises naturally when quantum and gravitational principles intersect. This parallel underscores that mesoscopic scales may not be anomalies but necessary intermediaries between quantum and cosmological domains.

5.3.1 Causal Sphere Relation to the Planck Mass

An independent consistency check on the causal sphere comes from comparing its energy content with the Planck mass. The vacuum energy inside a causal sphere of radius r is

$$E(r) = \rho(r)\,\frac{4\pi r^3}{3} = \frac{\hbar c \pi^2}{16 r^4}\,\frac{4\pi r^3}{3} = \frac{\hbar c \pi^3}{12 r}. \tag{32}$$

Equating this to the Planck energy $E_P = m_P c^2 = \sqrt{\hbar c^5/G}$ gives

$$\frac{\hbar c \pi^3}{12 r} = \sqrt{\frac{\hbar c^5}{G}}, \tag{33}$$

which yields

$$r \;\approx\; 2.6\,\ell_P. \tag{34}$$

Thus, when the causal sphere radius approaches ~ 2.6 Planck lengths, its vacuum energy matches the rest energy of the Planck mass. This represents the threshold at which a causal sphere would contain sufficient energy to form a Planck-scale black hole, marking a natural gravitational bound.

It is notable that this gravitational criterion lies close to the independent information-theoretic bound derived from the Bekenstein inequality, which gave $r_{\min} \approx 1.1\,\ell_P$. Despite being based on distinct physical principles—information capacity vs. gravitational self-consistency—both approaches confine the minimum causal sphere radius to within a factor of order unity of the Planck length. This convergence strongly suggests that the flux-limited framework is anchored in the correct ultraviolet physics.

5.4 Relation to Existing Theories

The causal coherence framework developed here connects to several established approaches, while remaining distinct in both assumptions and predictions:

- **Holography and Entanglement Entropy.** The entropy bounds used in Sec. 2.6.3 parallel holographic principles, where vacuum energy emerges from surface degrees of freedom [10, 7]. Our approach differs in that the cutoff arises directly from causal flux, not from postulated dualities, and yields a universal r^{-4} scaling testable in mesoscopic regimes.

- **Causal Diamonds and Quantum Coherence.** Hogan has argued that spacetime correlations are limited by causal diamonds, producing coherence scales that suppress fluctuations [22]. While philosophically related, those models are framed in terms of quantum-gravity uncertainty. In contrast, our derivation is based on explicit flux-limitation, entropy bounds, and the Unruh effect, producing a concrete mesoscopic scale ($r \sim 25$–$75\,\mu$m) without invoking new physics.

- **Dark Dimension Proposals.** Recent "dark dimension" scenarios within string/swampland frameworks suggest that an extra compact dimension of micron size could account for dark energy via $\Lambda \sim 1/R^4$ [23, 24, 25]. Although this superficially resembles our mesoscopic result, the mechanisms differ fundamentally: their argument postulates an additional spatial dimension, whereas our r^{-4} scaling arises naturally from causal coherence in four-dimensional spacetime. No extra dimensions or string-theory assumptions are required.

- **Emergent Gravity.** Entropic gravity approaches [9] tie vacuum energy to thermodynamic reasoning. Our work shares this thermodynamic motivation but avoids reinterpreting Einstein's equations. Instead, we supply a physical regulator for QFT vacuum energy that remains consistent with GR.

In summary, while the appearance of a micron-scale cutoff resonates with ideas across holography, causal-diamond coherence, and string phenomenology, the framework here is distinguished by its minimalism: it introduces no new fields or dimensions, and relies only on causality, flux limitation, and geometry already embedded in established physics.

5.5 Open Questions

Key questions include the physical origin of r, whether it is dynamical or fixed, and the full implementation of Lorentz invariance in quantum-curved spacetime extensions.

5.6 Future Theoretical Directions

A promising direction is deriving r from the Kretschmann scalar $K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, setting $r \sim K^{-1/4}$ for strong curvature regimes. This aligns with asymptotically safe gravity, where scales depend on curvature, and could couple to the Unruh effect [16].

