Concise golden formula for the fine-structure constant (α) — math-only excerpt

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$$\alpha = \left(\left[(\sqrt{N})^N + (N+1)^N \right]^{\varphi} e^{N+1} \right)^{-1/\pi}, \quad N = 4$$

Fig. 1. Concise formula for the fine-structure constant.

Definition (variant for N = 4): Let φ = (1 + √5)/2. Define S(N) = (√N)^N + (N + 1)^N. The proposed relation is $\alpha(N) = ([S(N)]^{\varphi} \cdot e^{N+1})^{-1/\pi}$. In what follows we set N = 4.

Symbols: α — fine-structure constant (dimensionless); φ — golden ratio; e — base of the natural logarithm; π — Archimedes' constant; N — positive integer.

Numerical evaluation (N = 4): S(4) = (√4)^4 + 5^4 = 16 + 625 = 641. Hence $\alpha = (641^{\varphi} \cdot e^{5})^{-1/\pi}$. Numerically: α ≈ 0.007297352568, so $\alpha^{-1} \approx 137.035999113$.

Comparison with CODATA (2018): $\alpha^{-1}_{CODATA2018} = 137.035999084$. Relative differences: Δα/α ≈ −0.21 ppb; $\Delta(\alpha^{-1}) / \alpha^{-1} \approx +0.21$ ppb.

Mathematical properties: (i) α(N) ∈ (0, 1) for N ≥ 1; (ii) due to the negative exponent, α(N) decreases with increasing argument inside the parentheses; (iii) sensitivity to N is driven by the growth of S(N) and of e^{N+1}; (iv) for N = 4 the term (N + 1)^N = 5^4 dominates S(N); (v) the function is smooth and differentiable in (φ, π, e); (vi) the expression uses only mathematical constants, hence it is unitless.

Editorial note: This excerpt intentionally contains mathematics only (definition, numeric evaluation, formal properties) with no speculative physical discussion.

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