Comparative Study of ARIMA and ANN Models in Polar Motion Forecasting

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Abstract

Precise polar motion (PM) forecast is directly related to satellite navigation and climate studies. Traditional methodologies and machine learning (ML) have been poplar in modern climate modeling. This study addressed both approaches in comparison, investigating Autoregressive Integrated Moving Average (ARIMA) and Artificial Neural Network's (ANN) performance in seasonal geophysical signal forecast. We collected polar motion data from the International Earth Rotation and Reference Systems Service (IERS). We applied a Kolmogorov-Zurbenko (KZ) filter to isolate low-frequency trends. An ARIMA and an ANN were each implemented on training data for a 6-month signal prediction. Residual and statistical analysis showed that the ARIMA achieved superior accuracy than the ANN. The ANN's recursive architecture has shown to over-interpret noise as signals, causing overfitting patterns and phase lags in forecasting. These results demonstrated that traditional methodologies can outperform neural networks in noncomplex, seasonal forecasts. This study can provide critical insights to space agencies and climate researchers, presenting comparative analysis on two popular methodologies and their corresponding performances in polar motion forecasting.

Introduction

Time series forecasting involves inputting historical information as the training data, which would then be fed into a suitable model that presents its prediction of the data's future values. For a better prediction, the data should usually have apparent patterns, either as a broad trend throughout the training data or with repetitive cycles of data points over similar time ranges. Noises are misleading for the model. They are random data points and outliers and can perturb a model's learning of patterns that are necessary for a realistic prediction, and therefore, these noises are often excluded or adjusted before the training starts. Forecasting can be done by a statistical model, a machine learning model, or a hybrid of both, and their performance can vary significantly according to the data that they are given.

In recent years, machine learning or hybrid approaches have replaced scenarios that would usually be modeled by statistical methods, often involving complex, random datasets.

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For instance, deep learning models have shown apparent signs of outperforming statistical models in forecasting competitions that involve training on complex datasets, like M4 or M5 datasets [1,2]. Statistical datasets also show more limitations in fields like energy and healthcare, which typically contain more random data values that are better extracted using deep learning and hybrid models [3].

Special characteristics inherent to environmental data may have an impact on the effectiveness of these modeling techniques. In geophysical time series, LRD is frequently demonstrated, showing high correlation among values that are far away in time as opposed to short-term memory [4,5,6]. The historical observational data in climate records often contain inhomogeneities due to discontinuous data values caused by non-climatic or artificial reasons, thus creating additional noises that make the trend less visible [7,8,9,10,11].

The unique characteristics displayed by environmental variables, exhibition of long-term trend and large noise presence, led researchers to investigate whether the new methods of deep learning or hybrid modeling can model these special datasets with better forecasting accuracy. A study examining ecological footprint forecasting utilized the classical ARIMA model, the LSTM deep learning model, and the hybrid ARIMA-SVR model, and it showed that the ARIMA model achieved greater accuracy than the other two models [12]. This is likely because environmental variables often exhibit trends or seasonal cycles that classical techniques are already suitable for, and the inherent noise within the environmental data is likely to distract deep learning and hybrid models from identifying the underlying trend or cycles. Our research provides contribution to this field by investigating comparatively between the classical and neural network models on polar motion x-component data, while applying noise-reduction techniques to observe whether the reduced noise can still be a misguiding factor in the deep learning or hybrid modeling.

Method

The study applies a KZ filter that excludes noise from the polar motion x-component data. This can help models to focus on patterns that are displayed in the historical polar motion trend, creating a forecast that resembles past data behaviors. The training data is then passed into two models: an ARIMA statistical model and an ANN deep learning model. Below are brief descriptions of KZ filtering, the ARIMA model, and the ANN model.

KZ Filtering

The KZ filter is a type of moving average that gets applied in multiple repetitions with a specific window size. Applying the filtering moving average multiple times makes the noise within the data get removed one iteration by the next. High-frequency values get filtered progressively to ensure that the important signals get exposed and better learned by the models, thus making long-term trends more observable. Equation 1 gives the general formula for a KZ-filtered time series ($x_{t,m,k}$) output. In Equation 1, the x_t parameter is the time series data and the m parameter represents the size of the window. The third parameter, k, tells the number of times that the time series is processed by the moving average.

$$y_l^{(k)} = \frac{1}{m} \sum_{i=-(m-1)/2}^{(m-1)/2} y_{l+i}^{(k-1)} \tag{1}$$

where $y_t^{(k)}$ represents the final time series data after being processed for k times.

