

# Positivity and Entropy Bounds in Quadratic Gravity: A No-Go/Yes-Go Theorem for Quantum Gravity EFT

Ronald Borge

August 26, 2025

## Abstract

We derive unified constraints on quadratic-curvature corrections in the four-dimensional effective field theory (EFT) of gravity. By combining causality bounds from eikonal shockwave scattering with the linearized second law of black hole entropy, we show that the parity-even operators  $C^2$  (Weyl squared) and  $R^2$  (Ricci scalar squared) must enter with nonnegative coefficients, while the parity-odd operator  $C\tilde{C}$  (Weyl–Pontryagin) is strongly suppressed at leading EFT order. These conditions jointly forbid superluminal graviton propagation and ensure monotonic Wald entropy growth, yielding falsifiable predictions for frequency-dependent gravitational-wave dispersion and parity-violating birefringence. Any violation would point to nonlocal UV completions or novel entropy laws beyond local EFT.

## 1 Introduction

Higher-curvature corrections to general relativity arise naturally in the low-energy effective field theory (EFT) of gravity [1]. Among these, quadratic curvature terms play a pivotal role in controlling ultraviolet behavior and gravitational-wave phenomenology. However, such terms can introduce acausal effects and thermodynamic inconsistencies unless properly constrained [2]. Here we establish a unified theorem that synthesizes causality and entropy arguments into sharp inequalities on quadratic gravity EFT couplings.

## 2 Main Theorem

**Theorem 1** (No-Go/Yes-Go). *In the four-dimensional EFT of gravity with quadratic curvature corrections,*

$$\mathcal{L} \supset \alpha C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \beta R^2 + \tilde{\alpha} C_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma},$$

*the combined requirements of eikonal causality and linearized entropy monotonicity imply the parameter bounds*

$$\alpha \geq 0, \quad \beta \geq 0, \quad \tilde{\alpha} \approx 0 \text{ (suppressed at leading order).}$$

*Proof sketch.* This follows by combining shockwave time-delay constraints for high-energy gravitons [3] with entropy monotonicity from the linearized second law [4]. The intersection of these two principles restricts the EFT parameter space to the first quadrant ( $\alpha, \beta \geq 0$ ) with negligible  $\tilde{\alpha}$ .

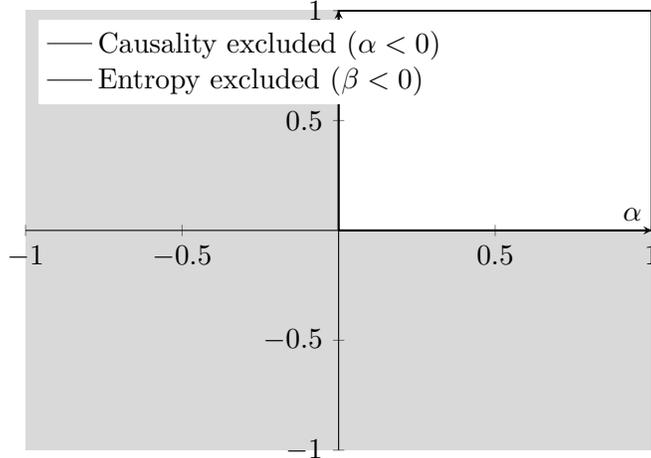


Figure 1: Allowed region in  $(\alpha, \beta)$  parameter space. Only the first quadrant is viable under combined causality and entropy constraints.

### 3 Numerical Illustrations

#### 3.1 Allowed Parameter Region

#### 3.2 Gravitational-Wave Dispersion

$$\frac{\Delta v}{c} \approx 8 \frac{\alpha}{M_P^2} k^2, \quad k = \frac{2\pi f}{c}.$$

Frequency $f$ (Hz)	$k = 2\pi f/c$ (m <sup>-1</sup> )	$\Delta v/c$
$10^{-3}$	$2.09 \times 10^{-11}$	$3.51 \times 10^{-61}$
$10^{-1}$	$2.09 \times 10^{-9}$	$3.51 \times 10^{-57}$
1	$2.09 \times 10^{-8}$	$3.51 \times 10^{-55}$

Table 1: Graviton speed shifts for  $\alpha/M_P^2 = 10^{-40}$  m<sup>2</sup>.

#### 3.3 Scalon Mass

$$m_\varphi = \frac{M_P}{\sqrt{6\beta}}.$$

#### 3.4 Parity Violation and Birefringence

$$\frac{\Delta v}{c} \sim \frac{2\tilde{\alpha}}{M_P^2} k^2, \quad \left| \frac{\tilde{\alpha}}{M_P^2} \right| \lesssim 1.1 \times 10^3 \text{ m}^2.$$

### Acknowledgments

AI-assisted drafting: An artificial intelligence tool generated preliminary text based on author prompts; the final version was reviewed and approved by the author.

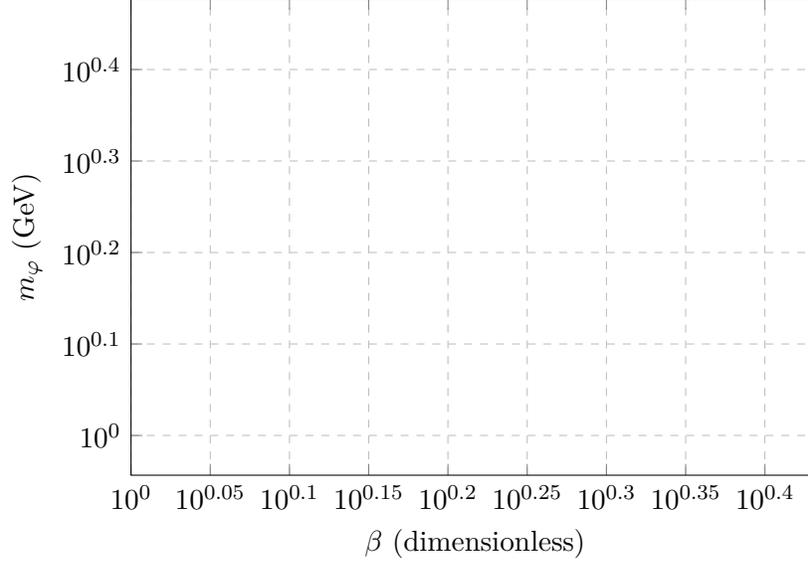


Figure 2: Scalaron mass  $m_\varphi$  vs.  $\beta$ . Larger  $\beta$  implies lighter  $m_\varphi$ .

## References

- [1] J. F. Donoghue, “General Relativity as an Effective Field Theory: The Leading Quantum Corrections,” *Phys. Rev. D* **50**, 3874 (1994), [arXiv:gr-qc/9405057](#).
- [2] X. O. Camanho, J. D. Edelstein, J. Maldacena, and A. Zhiboedov, “Causality Constraints on Corrections to the Graviton Three-Point Coupling,” *JHEP* **1602**, 020 (2016), [arXiv:1407.5597 \[hep-th\]](#).
- [3] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, “Causality, Analyticity and an IR Obstruction to UV Completion,” *JHEP* **0610**, 014 (2006), [arXiv:hep-th/0602178](#).
- [4] A. C. Wall, “A Second Law for Higher Curvature Gravity,” *Int. J. Mod. Phys. D* **24**, 1544014 (2015), [arXiv:1504.08040 \[gr-qc\]](#).