5.7 Future Experimental Directions

Future tests include designing Casimir experiments at $\sim 75\,\mu\text{m}$, analog simulations of foam dynamics, and cosmological surveys for curvature effects.

5.8 Exploratory Applications: Black Holes and the Big Bang

The coherence radius $r \sim K^{-1/4}$, where $K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Kretschmann scalar, suggests applications in high-curvature regimes like black holes and the early universe. Nonlinear Einstein field equations require careful backreaction analysis, so claims remain tentative, but these regimes may reveal new quantum-gravity principles.

5.8.1 Black Hole Interiors

For a Schwarzschild black hole ($r_s = 2GM/c^2$), $K = 48r_s^2/r^6$. The model suggests reverse compactification: - Far outside ($r \gg r_s$): Low curvature, large r, small ρ, consistent with cosmological dark energy. - Near horizon ($r \approx r_s$): Increased curvature, smaller r, higher ρ. - Inside ($r < r_s$): High curvature, $r \to 0$, but quantum effects cap at $r_{\min} \approx 1.1\,\ell_P$ ($\approx 1.8 \times 10^{-35}\,\text{m}$), yielding a finite ρ. These trends indicate possible singularity resolution, but numerical GR simulations are needed to confirm backreaction effects.

5.8.2 Big Bang and Cosmic Expansion

In the FLRW metric, $K \sim H^4$ for the early universe: - Singularity ($t \to 0$): $K \to \infty$, $r \to 0$, but $r_{\min}$ caps ρ to a finite value, suggesting a bounce. - Inflation ($t \sim 1 \times 10^{-36}\,\text{s}$): Constant H, K constant, $r \sim 1/H$, ρ constant, driving expansion. - Radiation/matter era: $K \sim 1/t^4$, $r \sim t$, $\rho \sim 1/t^4$, diluting with expansion. - Late dark energy: r stabilizes, ρ constant. Derivation: For the radiation era, $a \sim t^{1/2}$, $H = 1/(2t)$, $K \approx 3/(32t^4)$, and $r \sim t$. This avoids a singularity, supports inherent inflation, and allows evolving dark energy if r grows slowly.

5.9 CERN Experiment Proposal

To test the foam, we propose using CERN's LHC with heavy-ion collisions (Pb-Pb at 5.02 TeV), creating quark-gluon plasma (QGP) at $\sim 1 \times 10^{-15}\,\text{m}$, tapping intermediate foam layers. We predict $\sim 1-5\%$ entropy density anomalies (s/T^3 deviates from constancy) due to foam granularity, manifesting as non-thermal photon or pion yields and multiplicity fluctuations. Analysis with ALICE data is low-cost, requiring no new hardware.

6 Casimir Test of Causal Coherence

6.1 Prediction (Foundational Model)

In the causal-coherence framework, vacuum modes with plate-normal wavelength $\lambda_z = 2d$ longer than the coherence length L are suppressed. With $L \simeq 75\,\mu$m, the onset of a deviation from the standard Casimir force occurs when the fundamental mode crosses the macro layer, i.e.,

$$\lambda_z = 2d \gtrsim L \quad \Rightarrow \quad d \gtrsim L/2 \approx 38\,\mu\text{m}.$$

Let $P_0(d) = -\pi^2\hbar c/(240d^4)$ denote the ideal $T = 0$, perfect-conductor pressure. The model predicts a fractional suppression

$$\frac{\Delta P}{P_0}(d) \equiv 1 - \frac{P(d)}{P_0(d)}$$

that rises rapidly near $d \simeq L/2$ and then grows with d as increasingly long-wavelength modes are filtered out. In the simplest (sharp) implementation, $\Delta P/P_0 \to 1$ for $d \gg L$. A more physical smooth filter yields the same onset location with a gentler rise; a one-parameter roll-off can be used to fit data without overfitting. Decisive window: $d \approx 30$–$80\,\mu$m. Below $\sim 30\,\mu$m the effect is small; above $\sim 80\,\mu$m the absolute force becomes very weak and backgrounds dominate.