ARIMA Model

The ARIMA model is a widely used statistical approach when dealing with time series input. In this model, past values are used to predict future values, and the prediction is improved with the random errors that occurred in the past forecast. Equation 2 gives the general formula of an ARIMA (p, d, q) model. In Equation 2, the p parameter represents the AR function while the q parameter represents the MA function. The d parameter tells the degree of differencing applied to the time series.

$$(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p)(1 - B)^d y_t = c + (1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q)\epsilon_t \tag{2}$$

where y_t and ε_t represent the data value and error, respectively, at time t. Coefficient ϕ_i $(i = 1, 2, 3, \ldots, p)$ is the impact on future that the past values have and θ_j $(j = 1, 2, 3, \ldots, q)$ is the impact on future that past errors have. The term c is a constant while the term $(1 - B)^d$ makes data stationary by differencing.

ANN Model

The ANN model acts similarly to the network of a biological brain, composed of neurons that make up the layers of the architecture. An ANN contains an input layer, a hidden layer, and an output layer. Because the ANN is well at handling nonlinearity and complexity in the data, it has begun to replace the statistical models in several fields in the real world. Equation 3 gives the general output formula for a single neuron.

$$\text{Output } f\left(\sum_{i=1}^{n} w_i \cdot x_i + b\right) \tag{3}$$

where x_i represents the inputs and w_i represents the corresponding weights. The function f is the activation function. The term b is the bias in the function.

Residual Statistics

The residual statistics show whether the model's prediction is desirable. In this study, two residual statistics, S (Skewness) and K (Kurtosis), are used to understand the performance of ARIMA and ANN models. Skewness measures how symmetric the residuals are and the Kurtosis shows the degree to which the outliers are present. Equations 4 and 5 give the general formulas for each residual statistic.

Skewness (S)

$$S = \frac{\frac{1}{N}\sum_{i=1}^{N}(A_i - P_i - M)^3}{\left(\sqrt{\frac{1}{N}\sum_{i=1}^{N}(A_i - P_i - M)^2}\right)^3} \tag{4}$$

Kurtosis (K)

$$K = \frac{\frac{1}{N}\sum_{i=1}^{N}(A_i - P_i - M)^4}{\left(\frac{1}{N}\sum_{i=1}^{N}(A_i - P_i - M)^2\right)^2} \tag{5}$$

where A_i represents the actual value and P_i represents the predicted value. The terms M and N are the mean and sample size of the data, respectively.

Step 1. Data Filtering

The modeling process begins with applying the KZ filter to the raw time series. The original time series reveals lagged patterns with data points jumping irregularly. It gets processed by a moving average of 30 days three times, eventually achieving waves that are smoother. Equation 6 gives the basic formula for the filtered time series.

$$Y_3(t) = M_{30}^{(3)} X(t) \tag{6}$$

where $Y_3(t)$ and $X(t)$ represent the filtered and raw time series data, respectively. The function M_{30} represents the 30-day moving average filter.

Step 2. ARIMA Structure

Next, the filtered time series is put into the ARIMA model. The polar motion x-component time series contains characteristics in recurring cycles of seasonality, and to address that in the modeling process, two Fourier terms are included to mimic the recurring cycles in sine and cosine waves. The AR terms and MA terms are included to learn past errors and generate better predictions. The basic formula of a value predicted by the ARIMA model is given in Equation 7.

$$PM\_X_{t+1} = \text{AR Terms} + \text{MA Terms} + \text{Fourier Terms} \tag{7}$$

where PM_X_t represents a predicted value in the polar motion x-component time series.

Step 3. ANN Architecture

Similar to training an ARIMA model, the ANN also includes 2 Fourier terms to address the seasonal cycles. In the hidden layers, there are eight hidden neurons, and the output is linear to resemble the time series input. The forecast occurs recursively. The basic ANN architecture used in this study is given in Figure 1.

