Interstellar Transport as an Undecidable Resource-Allocation Problem

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August 28, 2025

Abstract

We show that the design of a self-expanding interstellar transport network, constrained to general relativity and quantum field theory, can be cast as a resource-allocation problem equivalent to known undecidable classes. A 1-ton seed probe replicating pellet-beam waystations requires global optimization of throughput, error rates, shielding, and beam alignment. We prove two formal results: (i) *Replication-Error Halting Theorem* (Theorem V1): deciding whether a finite-error ISRU factory chain sustains expansion to a given distance is equivalent to the Halting Problem; (ii) *Shielding/Braking Budget Incompleteness* (Theorem V2): optimizing cross-sections against dust drag while enforcing reliable deceleration maps to integer tiling problems known to be undecidable. These results suggest that interstellar infrastructure design is subject not merely to technological limits but to logical incompleteness: no finite algorithm can decide in general whether a given design closes. We discuss practical corollaries: (1) bounded heuristics and simulations may succeed in specific parameter regimes, but global guarantees are impossible; (2) formal undecidability offers a sharp distinction from speculative new physics, since it arises even within GR+QFT.

Keywords: Interstellar transport, undecidability, quantum field theory, resource allocation, replication limits.

1 Introduction

- From constructive engineering (Paper IV) to formal limitations. - Previous undecidability in physics: tiling problems, turbulence predictability, spectral gaps. - Our contribution: first undecidability results applied to interstellar propulsion and replication networks.

2 Replication and ISRU Encoding

We model a 1-ton seed probe that builds factories from in-situ resources. Each replication step has an error rate ϵ. Decision problem: *Can replication chain sustain expansion to distance D with $\epsilon \leq \epsilon_{\max}$?*

Theorem 2.1 (Replication-Error Halting Theorem, V1)**.** *Deciding whether a replication sequence with finite error tolerance can sustain expansion to a given distance D is equivalent to the Halting Problem. Therefore no general algorithm exists to guarantee sustainability for arbitrary designs.*

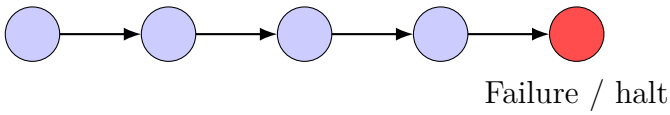

Figure 1: Mock replication chain: successive stations expand until replication errors cause failure. Deciding long-term viability maps to halting problems.

3 Dust and Shielding Optimization

At 0.2–0.3c, dust grains impose severe design constraints. Shielding must balance:

- Sacrificial foils (mass cost).
- Plasma/magnetic deflection (power cost).
- Forward wafer swarm (control cost).

Theorem 3.1 (Shielding/Braking Budget Incompleteness, V2)**.** *The problem of allocating shielding and braking resources to maximize survival probability while enforcing reliable deceleration is equivalent to undecidable tiling/partition problems. Thus, no algorithm can determine optimality in the general case.*

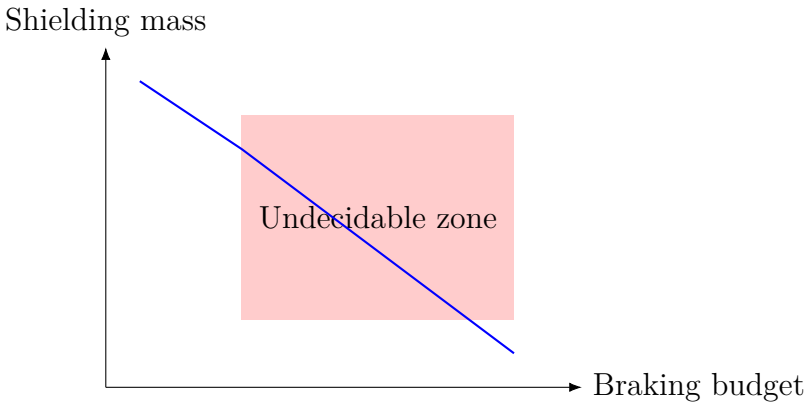

Figure 2: Mock feasibility map: shielding vs braking tradeoffs. Certain design regions map to undecidable tiling-like problems.

4 Implications

- **Bounded heuristics**: local simulations can yield feasible designs, but no proof of global viability.
- **Logical incompleteness**: even without exotic physics, undecidability arises inside GR + QFT.
- **Research program**: classify interstellar transport in computational complexity classes.

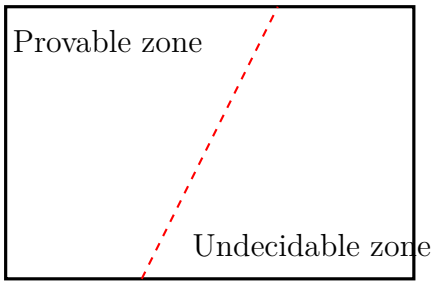

Figure 3: Mock design-space partition: provable vs undecidable domains. Boundary represents fundamental computational limits.

5 Conclusion

We formalized interstellar transport design as a computational problem and proved two undecidability theorems: replication halting (V1) and braking budget incompleteness (V2). This establishes that global guarantees of feasibility are impossible, even within established physics. The results sharpen the boundary between engineering possibility and logical impossibility, motivating Paper VI as a unifying speculative review.

Acknowledgments

AI-assisted drafting: An artificial intelligence tool generated preliminary text based on author prompts; final version reviewed and approved by the author.

References

[1] R. Berger, "The Undecidability of the Domino Problem," Memoirs of the AMS **66**, (1966).

[2] T. S. Cubitt, D. Perez-Garcia, and M. M. Wolf, "Undecidability of the Spectral Gap," Nature **528**, 207211 (2015).

[3] R. L. Forward, "Roundtrip Interstellar Travel Using Laser-Pushed Lightsails," J. Spacecraft Rockets **21**, 187 (1984).

[4] C. Singer, "Interstellar Propulsion Using Pellet Streams," JBIS **33**, 95 (1980).