6.2 Optional "Foam Layers" (Exploratory)

If the vacuum foam is information-granular (integer nat layers), the same suppression acquires a small residual floor $\alpha \in [0, 1)$ from deeper layers and may show sub-percent kinks when $2d$ crosses submultiples of L:

$$d_m \simeq \frac{L}{2m} \quad (m = 1, 2, \ldots) \quad \Rightarrow \quad 38, 19, 12.6, 9.4\,\mu\text{m}, \ldots$$

We treat these micro-features as exploratory targets (likely $\lesssim 1\%$) because thermal/material broadening and systematics can mimic ripples. The robust, falsifiable signature is the macro-layer onset at $d \simeq L/2$ and the large-d saturation set by α (if $\alpha > 0$).

6.3 Experimental Strategy and Current Limitations

State-of-the-art Casimir experiments achieve percent-level precision out to the micron range using AFM/torsional oscillators in sphere–plane geometry and careful Lifshitz modeling, but the tens-of-microns regime is sparsely covered. Measurements at 0.7–7 µm have observed the thermal Casimir contribution and highlighted the Drude–plasma prescription issue; beyond $\sim 10\,\mu$m, electrostatic patch potentials and alignment/bow limit sensitivity. Differential/isoelectronic techniques with conductive overlayers have been proposed and demonstrated to suppress patch forces and enhance thermal-range sensitivity—an approach likely needed here.

Practical recipe for the 30–80 µm window

- Geometry: sphere–plane (large-R sphere to boost signal), ultra-flat Au (or Si/Au differential) surfaces.
- Mitigations: isoelectronic overlayer to cancel patches; Kelvin-probe mapping of residual patches; UHV, temperature stabilization; in-situ UV/Ar-ion cleaning.
- Readout: quasi-dynamic lock-in around a mean gap (kHz–MHz nanometer modulation) to measure $\partial P/\partial d$, reducing noise.

Challenges include scaling gap sizes, managing thermal noise, and achieving sub-pN resolution. Current MEMS-based setups could be adapted with custom spacers or piezoelectric actuators.

6.4 Expected Outcomes and Significance

A confirmed suppression at $d \approx 38\,\mu$m would validate the causal coherence model, offering a new vacuum energy cutoff. Failure to detect it would constrain L or suggest alternative mechanisms, refining the theory. This test bridges quantum and cosmological scales, with implications for quantum gravity and dark energy dynamics.

6.5 Summary of Predictions

The primary prediction is a measurable Casimir force reduction starting at $d \simeq 38\,\mu$m, with potential sub-percent oscillations from foam layers, testable with existing technology upgrades.

7 Conclusion

7.1 Summary of Contributions

This work proposes a novel solution to the cosmological constant problem by introducing a causal coherence cutoff, yielding

$$\rho = \frac{\hbar c \pi^2}{16 r^4},$$

which reconciles quantum field theory predictions with observed dark energy density at $r \approx 75\,\mu$m. The entropic horizon interpretation, bounding r from $\sim 1.1\,\ell_P$ to $75\,\mu$m, resolves singularities and supports a dynamic vacuum consistent with DESI data. The multilayered cosmic foam model, rooted in the geometric mean of Planck and cosmic scales, visualizes this evolution, linking quantum mechanics and general relativity through causal flux limitation.

7.2 Implications for Physics

This framework unifies quantum and gravitational scales, suggesting spacetime emerges from coherence constraints. It offers a geometric quantization basis, with testable predictions in Casimir experiments, black hole interiors, and cosmic expansion dynamics. The same cutoff principle that yields dark-energy density at mesoscopic scales also recovers QED phenomena like the Lamb shift at atomic scales, suggesting that vacuum fluctuations across vastly different regimes share a common causal structure.

7.3 Future Directions

Future work should explore dynamical r from curvature scalars, refine Casimir tests at mesoscopic scales, and simulate foam evolution with GR backreaction. Interdisciplinary efforts with particle physics (e.g., CERN) and cosmology could further validate this approach.

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