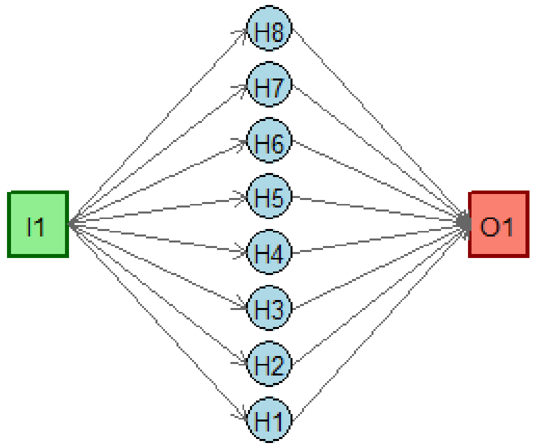

Figure 1: The ANN achitecture with 1 input layer (I1), 8 hidden neurons (H1, H2, H3,..., H8), and 1 output layer (O1).

Step 4. Prediction

The ARIMA and ANN models trained in the previous steps are then used for forecasting. After finalizing the models, their performance is evaluated by the residual statistics.

Regarding the libraries and packages used, the `forecast` package is used to train the ARIMA model, while the ANN model is trained by the `nnet` library. Code development and model training are done on RStudio 2024.12.1+563 "Kousa Dogwood" Release and with R version 4.4.3.

Results

The polar motion x-component dataset is selected from the International Earth Rotation and Reference Systems Service (IERS). For one-to-one comparison between the two models, the residual analysis and statistics are used. Section 2.4 contains a brief description of the residual statistics used in this study.

The way in which Earth's axis would change has been a great wonder for physicists and astronomers for a long time. The International Latitude Service (ILS) and its six observatories have been observing and keeping records of polar motion in the United States, Japan, Italy, and Turkmenistan since the beginning of the twentieth century, and it had led scientists and researchers to later discover the fourteen-month Chander wobble and the annual wobble [13]. Due to polar motion's influence on a range of fields, including geodesy, climate, and space navigation. Yet studying the underlying patterns and relationships of polar motion is difficult because the methods for recording polar motion often introduce considerable biased and errors during measurements [14,15], and the data has a high degree of complexity, particularly the x-component of polar motion that includes multiple trends and cycles that occur simultaneously, thus making it especially complicated to differentiate among the singals [16,17].

The polar motion x-component time series used in this paper expands over 4 yearly observations, from 2020 to 2024. A 80 to 20 train-test split is done on the dataset. Similar to the previous studies done on polar motion modeling. we utilized similar extraction and preprocessing techniques to prepare our time series [18,19,20,21].

The figures and statistics suggest that the ARIMA model performs better than the ANN model.

Figure 2 shows that ARIMA closely follows the historical data's seasonal turning points, while ANN exhibits 1 to 2 month delay in capturing these reversals. ARIMA maintains a signal range similar to the historical data (125–225 mas), while ANN gives more extreme swings (85–225 mas range compared to historical data's 110–210 mas range). ARIMA predicts a moderate downturn consistent with the KZ-filtered trend, while ANN projects an unrealistic sharp rebound.

Figre 3 shows that ARIMA's Temporal Residual scatters randomly around zero, while ANN's contains clusters of high and low values that are not random nor ideal. ARIMA's Residual Distribution shows a symmetric bell curve that matches closely with a normal curve, while ANN's is right-skewed with fat tails and contains too many extreme values. ARIMA's Residuals vs Fitted Values is evenly spread at all levels, while ANN's has spread that's wider at extremes.

Table 1 shows that ARIMA's skewness is ideally near zero, while ANN's skewness suggests greater positive errors and biased predictions. ARIMA's Kurtosis is close to three, resembling ideally to a normal distribution, while ANN's Kurtosis is much higher than three, indicating more outliers and worse predictions.

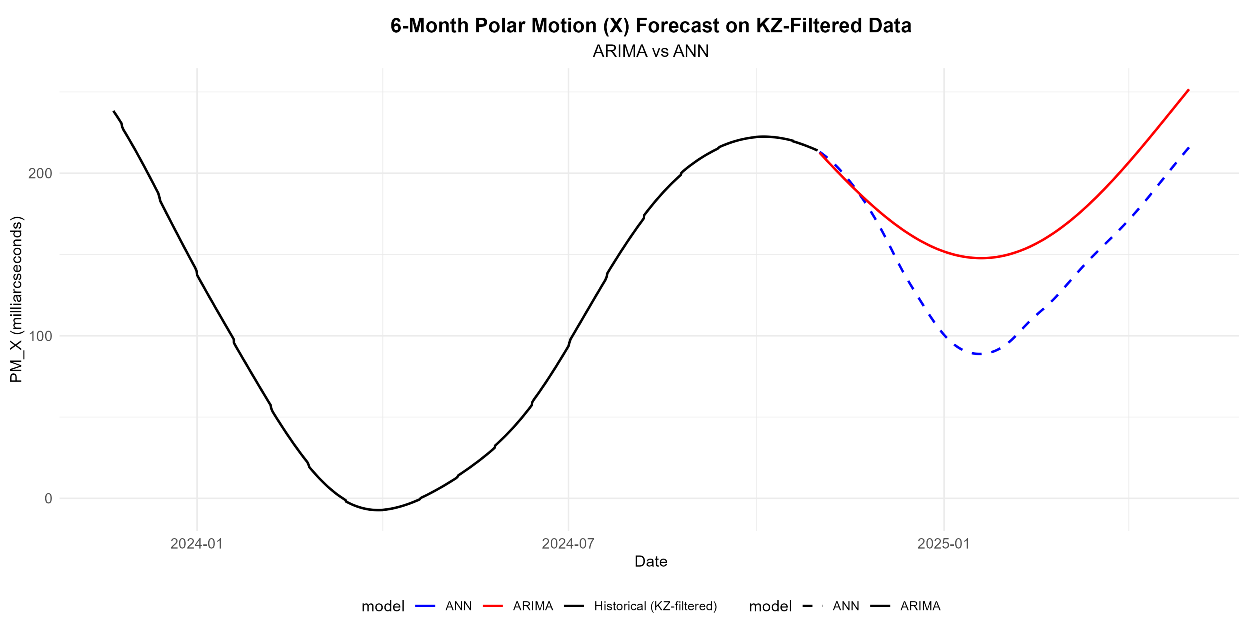

Figure 2: 6-month PM_X forecast comparison between ARIMA and ANN.

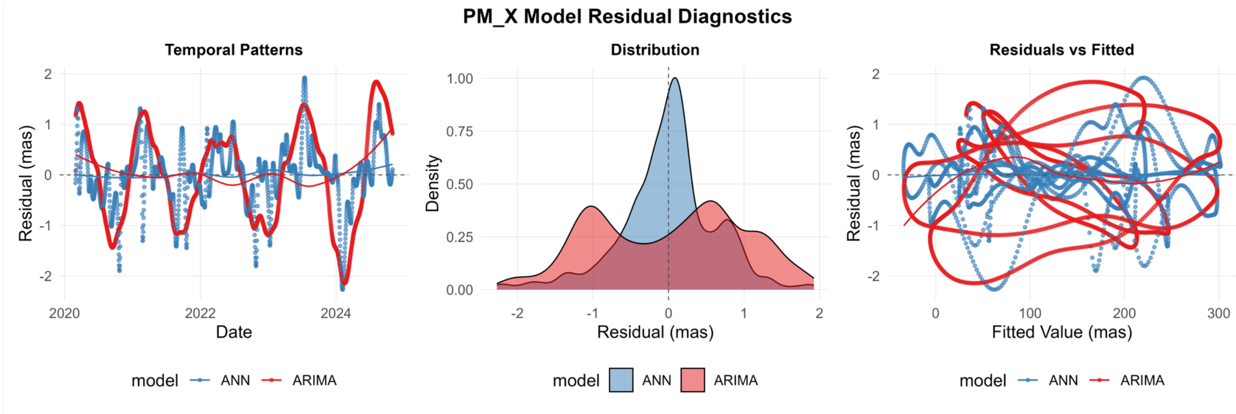

Figure 3: Comparison of ARIMA and ANN residuals.

Model	Skewness (S)	Kurtosis (K)
ARIMA	0.15	2.9
ANN	0.82	4.1

Table 1: Residual Statistics.

Conclusion

Previous studies have shown that classical statistical models perform better than deep learning and hybrid models on forecasting with environmental variables. Our study has shown similar findings using the classical ARIMA model and the deep learning ANN model, specifically with polar motion x-component data. The results show that the ARIMA performs more superior than the ANN model on: greater similarity to the observational data in the six-month forecast and more ideal residual plots and statistics. These findings provide additional insights into the classical vs. modern method comparison for researchers studying the space and climate. We have utilized a KZ filter as the primary and only way to address the inherent issue of significant noise and error within environmental datasets. Many environmental researchers have studied and proposed different ways to reduce noise within data and explore classical and modern models in comparison. Our study provides contributions to this line of research and has discovered results similar to the previous studies.

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